

## REFERENCE 173

**B. G. DUBOVSKII, A. V. KAMAEV, G. M. VLADYKOV, F. M. KUZNETSOV, V. Z. NOZIK, Yu. D. PALAMARCHUK, G. A. POPOV, AND V. V. VAVILOV,**  
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## INTERACTION OF SUBCRITICAL REACTORS\*

B. G. DUBOVSKII, A. V. KAMAEV, G. M. VLADYKOV,  
F. M. KUZNETSOV, V. Z. NOZIK, YU. D. PALAMARCHUK,  
G. A. POPOV and V. V. VAVILOV

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**Abstract**—The article explains the method of ‘equivalent dimensions’ for calculating the interaction of subcritical reactors. The substance of the method is to determine ‘equivalent dimensions’ of a system of interacting reactors and to compare the equivalent geometric parameter with the parameter of the critical system. The results are given of experiments on the study of interaction, and these agree satisfactorily with calculated data.

WHEN examining the nuclear safety of various systems containing fissile materials, the interaction of the elements of which the given system is composed must be taken into account. The problem of the interaction of a large number of subcritical reactors is so complex that it is sometimes quite impossible to find an accurate solution. It must be emphasized that the problem of interaction between elements is encountered very frequently in the production, storage and transportation of fissile materials.

In examining the interaction of reactors, both in the Soviet Union<sup>(1,2)</sup> and abroad<sup>(3)</sup>, attempts are being made, not to find an accurate solution, but to find reliable methods of evaluation which will give a guaranteed margin in determining the safety of an interacting system. The method described below, known as the method of ‘equivalent dimensions’, is an attempt to satisfy these requirements.

Let us assume that a certain collection of subcritical assemblies with certain definite nuclear properties and geometric parameters can be replaced by a single reactor with equivalent Laplacian and with changed geometrical characteristics but the same nuclear characteristics as previously. We shall try to write an equation for determining equivalent geometric dimensions from the general considerations.

Effective augmentation of the dimensions of an equivalent reactor depends upon the probability  $P_{ij}$  that a neutron emerging from the  $i$ -th assembly will enter the  $j$ -th assembly. This probability is determined by the solid angle  $\Omega_{ij}$  subtended by the surface of the  $j$ -th assembly at a point on the  $i$ -th assembly. The solid angle must be computed with a weighting equal to the angular distribution function of the neutrons emerging from the assembly. We shall determine the association between the equivalent dimension and the solid angle.

1. We shall examine the case when  $n$  identical assemblies in the form of parallelepipeds are arranged in a line at an identical distance  $d$  apart and have neither a common nor an individual reflector (Fig. 1a). It is expedient in the given case for the equivalent reactor to have dimensions  $a \times b \times c_{\text{equiv}}$  i.e. only the dimension in the direction of

\* Translated by J. J. CORNISH from *Atomnaya Energiya* **16**, 16 (1964).

the interaction changes. The unknown relation must satisfy the obvious conditions:

- (i) at  $d \rightarrow 0$   $c_{\text{equiv}} = nc$ ,
- (ii) at  $d \rightarrow \infty$   $c_{\text{equiv}} = c$ ,
- (iii) at  $n \rightarrow \infty$   $c_{\text{equiv}} \rightarrow \infty$  for any fixed  $d$ .

If the solid angle is normalized in such a way that at  $d \rightarrow 0$   $\Omega = 1$  the aforesaid conditions are satisfied by the following equation for finding the equivalent dimension

$$c_{\text{equiv}}^k = (nc)^k \Omega + c^k (1 - \Omega) \quad (1)$$

for positive values of exponent  $k$ . If it is further required that the increase in dimension  $\Delta_c = c_{\text{equiv}}^{n+1} - c_{\text{equiv}}^n$  (where  $n+1$  and  $n$  are the number of assemblies in a linear

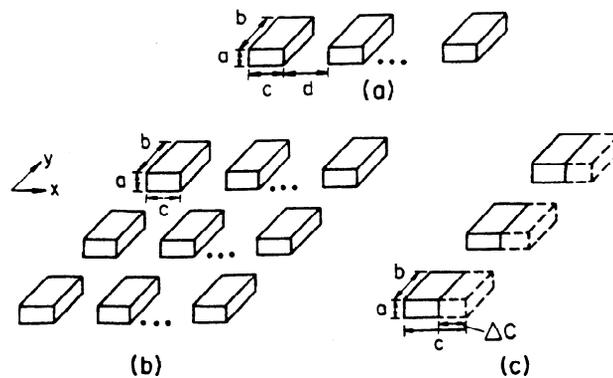


FIG. 1.—Arrangement of interacting reactors.

chain) should be independent of the quantity of interacting assemblies, this will mean an unambiguous selection of exponent  $k$ , namely a value  $k = 1$ , since

$$\Delta c_{\text{equiv}} = c^k \Omega n^{k-1} c_{\text{equiv}}^{1-k} \Delta_n$$

is independent of  $n$  only at  $k = 1$ .

