ORNL-CDC-3, C. I
UC-46 — Criticality Studies

ESTIMATES OF MAXIMUM SUBCRITICAL DIMENSIONS OF SINGLE FISSILE METAL UNITS

W. H. Roach and D. R. Smith

Los Alamos Scientific Laboratory

Printed in the United States of America. Available from Clearinghouse for Federal
Scientific and Technical Information, National Bureau of Standards,
U.S. Department of Commerce, Springfield, Virginia 22151
Price: Printed Copy $3.00; Microfiche $0.65

ORNL-CDC-3
UC-46 Criticality Studies

ESTIMATES OF MAXIMUM SUBCRITICAL DIMENSIONS OF SINGLE FISSILE METAL UNITS

W. H. Roach and D. R. Smith

Los Alamos Scientific Laboratory
Los Alamos, New Mexico 87544

OCTOBER 1967

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee
operated by
UNION CARBIDE CORPORATION
for the
U. S. ATOMIC ENERGY COMMISSION

Contract No. W-7405-eng-26

ESTIMATES OF MAXIMUM SUBCRITICAL DIMENSIONS OF SINGLE FISSILE METAL UNITS

W. H. Roach and D. R. Smith

ABSTRACT

The maximum subcritical dimensions of water-reflected metal spheres, cylinders, and slabs of ^{235}U, ^{233}U, and ^{239}Pu, which will be of value in the specification of process, storage, and transport conditions for fissile materials, were determined using the DTF transport code with the Hansen-Roach 16-group cross sections. The effective neutron multiplication factor of single units which had been shown experimentally to be critical was first computed to evaluate the bias inherent in the calculations. These dimensions, which contain no safety factors other than those demanded by the uncertainty in the calculations, are internally consistent and derive from a common method of calculation and input data. Some individual values may be relaxed when based on well established experimental results.

Tabulations of criticality parameters which have been published (see, for example, Ref. 1) provide only a critical value for a particular parameter of interest. The Nuclear Safety Guide[2] contains a large number of useful safe parameters for a variety of plant processes and equipment, but, for some applications, is overly restrictive in recommended safety factors.

It is the purpose of the present note to attempt to establish subcritical bounds for several of the commonly utilized parameters. This information will be of particular value to those engaged in material processing when used in conjunction with realistic safety factors. Isotopically pure, unmoderated, water-reflected single units of ^{233}U, ^{235}U, and ^{239}Pu metal in simple geometry are considered primarily because the existing experimental data for such simple assemblies provide a base from which calculational extrapolations can be made.

The maximum subcritical dimensions of water-reflected metal spheres, cylinders, and slabs were determined after first calculating the critical values for these systems. All calculations utilized the Los Alamos DTF code,[3] which is an application of the Carlson S_n method with the transport approximation. Both S_8 and S_{16} angular approximations were used, and results were extrapolated to S_∞ to minimize geometrically dependent calculational errors. The variation of the effective neutron multiplication factor (k_{eff}) with metal radius was also obtained. All assemblies were calculated using Hansen-Roach 16-group cross sections[4] and a 20-cm-thick water reflector. Radii of spheres and of cylinders of infinite length and the thickness of slabs infinite in two dimensions were determined for ^{233}U, ^{235}U, and ^{239}Pu based on densities of 18.66 g/cm^3, 18.82 g/cm^3, and 19.70 g/cm^3, respectively. Results are presented in Table 1.

---

1. H. C. Paxton, J. T. Thomas, D. Callihan, and E. B. Johnson, "Critical Dimensions of Systems Containing U^{235}, Pu^{239}, and U^{233}," USAEC Report TID-7028 (1964).
2. Subcommittee 8 of the American Standards Association Sectional Committee N6 and Project 8 of the American Nuclear Society Standards Committee, Nuclear Safety Guide, USAEC Report TID-7016, Rev. 1 (1961).
3. B. G. Carlson et al., "DTF Users Manual," UNC Phys/Math-3321, Vol. I (1963), Vol. II (1964).
4. G. E. Hansen and W. H. Roach, "Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies," LAMS-2543, Los Alamos Scientific Laboratory (1961).

