REFERENCE 149

H. L. REYNOLDS, "CRITICAL MEASUREMENTS AND CALCULATIONS FOR ENRICHED-URANIUM GRAPHITE-MODERATED SYSTEMS," IN PROCEEDINGS OF THE SECOND INTERNATIONAL CONFERENCE ON THE PEACEFUL USES OF ATOMIC ENERGY, GENEVA, 1958 (UNITED NATIONS, NEW YORK, 1958), VOL. 12, PP. 632-642.

Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy

Held in Geneva
1 September - 13 September 1958

Volume 12
Reactor Physics

UNITED NATIONS
Geneva
1958

A/CONF.15/1 English, Vol. 12

UNITED NATIONS PUBLICATION

Sales No.: 58. IX. 2. Vol. 12

Price: $U.S.18.50; £6 12s. (stg.); Sw. fr. 80.00
(or equivalent in other currencies)

PRINTED IN GREAT BRITAIN

Critical Measurements and Calculations for Enriched-uranium Graphite-moderated Systems

By H. L. Reynolds*

Although natural uranium, graphite-moderated systems have received extensive treatment in the past, little consideration has been given to *enriched* uranium, graphite-moderated systems. Zimmerman[1] has reported critical measurements on a single enriched uranium, graphite-moderated, graphite-reflected system. This paper presents the experimental results of the above laboratories on a variety of enriched uranium, graphite-moderated systems, both with and without reflectors. Also included are the results of the multigroup transport, and one- and two-dimensional diffusion calculations used to interpret the data.

In general the UCRL systems were bare or one-dimensional in order to simplify the analysis. Efforts were made to obtain systems as close as possible to an idealized system containing only a homogeneous mixture of moderator and fuel with all extraneous factors removed. The LASL experiments were carried out with more complex geometries approaching more closely potential reactor systems. All of the assemblies utilized heterogeneous arrangements of moderator and thin uranium foils.

The bare systems range in carbon-to-uranium atomic ratios (C/U) from 300/1 to 2500/1. These systems are not truly thermal and are in a range where critical mass is extremely sensitive to size or buckling. Experiments were performed for these systems to determine accurately the effects of extraneous factors such as room return, control-rod void spaces, nonhomogeneity of fuel loading, moderator block porosities and poison content. The reflectors include graphite and beryllium in one-, two- and three-dimensional arrangements.

The constants used in the multigroup calculations are presented in tabular form. Eighteen energy groups were used. The same constants were used in the transport and diffusion calculations to allow comparison of the two methods of calculation.

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UCRL EXPERIMENTS

Method

The reactors were assembled upon a low-mass stacking table that consists of an 8×8×1 ft aluminum honeycomb slab rigidly supported by an aluminum stand. The honeycomb is made of 2S aluminum and has a density of 3 lb/ft^3. The top of the table is 4 ft above the floor, to reduce neutron return from the floor as much as possible.

A braced post-and-beam aluminum frame rigidly suspends the control unit over the low-mass table. The control unit is a 2S aluminum canister with four channels in X-cross section (each ½ by 6 in. exterior dimension) mounted beneath a manual-electric drive assembly. Each channel guides a ¼-in. thick, 4-ft long, boral element. In more recent experiments the boral has been replaced by cadmium so that the void left in the reactor by the control system could be reduced. Three of the 5-in. wide elements are safety rods and the fourth 1-in. element is the control rod. A tube through the canister axis permits insertion of neutron sources into the reactor.

Nominally 1- and 2-mil thick foils containing† 93.2% U^{235} in 5¼-in. squares were used for fuel. All foils were coated with a fluorocarbon plastic to prevent oxidation and reduce erosion. There is an average of 0.92 grams of plastic per foil. The moderator material is ATJ graphite‡ machined in ½-in. thick, 6×6 in. solid and grooved square plates with a 0.010 in. recess milled in the top to accommodate the fuel foils (see Fig. 1). The grooved plates have a 40% void volume and were used as pallets to implement the moderator stacking. Generally one grooved plate per eleven ordinary plates was used. A core lattice of a 2-mil foil on each graphite plate gives a nominal carbon-to-U^{235} ratio of 600 : 1. Nonporous graphite plates were placed on end to plug the ½ in. canister extension to the core edge.

Standard QMV solid beryllium was used for reflector material. The beryllium density is 1.84 g/cm^3 and has been machined into a selection of convenient-size parallelepipeds.

A stacked parallelepiped system is shown in Fig.

† The term "oralloy" will be used for this material.
‡ Made by National Carbon Company.