Thus, the suggested equation for determining the equivalent dimension has the form

$$c_{\text{equiv}} = c[1 + (n - 1)\Omega] \quad (2)$$

where the first term on the right-hand side is the true dimension, the second the addition to the dimension due to interaction.

2. We shall examine a plane lattice of identical parallelepipeds (see Fig. 1b) which we shall replace by a single equivalent parallelepiped with sides  $a \times b_{\text{equiv}} \times c_{\text{equiv}}$ . In determining  $b_{\text{equiv}}$  and  $c_{\text{equiv}}$  two successive approximations are possible. In the first approximation each of the chains along the  $x$ - and  $y$ -direction is examined independently. Then

$$c_{\text{equiv}} = c[1 + (n_x - 1)\Omega_x] \quad (3a)$$

$$b_{\text{equiv}} = b[1 + (n_y - 1)\Omega_y] \quad (3b)$$

where  $n_x$  and  $n_y$  are the number of assemblies in the chains along the  $x$ - and  $y$ -directions,  $\Omega_x$  and  $\Omega_y$  are the solid angles between closest assemblies in the  $x$ - and  $y$ -directions.

In the second approximation, the diagonal interactions are allowed for artificially. The equivalent dimensions are determined from equation

$$c' = c[1 + (n_x - 1)\Omega_x]. \quad (4a)$$

This corresponds to 'bunching' of all the assemblies in the  $x$ -direction and transforming the initial system into the system shown in Fig. 1(c).

Now we find

$$b_{\text{equiv}} = b[1 + (n_y - 1)\Omega_y'], \quad (4b)$$

where  $\Omega_y'$  is the solid angle on surface  $a \times c'$ .

In order to determine  $c_{\text{equiv}}$  we shall 'bunch' all the assemblies in the  $y$ -direction:

$$b' = b[1 + (n_y - 1)\Omega_y], \quad (5a)$$

then

$$c_{\text{equiv}} = c[1 + (n_x - 1)\Omega_x'] \quad (5b)$$

where  $\Omega_x'$  is the solid angle on surface  $a \times b'$ .

3. We shall examine a space lattice of parallelepipeds. In the first approximation the equivalent dimensions are found from the following scheme:

$$a_{\text{equiv}} = a[1 + (n_z - 1)\Omega_z] \quad (6a)$$

$$b_{\text{equiv}} = b[1 + (n_y - 1)\Omega_y] \quad (6b)$$

$$c_{\text{equiv}} = c[1 + (n_x - 1)\Omega_x]. \quad (6c)$$

The equivalent dimensions can also be found by the method of successive approximations. Usually the first approximation is quite sufficient, in spite of the fact that in this case the diagonal interactions are neglected.

The scheme for computing the equivalent dimensions given above is fairly general, since each irregular permutation of identical assemblies may be rearranged into a regular one by forming a more reactive configuration. Spherical and cylindrical assemblies may be replaced by the parallelepipeds described. In this case the procedure for computing the solid angle from a point onto plane  $a \times b$  is simple:

$$\Omega = \frac{2}{\pi} \arctan \frac{ab}{2d\sqrt{a^2 + b^2 + 4d^2}}. \quad (7)$$

The expression for  $\Omega$  is obtained on the assumption that the emergence of neutrons from the surface of the assembly is both isotropic and homogeneous. This means that the weighting function of the angular distribution of the escaping neutrons is equal to unity and an identical number of neutrons escapes from any point on the surface. The latter provides a substantial increase in the margin of safety, compensating the error due to the assumption of isotropic escape.