Table 1. Calculated Critical Dimensions of Water-Reflected Metal Units

Critical Parameter (cm)	S_8	S_{16}	Estimated S_∞	$\frac{\Delta k / \Delta r}{r}$
^{233}U sphere radius	4.552	4.576	4.60	0.75
^{233}U cylinder radius	2.524	2.538	2.55	0.59
^{233}U slab thickness	0.6296	0.6288	0.628	0.21
^{235}U sphere radius	6.464	6.496	6.52	0.71
^{235}U cylinder radius	3.777	3.794	3.81	0.56
^{235}U slab thickness	1.494	1.494	1.49	0.22
^{239}Pu sphere radius	4.024	4.047	4.06	0.77
^{239}Pu cylinder radius	2.265	2.280	2.29	0.67
^{239}Pu slab thickness	0.723	0.724	0.725	0.29

The bias inherent in the calculations was evaluated by computing the values of k_{eff} of spheres shown experimentally to be critical. This bias is reflected in the degree of subcriticality of the values of the parameters listed in Table 2 in the following manner.

Table 2. Subcritical Parameters for Single Units of Water-Reflected Metal

Parameters	^{233}U	^{235}U	^{239}Pu
k_{eff} (S_∞)	0.97	0.98	0.97
Mass (kg)	6.75	20.1	4.9
Sphere radius (cm)	4.4	6.3	3.9
Cylinder radius (cm)	2.3	3.7	2.2
Slab thickness (cm)	0.54	1.35	0.65

The water-reflected spherical critical mass of uranium enriched to 93.5% in ^{235}U at a uranium density of 18.8 g/cm^3 has been reported[5] as 22.8 kg of ^{235}U. The value of k_{eff} from an S_8 calculation of this unit

5. G. A. Graves and H. C. Paxton, "Critical Masses of Oralloy Assemblies," Nucleonics 15, No. 6, 90 (June 1957).

was 1.006. The ^{235}U sphere radius from S_8 calculations, in Table 1, is therefore low by a factor of 1.008, which is essentially the margin between S_8 and S_∞. There is, therefore, no significant bias in the S_∞ value of the sphere radius, and the S_∞ values for cylinders and slabs were assumed valid. The subcritical values quoted in Table 2 for ^{235}U correspond to k_{eff} equal to 0.98 from S_∞ calculations and represent a margin below criticality of about 2% in k_{eff}.

There are no data available describing critical water-reflected metal spheres of plutonium or of ^{233}U. There are, however, measurements of critical dimensions of unreflected metal spheres and of unreflected solution spheres, so that one may evaluate the applicability of the ^{233}U and ^{239}Pu cross sections for both fast and moderated assemblies.

The reported[6] critical radius of an unreflected metal sphere, composed of 98.14% ^{233}U, 1.24% ^{234}U, 0.02% ^{235}U, and 0.60% ^{238}U at a density of 18.44 g/cm^3, was 5.965 cm. The calculated value of k_{eff} of this sphere, using the S_8 approximation, was 1.006. A 16.0 cm radius unreflected sphere of $^{233}UO_2F_2$ solution was critical[7] at a concentration of 67.1 g of ^{233}U per liter. The corresponding calculated k_{eff} was 0.995. The value of k_{eff} from an S_8 calculation of a critical water-reflected metal sphere would be expected to lie between these two values, so any calculated value less than 0.995 is indicative of a subcritical unit. The 1.04% difference in critical radius calculated by S_8 and by S_∞ corresponds to a difference in k_{eff} of 0.75 x 1.04 = 0.78%, hence an S_∞ value of k_{eff} below 0.987 would indicate subcriticality. A margin of 1.7% in k_{eff} is considered adequate to provide confidence in the subcriticality of these units and therefore values quoted in Table 2 for ^{233}U correspond to a calculated k_{eff} (S_∞) of 0.97.

---

6. G. E. Hansen, "Status of Computational and Experimental Correlations for Los Alamos Fast Neutron Critical Assemblies," Proceedings of the International Atomic Energy Conference on Physics of Fast and Intermediate Reactors, Vienna, August 3-11, 1961, Vol. I, 445, International Atomic Energy Agency, Vienna (1962).
7. J. K. Fox, L. W. Gilley, and E. R. Rohrer, "Critical Mass Studies, Part VIII, Aqueous Solutions of U^{233}," ORNL-2143, Oak Ridge National Laboratory (1959).