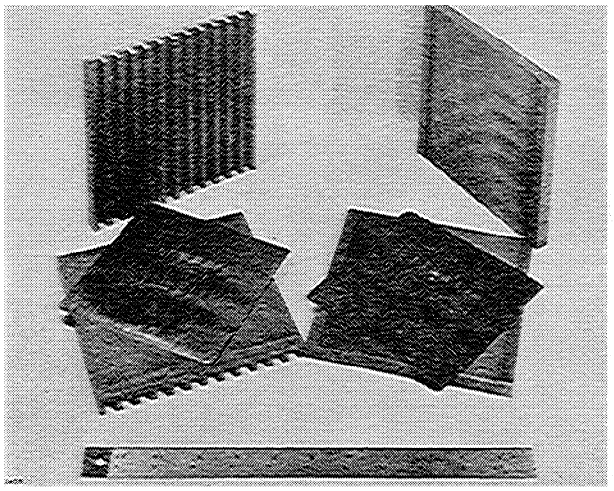


Figure 1. Photograph of foils and graphite used in UCRL system

2A. The object on the left is a movable paraffin slab. The bodies of personnel working on the assembly of the unreflected systems reflect neutrons and thus increase the reactivity of the system. The paraffin slab, moved from the control room, was used to assure that a contemplated manual change in reactor loading could be accomplished without driving the assembly critical during the loading operation. A pseudo-octagonal system is shown in Fig. 2B, giving a better view of the control system.

Results

Approximately 70 different graphite-moderated assemblies have been studied. The majority of the assemblies were made to measure the effect of extraneous factors so that the reactors could be reduced to the simplest possible systems, allowing accurate comparison with calculations. Ten different basic geometries were studied. The data for these basic systems is given in Table 1. The corrections which were applied to the original data to obtain homogeneous fueled systems, without poisons and isolated from the surrounding environment, column 4, are given below.

Corrections and Special Measurements

Poisons in Assemblies

The poisons consist of the aluminum guides for the control vanes, the plastic coating on the fuel foils, and the impurities in the moderator. The control rod guides amount to 180 g of 2S aluminum per inch of reactor height. Elementary calculations indicate that the effect of this amount of aluminum would be undetectable. Half of the void in Assembly 3 was filled with 500 g per in. of aluminum. The change in critical height was less than 0.1 in. substantiating the calculations.

The fluoroplastic $(CF_2)_n$ was chemically analyzed and no impurities in a concentration greater than 1 part per million were found. Magnetic resonance tests indicated that hydrogen was present to less than 3 parts per thousand. The fluoroplastic may thus be ignored.

The results of a chemical analysis of the graphite indicate the boron content is between 10 to 15 parts per million. Pulsed neutron measurements on the graphite give a value of 13 parts per million. The impurities present result in a thermal capture cross section several times larger than that of the carbon and should be included in calculations.

Porosity in Blocks

The simplest method of building up the reactor is with entire horizontal planes of low-density blocks appearing every 6 inches in height. One assembly

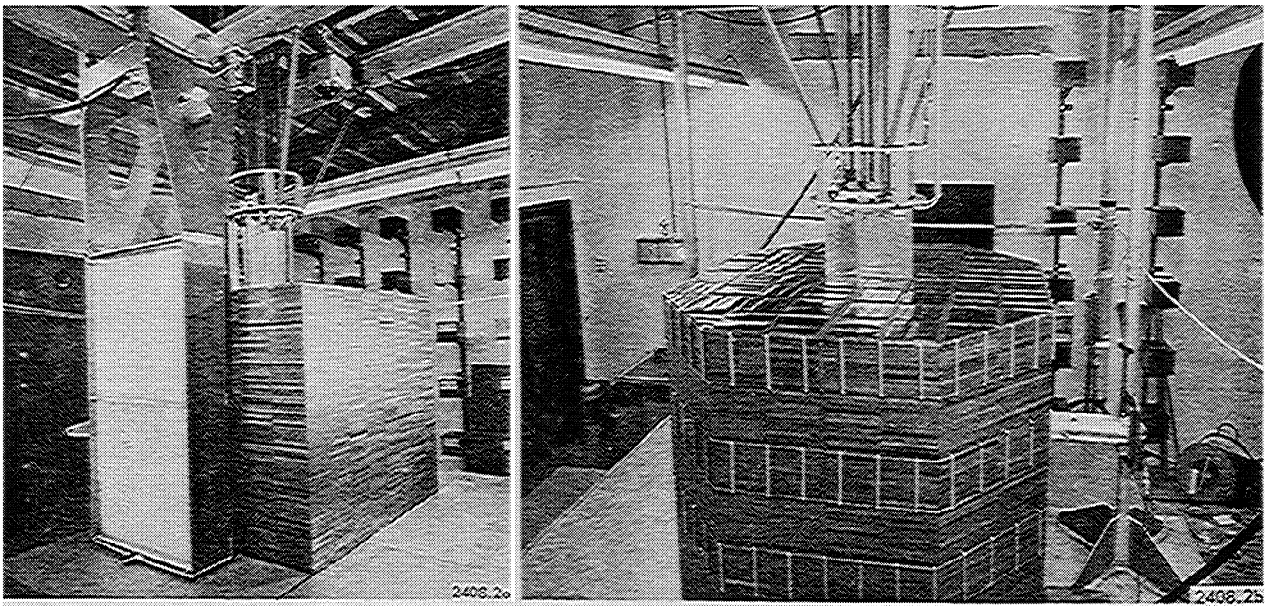


Figure 2. Photographs of UCRL systems. (a) Rectangular system showing "paraffin man". (b) Octagonal system showing vane guides