For cylindrical reactors, which are encountered frequently, more accurate equations must be used for computing the solid angle. When there is interaction at the ends of the cylinders, the solid angle from a point onto a circle of radius  $R$  is computed on the aforesaid assumptions from

$$\Omega = 1 - \frac{d}{\sqrt{d^2 + R^2}} \quad (8)$$

whilst the solid angle from a point onto the lateral surface of a cylinder is computed from

$$\Omega \approx \frac{1}{2\pi} \left[ \frac{h}{\left(\frac{h}{2}\right)^2 + d^2} + \frac{h}{\sqrt{\left(\frac{h}{2}\right)^2 + d^2 + 2dr}} \right] \arcsin \frac{R}{d+R} \quad (9)$$

where  $d$  is the shortest distance from the point to the surface,  $h$  the height of the cylinder,  $R$  its radius.

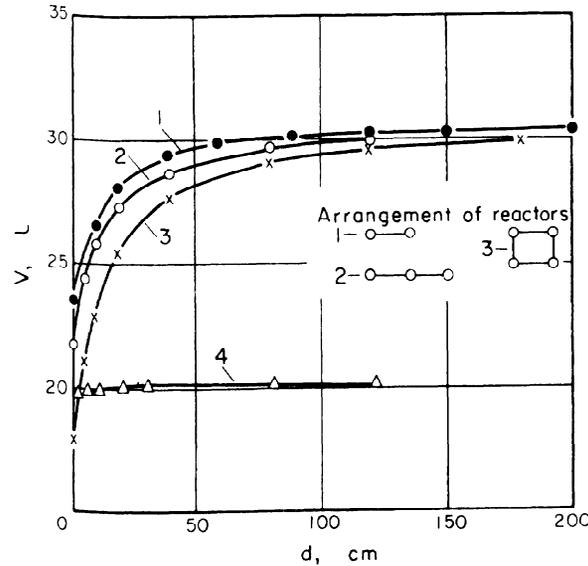


FIG. 2.—Interaction of reactors in the form of cylinders. 1,2,3: Two, three and four reactors respectively; 4: Two reactors with a water reflector 5 cm thick on the end and lateral surfaces. (Uranium concentration in uranyl nitrate solution 71 g/l.)

After determining the equivalent dimensions the system obtained must be transformed into a sphere by equating the geometric parameters

$$\frac{\pi^2}{(a_{\text{equiv}} + 2\lambda)^2} + \frac{\pi^2}{(b_{\text{equiv}} + 2\lambda)^2} + \frac{\pi^2}{(c_{\text{equiv}} + 2\lambda)^2} = \frac{\pi^2}{(R_{\text{equiv}} + \lambda)^2} \quad (10)$$

where  $\lambda$  is the extrapolation distance.

To solve the problem of the criticality of a given system of assemblies one must compare the experimentally determined critical mass of a sphere  $m_{\text{cr}}$  and the mass in the volume of the sphere obtained  $m_{\text{equiv}}$  then the subcriticality condition will take the form

$$m_{\text{cr}} > m_{\text{equiv}} = \frac{4}{3} \pi R_{\text{equiv}}^3 \rho \quad (11)$$

where  $\rho$  is the known density of the core medium in the given system.

The method under examination may be used for calculating the interaction of subcritical assemblies entirely located in a moderator. In this case, the association of type (2) has the form

$$c_{\text{equiv}} = c \left[ 1 + (n-1)\Omega e - \frac{d}{M} \right] \quad (12)$$

where  $M^2$  is the migration area.

In addition,  $m_{cr}$  in the subcriticality condition (11) must be replaced by the critical mass of a sphere with a reflector made of the given moderator.

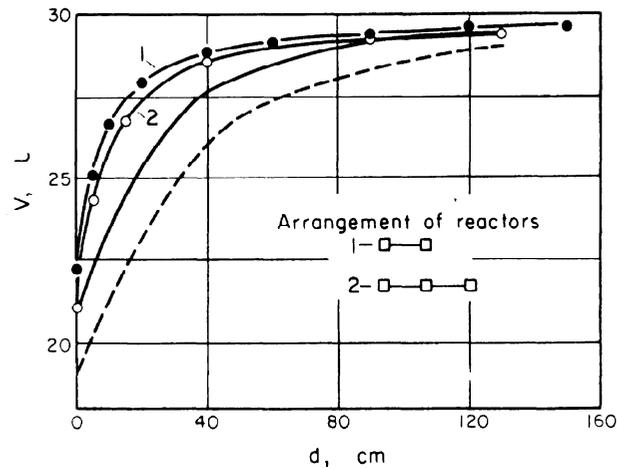


FIG. 3.—Interaction of reactors in the form of parallelepipeds with a square base  $a = b = 300$  mm.