The critical unreflected plutonium sphere was reported[8] to have a mass of 16.28 kg of ^{239}Pu at a density of 15.44 g/cm^3; the calculated k_{eff} (S_8) was 0.999. The calculated value of k_{eff} (S_8) of an unreflected spherical critical plutonium solution[9] was 1.015, so the critical water-reflected plutonium sphere should have k_{eff} greater than 0.999 in S_8, or greater than 0.992 in S_∞. The values quoted in Table 2 correspond to k_{eff} (S_∞) equal to 0.97 and provide a margin of at least 2.2% in k_{eff}.

The authors wish to acknowledge the helpful comments and criticism of Dr. Gordon E. Hansen and the invaluable computational assistance of Mrs. Laura Stone of the Los Alamos Theoretical Division.

---

8. G. A. Jarvis, G. A. Linenberger, J. D. Orndoff, and H. C. Paxton, Nucl. Sci. Eng. 8, 525 (1960).
9. F. E. Kruesi, J. O. Erkman, and D. D. Lanning, "Critical Mass Studies of Plutonium Solutions," HW-24514 (Del.), Hanford Atomic Products Operation (1962).

INTERNAL DISTRIBUTION

1. L. S. Abbott
2. R. G. Affel
3. F. T. Binford
4. F. R. Bruce
5- 6. Dixon Callihan
7. R. Gwin
8. E. B. Johnson
9. W. H. Jordan
10. F. Kertesz
11. D. W. Magnuson
12-13. F. C. Maienschein
14. J. H. Marable
15. J. T. Mihalczo
16. J. P. Nichols
17. A. M. Perry
18. S. J. Raffety
19. R. K. Reedy, Jr.
20. J. T. Thomas
21. J. W. Wachter
22. J. W. Webster
23. G. Dessauer (Consultant)
24. B. C. Diven (Consultant)
25. M. L. Goldberger (Consultant)
26. M. H. Kalos (Consultant)
27. L. V. Spencer (Consultant)
28- 30. Central Research Library
31- 32. ORNL Y-12 Technical Library
Document Reference Section
33-342. Laboratory Records
343. Laboratory Records, ORNL R.C.

EXTERNAL DISTRIBUTION

344. F. W. Barclay, Whiteshell Nuclear Research Establishment, Pinawa, Manitoba, Canada
345. G. H. Bidinger, US AEC, Div. of Materials Licensing, Washington, D.C. 20545
346. P. A. Birkhofer, Institut Für Mess-und Regelungstechnik, Technische Hochschule München, München, Germany
347. C. L. Brown, Battelle Memorial Institute, Box 999, Richland, Wash. 99352
348. R. L. Brunnenmeyer, Bechtel Corp., 220 Bush St., San Francisco, Calif. 94119
349. F. R. Charlesworth, Ministry of Power, Thames House, Millbank, London, England
350. J. H. Chalmers, UKAEA Health and Safety Branch, Risley, England
351. R. B. Chitwood, US AEC, Div. of Compliance, Washington, D.C. 20545
352. H. K. Clark, E. I. du Pont de Nemours, Aiken, S. C. 29801
353. E. D. Clayton, Battelle Memorial Institute, Box 999, Richland, Wash. 99352
354. D. F. Cronin, United Nuclear Corp., Box 1883, New Haven, Conn. 06500
355. J. T. Daniels, UKAEA Health and Safety Branch, Risley, England
356. D. M. Dawson, GE APD, San Jose, Calif. 95125
357. F. A. De Troy, Metallurgie Hoboken, Olen (Antwerp) Belgium
358. K. W. Downes, Brookhaven National Laboratory, Upton, N. Y. 11973
359. D. L. Dunaway, National Lead Co. of Ohio, Box 39158, Cincinnati, Ohio 45239
360. T. C. Engelder, Babcock & Wilcox Co., Box 1260, Lynchburg, Va. 24505