Table 1. Data for the Basic UCRL Systems

Experiment No.	Core size, inches ±0.03	Critical height, inches ±0.1	Corrected critical height, inches ±0.3	Reflector thickness[a] inches	C/U^{235} atomic ratio	Foil thickness, mils	Mass U^{235}, kg	Critical buckling $\pm 0.1\ cm^{-2}$
U1	42.50 × 42.50	47.1	44.4	—	297	4	149.7	23.07×10^{-4}
U2	48.50 × 48.50	40.1	37.6	—	600	2	82.26	22.26×10^{-4}
U3	48.50 × 48.50	42.3	40.3	—	1200	2	43.57	21.01×10^{-4}
U4	48.50 × 48.50	47.6	44.7	—	2340	1	25.14	19.42×10^{-4}
U4 A	60.50 × 60.50	—	51.2	—	10500	1	9.77	12.6×10^{-4}
U5	48.50 × 36.50	42.8	40.7	6.0 C on 2 sides	1200	2	32.97	
U6	36.50 × 36.50	43.5	41.3	6.0 C on 4 sides	1200	2	25.16	
U7	48.50 × 36.50	48.1	45.1	3.0 Be on 2 sides	2340	1	19.05	
U8	48.50 × 36.50	41.0	38.4	6.0 Be on 2 sides	2340	1	16.23	
U9	36.50 × 36.50	36.8	34.5	6.0 Be on 4 sides	2340	1	10.91	

[a] Reactor base is square in all cases except Experiment U7 where base is 48.5 × 42.5 in.

Note: Graphite density is 1.645 g/cc except in U1 where density is 1.640 g/cc. If the total graphite weight were divided by the core volume average density would be slightly less because of the control void. Be density is 1.84 g/cc. Graphite density in U4 A is 1.70 g/cc and buckling is given for 1.645 g/cc.

was reloaded with the low-density blocks randomized so that no horizontal low-density plane existed, in order to determine if these planes caused neutron streaming. The critical height decreased by 0.1 in.

Room Return

It can be seen from Fig. 1 that the room walls are about 10 ft from the reactor edges. Neutron reflection from the walls and floor was investigated by surrounding all of the assemblies by a $\frac{1}{4}$ in. thick boral sheet which would effectively absorb all neutrons up to 15 ev in energy. The capture of these returning neutrons before they entered the reactor required an increase in critical height of approximately 0.1 in. Because of moderation in the concrete, an insignificant number of the returning neutrons would have energies higher than 15 ev. The boral reflects some of the fast neutrons. To determine how much this fast reflection was compensating for the capture of the thermal neutrons, $\frac{1}{4}$ in. Al sheets were placed on the wall side of the boral. The additional Al caused a decrease in critical height of 0.1 in. Therefore the critical heights for all of the bare systems were increased 0.2 in. to take account of room return.

U^{238}

The Oralloy foils contain approximately 6% of U^{238}. The resonance region absorption in U^{238} will decrease the buckling by several per cent and therefore is included in the calculations.

Control Voids

The space in the original assembly left open for the control rods consists of two crossed rectangles $\frac{1}{2}$ in. wide by 12 in. long, amounting to about 1% of the reactor volume. The effect of the void was determined in the following manner:

First the void that was located in the center of the assembly was moved toward the edge in 6 in. steps by restacking the core. With the center of the void located 6 in. from the edge of the assembly, the critical height was found; then a similar void was introduced in the same position at the opposite side of the assembly and a new height was determined. Thus the effect of the void was determined 6 in. from the edge and could then, if desired, be extrapolated to the center of the system with the previously measured critical height versus void position. In general only one arm of the cross was duplicated, since it was shown that two arms gave just twice the effect of one arm. In Table 2 the critical heights with and without the void correction are shown for the bare systems. Notice that with the void removed the over-all C/U value changes slightly because the amount of graphite per unit height has increased with no change in uranium per unit height.

Because of the recognized inaccuracies in the above procedure, recent experiments have been performed with cadmium vanes replacing the boral. The void for the Cd vanes was only 22.5% as large as the void for the boral vanes. Cd vanes were not used for Assembly 1 because they did not give adequate control. In Table 2 the critical height with the Cd vane system is also given. The final column is determined from the Cd measurements by assuming that the critical buckling can be extrapolated to zero void linearly. The error in making this assumption is small. The final column is used as the correct critical heights. It is estimated that the void removal introduces an error of ±0.2 in. in the critical height.