1,2: Two and three reactors respectively; solid line: calculation for two reactors; broken line: calculation for three reactors. (Concentration of uranium in solution 71 g/l.)

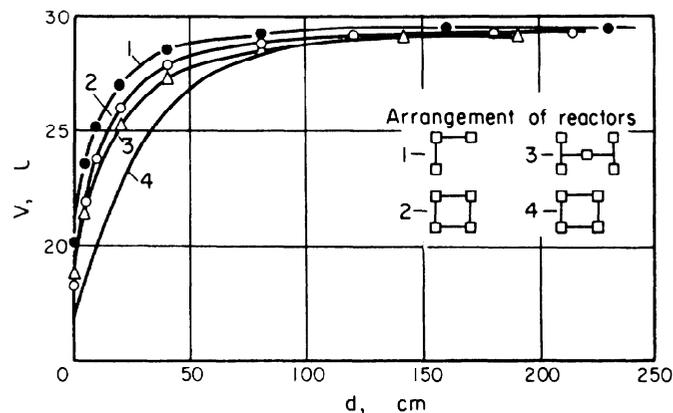


FIG. 4.—Interaction of reactors in the form of parallelepipeds with a square base  $a = b = 300$  mm.

1,2,3: Three, four and five reactors respectively; 4: Calculation (first approximation). (Uranium concentration in solution 71 g/l.)

#### EXPERIMENTAL RESULTS

In order to ascertain some rules for the interaction, experiments were conducted with subcritical reactors in the form of cylinders and parallelepipeds. The reactor core is an aqueous solution of  $\text{UO}_2(\text{NO}_3)_2$  salt with 90 per cent  $^{235}\text{U}$  enrichment. The cylinders and parallelepipeds are made from stainless steel 1.5–2 mm thick, the diameter of the cylinders is 30 cm, the base of a parallelepiped is 30 cm square.

Figures 2–4 give the results of experiments to determine the efficiency of the interaction of two, three, four and five reactors in air. The figures show the volume  $V$  of one reactor as a function of the distance  $d$  between them, on condition that the whole system of reactors is critical and the quantity of solution in all the reactors is identical.

An analogous relation for two cylindrical reactors 25.4 cm in diameter interacting

in water is shown in Fig. 5. For comparison, Figs. 2-5 also give curves calculated by the method of equivalent dimensions.

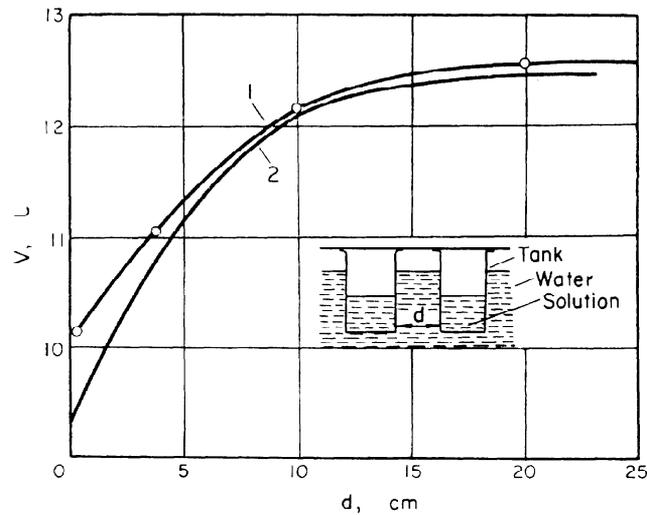


FIG. 5.—Interaction of two cylindrical reactors.  
1: Experiment, 2: Calculation. (Uranium concentration in solution 113 g/l.)