361. J. W. Flora, US AEC, Div. of Compliance, Denver, Colo. 80215
362. J. L. Forstner, E. I. du Pont de Nemours, Aiken, S. C. 29801
363. J. K. Fox, Idaho Nuclear Corp., Idaho Falls, Idaho 83401
364. T. A. Fox, NASA-Lewis Research Center, 21000 Brookpark Road, Cleveland, Ohio 44100
365. P. E. Hamel, Atomic Energy Control Board, Ottawa, Canada
366. G. E. Hansen, LASL, Box 1663, Los Alamos, N. M. 87544
367. H. F. Henry, Depauw University, Greencastle, Ind. 46135
368. D. L. Hetrick, University of Arizona, Tucson, Ariz. 85716
369. William Householder, Nuclear Fuel Services, Erwin, Tenn. 37650
370. Information Research Center, Battelle Memorial Institute, 505 King Ave., Columbus, Ohio 43201
371. W. A. Johnson, US AEC, Oak Ridge Operations Office, Oak Ridge, Tenn. 37830
372. Norman Ketzlach, Atomics International, Box 309, Canoga Park, Calif. 91304
373. G. R. Kiel, Isochem, Inc., Richland, Wash. 99352
374. Ryokei Kiyose, University of Tokyo, Bunkyoky, Tokyo, Japan
375. O. C. Kolar, Lawrence Radiation Laboratory, Livermore, Calif. 94551
376. R. C. Lane, UKAEA, Atomic Weapons Research Establishment, Aldermaston, Berkshire, England
377. P. R. Lecorche, CEN/SACLAY, Commissariat a l'Energie Atomique, BP n°2 - Gif sur Yvette, Seine et Oise, France
378. W. B. Lewis, Phillips Petroleum Co., Idaho Falls, Idaho 83401
379. Wesley Lewis, Nuclear Fuel Services, Inc., Box 124, West Valley, N. Y. 14171
380. C. D. Luke, US AEC, Div. of Materials Licensing, Washington, D.C. 20545
381. A. J. Mallett, Oak Ridge Gaseous Diffusion Plant, Oak Ridge, Tenn. 37830
382. J. D. McGaugh, APD Westinghouse Power Plant Div., Box 1585, Pittsburgh, Pa. 15230
383. J. E. McLaughlin, US AEC, 376 Hudson St., New York, N. Y. 10014
384. J. D. McLendon, Union Carbide Corp., Y-12 Plant, Oak Ridge, Tenn. 37830
385. C. B. Mills, LASL, Box 1663, Los Alamos, N. M. 87544
386. Jean Moreau, Commissariat a l'Energie Atomique, Lederc Leaux, France
387. W. G. Morrison, Idaho Nuclear Corp., Idaho Falls, Idaho 83401
388. C. N. Nichols, UKAEA, Harwell, Didcot, Berkshire, England
389. H. C. Paxton, LASL, Box 1663, Los Alamos, New Mexico 87544
390. W. A. Pryor, US AEC, Oak Ridge Operations Office, Oak Ridge, Tenn. 37830
391. K. H. Puechl, Nuclear Materials and Equipment Corp., Apollo, Pa. 15613
392. Radiation Chemistry Data Center, University of Notre Dame, Notre Dame, Indiana 46556
393. S. L. Reese, Nuclear Safety Associates, 7735 Old Georgetown Road, Bethesda, Maryland 20014
394. C. R. Richey, Battelle Memorial Institute, Box 999, Richland, Wash. 99352

395. Walter Schüller, GWK, Karlsruhe, Germany
396. C. L. Schuske, Dow Chemical Corp., Rocky Flats Plant, Golden, Colo. 80401
397. Raffaele Semonetta, CNEN, Rome, Italy
398. A. J. Smith, Lawrence Radiation Laboratory, Livermore, Calif. 94551
399. D. R. Smith, LASL, Box 1663, Los Alamos, N. M. 87544
400. R. L. Stevenson, US AEC, Div. of Materials Licensing, Washington, D.C. 20545
401. W. R. Stratton, LASL, Box 1663, Los Alamos, N. M. 87544
402. P. B. Suhr, Danish Atomic Energy Commission, Raskilde, Denmark
403. A. F. Thomas, UKAEA, Atomic Weapons Research Establishment, Aldermaston, Berkshire, England
404. Westinghouse Electric Corp., Bettis Atomic Power Laboratory, Box 179, West Mifflin, Pa. 15122 ATTN: Linda Tafel, Library
405. F. E. Woltz, Goodyear Atomic Corp., Piketon, Ohio 45661
406. D. P. Wood, US AEC, Sandia Base, Albuquerque, N. M. 87115
407. E. R. Woodcock, UKAEA Health and Safety Branch, Risley, England
408. B. J. Youngblood, US AEC, Div. of Compliance, 50 Seventh St., Atlanta, Ga. 30300
409. I. F. Zartman, US AEC, Washington, D.C. 20545

410-614. Given distribution as shown in TID-4500 under Criticality Studies (25 copies – CFSTI)