Although measurements of the void effects were not undertaken for the reflected systems, the percentage change in critical height was assumed to be the same as that for the unreflected systems with the same carbon-to-uranium ratios.

Homogenizing Fuel

The fuel thickness varied from 1 to 4 mils for the assemblies (see Table 1). Corrections must be made for the self-shielding and flux depression caused by the lumped fuel in order to arrive at a system with a homogeneous mixture of fuel and moderator which is amenable to calculation. A disadvantage factor[2] was obtained experimentally where the disadvantage

Table 2. Effects of Control Void Removal

Experiment No.	Critical height with void, in.	Critical height with void removed; estimated from boral measurements, in.	Critical height with Cd system in.	Estimated critical height with no void; from Cd measurements in.
U1	47.1	44.5	—	—
U2	40.1	38.5	38.2	37.7
U3	42.3	40.2	40.9	40.4
U4	47.6	45.4	45.4	44.8

factor times the actual amount of fuel in the lumped system gives the equivalent amount of fuel in the desired homogeneous system. Small oralloy foils of different thicknesses were irradiated in a uniform neutron flux region in the critical assembly of interest. The γ-emission intensity of the foils after irradiation was used as a measure of foil activation or absorption. The foil specific activity per unit thickness was plotted versus the foil thickness, and the curve was extrapolated and normalized to unity at zero foil thickness. This is the foil disadvantage factor versus foil-thickness curve. By choosing a thickness corresponding to the fuel foils, the appropriate disadvantage factor can be determined. Corrections were applied for the fraction of the fission fragments that escaped from the surface of the foils.[3]

The disadvantage factor so obtained is a sum of the effect of self-shielding in the foil and flux depression in the moderator. The flux depression for the measuring foils was not necessarily that of the fuel foils. In addition the correction for escaping fission fragments was large and subject to error.

Therefore, the effect of the fuel lumping was studied in another, more accurate manner. An assembly with a carbon-to-U^{235} atomic ratio of 1150 : 1 was loaded with fuel foils 1, 2, and 4 mils thick, with spacings of $\frac{1}{2}$, 1, and 2 in. respectively. The critical heights were 40.5, 40.6, and 40.8 in. respectively. Extrapolating to a homogeneous fuel mixture gives a critical height of 40.4 in., or a change in critical height of 0.2 in. because of using 2 mil foils. Using the C/U versus B^2 curve for the bare systems, it can be determined that the 1200 : 1 system has an effective C/U of about 1280. The effect of homogenizing the fuel as measured in this manner is $\frac{1}{6}$ of that obtained by using the disadvantage factor. Although the magnitude of the disadvantage factor is incorrect, the relative values for the various assemblies should be correct. Therefore the correction to the critical height is taken in proportion to the disadvantage factor based upon the change at a C/U of 1150 : 1.

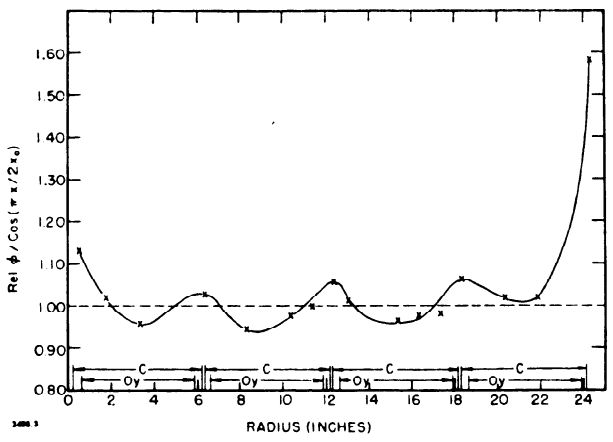


Figure 3. Oralloy foil activation in the plane of uranium foils for Assembly U3

The order of magnitude of the flux variation in the moderator in one dimension can be seen in Fig. 3. The activation of oralloy foils is shown as a function of the distance from the center of the reactor for a C/U value of 1200 : 1. The foils were $\frac{1}{2}$ in. from the nearest fuel foils. The foil activity has been divided by cos $(\pi X/2X_0)$, the flux expected for a bare reactor. The position of the carbon and the uranium foils is shown. The flux peaks between the foils by about 5%. The rise in the center of the reactor is due to the absence of fuel due to structural modifications necessary for the control void. The rise at the edge is due to the inadequacy of diffusion theory near a boundary.

Corrected Results

The results with the inclusion of all corrections are shown in Table 1. The critical heights have a probable error of ± 0.3 in. for the bare systems. The bucklings are correct to $\pm 0.1 \times 10^{-4}/cm^2$. The effects of the void and nonhomogeneous fuel were not measured for the reflected systems and therefore the corrections to the critical height made were the same percentage as those of the bare systems with the same carbon-to-uranium atomic ratio. The corrections for room return and block porosity were not made for the reflected systems. The critical height for the reflected systems has a probable error of ± 0.5 in.