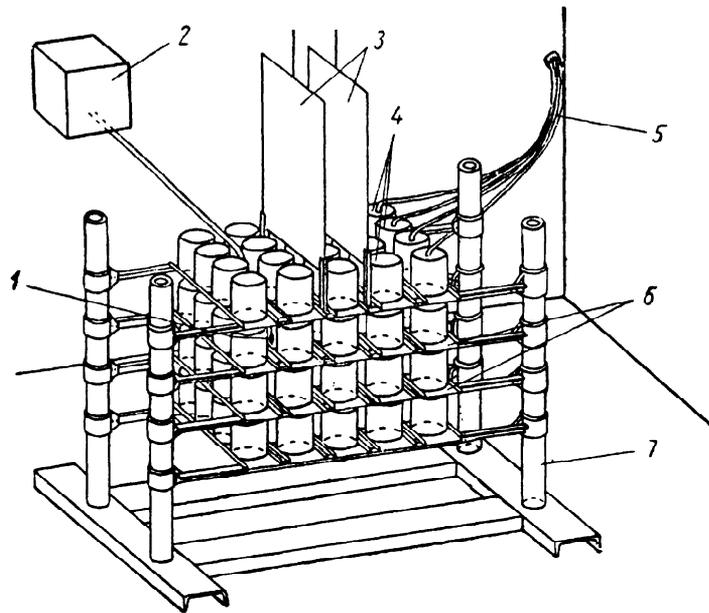


FIG. 6.—General view of the unit for studying the interaction of subcritical vessels arranged in a space lattice.

1: Neutron; 2: container for source; 3: cadmium screens; 4: vessels containing  $\text{UO}_2(\text{NO}_3)_2$  solution; 5: hoses for filling the vessels with solution; 6: shelf; 7: upright.

For studying the interaction of a large number of subcritical elements arranged in a space lattice 6-1. cylindrical glass vessels filled with an aqueous solution of  $\text{UO}_2(\text{NO}_3)_2$  salt to a level of 24 cm were employed. The vessels were  $\sim 18$  cm in diameter with a wall thickness of 0.5 cm. The uranium concentration in solution was 6 g/l.

The unit for studying the interaction of the vessels arranged in a space lattice is illustrated in Fig. 6. The uprights and the frame of the unit are made from steel, the

shelves on which the vessels were placed from aluminium. The results of the experiments are given in the Table 1.

TABLE 1.—INTERACTION OF VESSELS ARRANGED IN A SPACE LATTICE WITHOUT A REFLECTOR

Arrangement of vessels*	Number of vessels		
	How placed	Critical value	Calculation
In two planes ( $d_x = 1$ cm, $d_y = 4.5$ cm, $d_z = 12$ cm)	23 (in the first plane $4 \times 3 = 12$ , in the second $(4 \times 3) - 1 = 11$ )	23	12
In three planes ( $d_x = d_y = 6.5$ cm, $d_z = 12$ cm)	52 (in the first plane $4 \times 4 = 16$ , in the second $4 \times 5 = 20$ , in the third $4 \times 4 = 16$ )	$54 \pm 0.5$ (extrapolation)	18
In three planes ( $d_x = d_y = 4.5$ cm, $d_z = 12$ cm)	39 (in the first plane $3 \times 4 = 12$ , in the second $(3 \times 4) + 3 = 15$ , in the third $3 \times 4 = 12$ )	$39.5 \pm 0.3$ (extrapolation)	12
In four planes, ( $d_x = d_y = 9$ cm, $d_z = 11$ cm)	67 (in the first plane $4 \times 4 = 16$ , in the second $4 \times 4 = 16$ , in the third $(4 \times 5) - 1 = 19$ , in the fourth $4 \times 4 = 16$ )	$80 \pm 5$ (extrapolation)	36
In four planes, ( $d_x = d_y = 7.5$ cm, $d_z = 12$ cm)	64 (in each plane $4 \times 4 = 16$ )	$72 \pm 5$ (extrapolation)	27

\* Values  $d_x$  and  $d_y$  are the distances in the plane,  $d_z$  is the distance between planes; all distances are indicated with allowances for the walls, i.e. between solutions.

The neutron absorption in the structural materials of the unit was not taken into account in the calculation. If the neutron absorption is allowed for, the calculated values more nearly approach the experimental ones.

It is evident from comparing the results of calculations and experiments that in all cases the method of equivalent dimensions gives a margin of safety and can be used to evaluate the nuclear safety of interacting systems.

This same method was used to calculate experimental data on the efficiency of interaction of spheres of metallic uranium<sup>(4)</sup>. Calculations exhibited satisfactory agreement with experiment and gave a nuclear safety margin.

The interaction of several subcritical assemblies can be calculated to an accuracy of  $\sim 20$  per cent by the method of 'equivalent dimensions'; the interaction of a large number of assemblies may be evaluated with an accuracy of  $\sim 100$  per cent. In all cases calculations exaggerate the interaction efficiency.

In spite of the simplicity of the method described it is just as accurate as the approximate 'interaction parameter' and 'density analogue' method<sup>(4)</sup>.

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