LASL EXPERIMENTS

Method

The reactors were assembled in the assembly machine known as "Honeycomb" shown in Fig. 4. The machine consists of two stacks of aluminum tubes 3 in. square by 36 in. long with 0.047 in. walls. The aluminum alloy is 2S and is 99% Al. Throughout the assembly machine the average density of Al is 0.165 g/cm^2. The two stacks are brought together by remote control to form a 6-ft cubic matrix of tubes. Critical assemblies are built up by slipping fuel and

Table 3. Data for LASL Cylindrical Systems

Expt. no.	C U^{235}	Critical mass U^{235}, kg	Fuel region: Effective graphite density, g/cc	Fuel region: Radius, cm	Fuel region: Length, cm	Wall reflector: Effective Be density, g/cc	Wall reflector: Thickness, cm	East end reflector: Graphite thickness ΔFE, cm	East end reflector: Be thickness, cm	West end reflector: Graphite thickness ΔFW, cm	West end reflector: Be thickness, cm
L1	124	53.5	1.42	31.30	77.70	1.66	12.40	2.54	10.10	1.02	20.20
L2	1020	16.2	1.48	48.83	76.20	1.66	12.00	2.54	9.76	2.54	9.76
L3	412	30.1	1.50	40.56	81.28	1.66	13.08	—	10.16	—	10.16
L4	412	30.1	1.50	40.56	81.28	1.66	16.74	—	10.16	—	0
L5	412	30.1	1.50	40.56	81.28	1.66	29.90[a]	—	0	—	0
L6	380	11.9	1.42	25.25	76.20	1.66	24.71	2.54	10.16	2.54	20.32

Assembly 1 contained 16.3 kg U^{235} as 2-mil foils and the remainder as 5-mil foils.
Assemblies 2 and 6 contained 2-mil foils.
Assemblies 3, 4 and 5 contained 5-mil foils.
Al density of 0.165 g/cc throughout all assemblies.

[a] Extrapolated value; actual critical configuration with 27.58 cm Be + 5.75 cm graphite (ρ = 1.56).

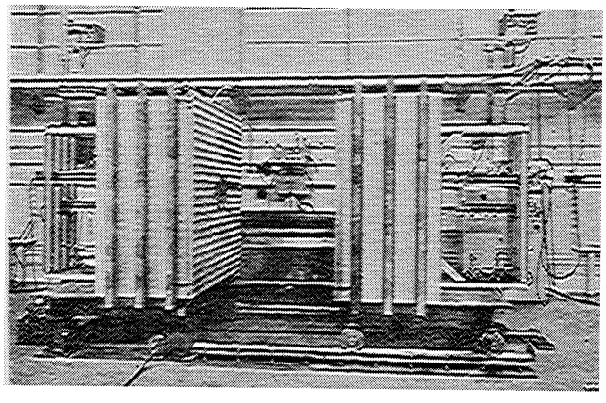


Figure 4. LASL assembly machine

reflector material into the aluminum tubes. Fuel subassemblies consist of graphite plates 16 in. long by 2.9 in. wide with 2- or 5-mil thick foils of 93.2% U^{235} interleaved to give the desired average carbon-to-uranium atomic ratio.

Four safety rods withdraw fuel subassemblies from the core portion of the assembly. Vernier control is obtained with two positionable fuel subassemblies.

Results

Two groups of experiments were performed, a series of graphite-moderated, beryllium-reflected pseudo-cylinders and a series of graphite-moderated, graphite-reflected, rectangular parallelepipeds.

Cylindrical Systems

The results for the cylindrical systems are given in Table 3. The regions of the reactors are shown in Fig. 5. The materials in each of the regions can be considered homogeneous. The graphite in regions ΔFE and ΔFW is at the same density as the core or region F. Because of the modular design of the assemblies, the cross sections are not exactly cylindrical. Cross sectional views of Assemblies 1, 2 and 3 are given in Fig. 6. The radii given in Table 3 are obtained by assuming the radius to be equal to that of a cylinder with the same area as that of the assembly.

A knowledge of the spatial variation of fission rates and fluxes in the assemblies was obtained by distributing small foils of oralloy, gold, and indium with and without cadmium covers throughout the systems. After irradiation the γ activity of the foils was measured. The effect of the nonuniform structure of the fuel has been minimized by placing detector foils throughout the assembly in similar positions relative to (but not adjacent to) fuel foils.

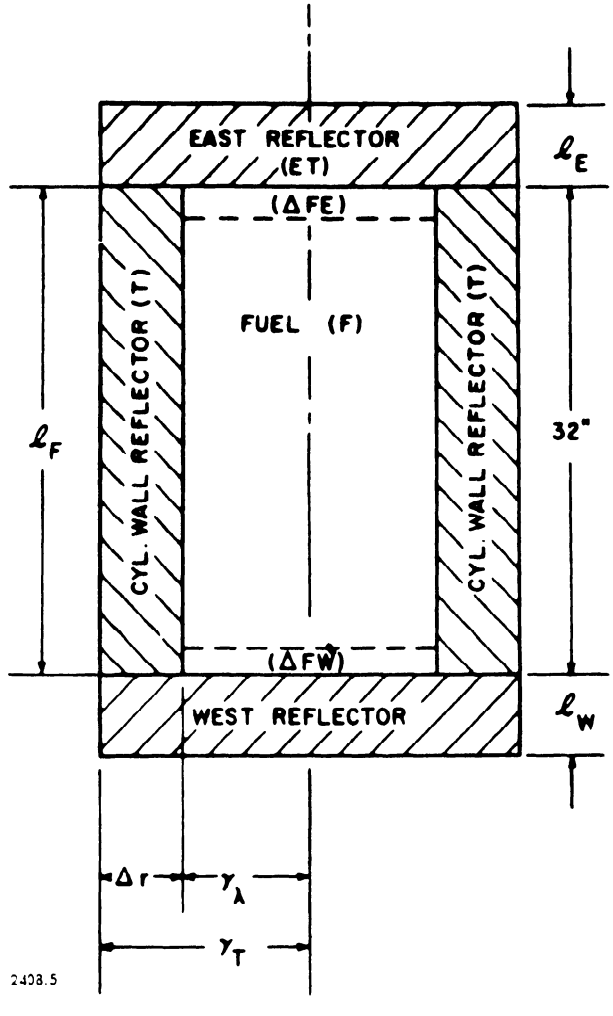


Figure 5. Regions of cylindrical reactor

Table 4. Summary of Critical Conditions for Graphite-Moderated Graphite-Reflected Systems

(Reflector graphite 30, 48 cm on all sides, $\rho = 1.55$ g/cm³ Al density = 0.165 g/cc)

Expt. no.	C Oy	Critical mass kg Oy	Graphite density (core), g cm³	Core dimensions, cm	Corrected k
LA1	6650	7.98	1.50	121.92 × 121.92 × 121.92	1.0666
LA2	4937	7.63	1.50	99.06 × 106.68 × 121.92	1.0465
LA3	3140	7.92	1.50	91.44 × 91.44 × 101.60	—
LA4	2365	8.84	1.50	83.82 × 83.82 × 101.60	—
LA5	4366	9.73	1.34	115.82 × 114.30 × 121.92	1.0480
LA6	2770	10.21	1.34	99.06 × 106.68 × 101.60	—

The perturbation of the flux due to the non-cylindrical cross section was investigated by mapping the relative number of fissions parallel to the core-reflector interface at two distances from the interface. Figure 7 shows foil locations and the relative fission data. The effect of corners can be observed in the traverse closest to the core-reflector interface.

The graphite used was CS* and contained less than 1 part per million of boron. The possibility of a reactivity contribution from water absorbed in the graphite was considered. An assembly was restacked after the graphite had been dried in a vacuum oven. The water content corresponded to the effect of 4 grams of lucite, indicating the graphite is essentially free of moisture.

Approximately 12% of the 2 mil foils in Assembly 2 were replaced by 5 mil foils. A Δk of 0.075% was measured. A 100% exchange is estimated to be worth 0.75%. If a linear effect is assumed, Assembly 2 with homogeneous fuel would have a k of 1.005. The corrected k for the other assemblies will be similar.

Rectangular Systems

All of these systems were rectangular parallelepipeds surrounded by a 1-ft thick graphite reflector on all six sides. The results are presented in Table 4.

The effect of self-shielding of the 1-mil oralloy foils was determined by replacing a number of 1-mil foils by 2-mil foils. A 100% replacement was estimated to be worth 6.66% in reactivity for Assembly 1. For Assemblies 2 and 5 this effect is 4.65%.

The graphite density in Assemblies 5 and 6 was reduced by removing a graphite plate 0.54 in. thick from each of the fuel subassemblies. Measurements were made to determine the effect of changing from one 0.54-in. gap in each of four tubes to one 2.16-in. gap in a single tube. The result of such a change over the entire core was to reduce the reactivity by 0.6%, and therefore demonstrated the existence of streaming effects. Thus a homogeneous graphite density would increase the reactivity by about 0.15%.

Assuming a homogeneous fuel and moderator distribution the k values for the assemblies are given in the last column of Table 4. The corrected k values for Assemblies 3, 4 and 6 are thought to be between 1.01 and 1.03.

An axial fission distribution for Assembly 1 is shown in Fig. 8. A cosine distribution and the ratio of the measured distribution to the cosine are also plotted for comparison.

* Also made by National Carbon Co.

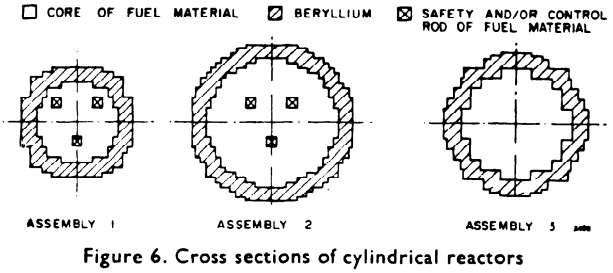


Figure 6. Cross sections of cylindrical reactors

CALCULATIONS

Method

Three types of calculations were used for comparison with the data. These are known as "Zoom", "Angie", and S_4, and are described below.

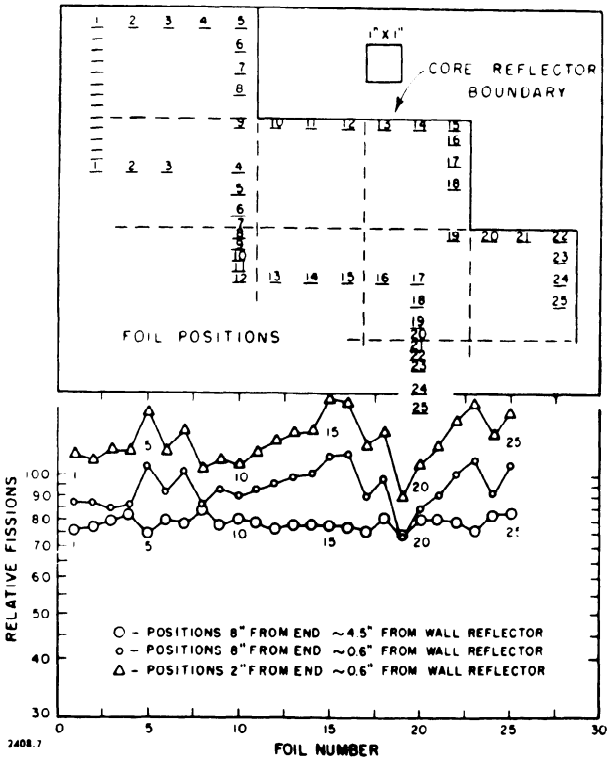


Figure 7. Activation of oralloy foils near edges of core of cylindrical reactor L3 to illustrate effects of irregular core edge

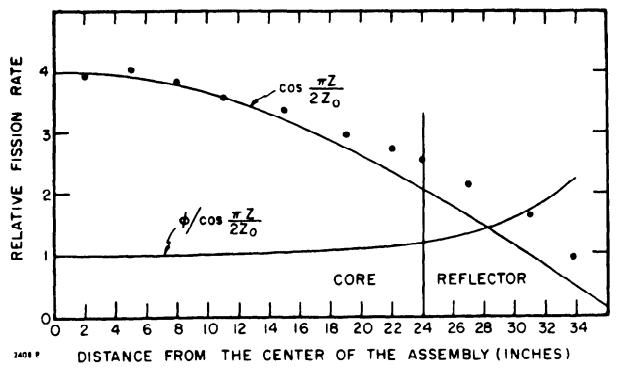


Figure 8. Oralloy foil activation for Assembly LA-1

Zoom

The Zoom calculation is an IBM 704 code that solves the one-dimensional neutron diffusion equation for slabs, cylinders, and spheres. The basic diffusion equation for energy group i is:

$$D_i\nabla^2\phi_i-(\Sigma_{c_i}+\Sigma_{f_i}+\sum_{j\neq i}\mu_i{}^j\Sigma_{e_i})\phi_i + \sum_{j\neq i}\mu_j{}^i\Sigma_{e_j}\phi_j+F_i\sum_j\Sigma_{f_j}\phi_j=0$$

where $\mu_i{}^j$ is the fraction of neutrons transferred from group i to j in one collision. A third-order differencing technique is employed. The moderator is represented as an ideal gas for the calculation of scattering and transfer coefficients in the thermal range, giving a mechanism whereby neutrons can gain as well as lose energy. A total of 90 energy groups can be employed. In this study 18 energy groups were used.

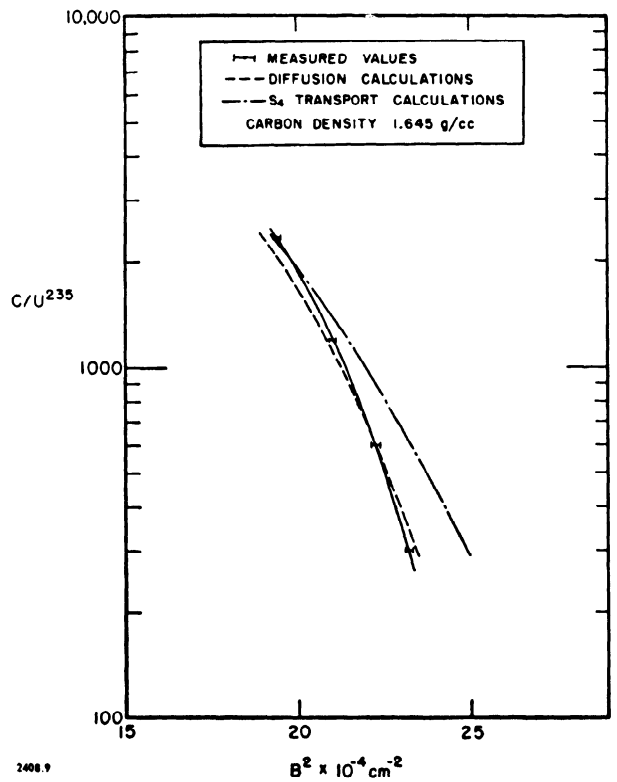


Figure 9. Carbon-to-uranium ratio vs. buckling for homogeneous bare systems

Angie

Angie is a two-dimensional IBM 704 code for plane and cylindrical geometry.[4] The differenced diffusion equation is solved by means of the Douglas iterative technique.[5] A maximum of 2000 mesh points and 18 energy groups is allowed. Scattering down a maximum of three energy groups or up one energy group is permitted per collision. This code has been used for all the two-dimensional systems. The constants used are the same as those in Zoom.

S_4

The S_4 calculation developed by Carlson,[6] is an approximate method for solving the transport equation. The angular space is divided into four segments. The angular flux in each segment is assumed to be linear. By an iterative procedure the angular fluxes are determined for all segments. At present the S_4 calculation can handle one-dimensional problems. The cross sections are based on those used in the diffusion codes so that a comparison between transport and diffusion calculations could be made.

Constants

The constants are listed in Table 5. In general they are obtained from BNL-325.[7] The transport cross sections were obtained from the absorption and elastic cross sections by means of the transport theory approximation, $\Sigma_{tr}=(\Sigma_s+\frac{1}{3}\Sigma_a)(1-\bar{\mu}_0)$.[8] The transport correction has been applied individually to each element although it should apply to the overall mixture. The error in adding the corrected transport cross sections linearly has been found to be less than 1% in the cross section. Only the absorption was considered in U^{238}. Although the U^{238} cross sections will change according to the dilution in the moderator due to resonance absorption, they were considered constant, since the effect of U^{238} is small in any case. The boron and aluminum were considered only as absorbing media. Inelastic events and slowing down in uranium were neglected. The production of neutrons in (n, 2n) events in Be is represented by negative absorption cross sections and appropriate changes in the transfer coefficients and elastic scattering cross sections. The transfer coefficients in the thermal range are obtained by assuming the moderator to be an ideal gas at room temperature and the flux distribution to be Maxwellian. The transfer coefficients in the higher groups are obtained from the logarithmic energy decrement and a flux proportional to the reciprocal of the energy. However, in the case of carbon the transfer coefficients in groups 17 and 18 were obtained from typical flux distributions in the carbon moderator. The fluxes were approximately flat in group 17 and proportional to E^{-4} in group 18.

It is not claimed that these are the best of all possible constants. Their usefulness can be determined from the following comparison with experiments.

Table 5. 18 Group Constants

Group no.	Upper energy, ev	U^{235} $\sigma_{elastic}$	$\sigma_{capture}$	$\sigma_{fission}$	$\sigma_{transport}$	Fission neutron distribution	U^{238} $\sigma_{capture}$	C Carbon + boron $\sigma_{elastic}$	$\sigma_{capture}$	$\sigma_{transport}$	Be $\sigma_{elastic}$	$\sigma_{capture}$	$\sigma_{transport}$	Al $\sigma_{capture}$	Transfer coefficients Carbon μ_i^i	μ_i^{i-1}	μ_i^{i+1}	Be μ_i^i	μ_i^{i-1}	μ_i^{i-2}	μ_i^{i+1}
1	$10^{-3/2}$	8.20	101	549	138	—	2.75	4.90	0.013	4.90	6.01	0.013	5.57	0.230	0.7637		0.2363	0.7300		0	0.2700
2	10^{-1}	9.50	62.8	344	91.0	—	2.50	4.72	0.007	4.46	6.00	0.007	5.57	0.129	0.8444	0.1556	0	0.8222	0.1778		0
3	$10^{-1/2}$	11.0	34.7	183	54.7	—	2.20	4.72	0.004	4.46		0.004	5.56	0.073	0.8629	0.1371		0.8205	0.1795		
4	1	12.5	17.0	82.9	32.5	—	2.30	4.72	0.002	4.46		0.002		0.041							
5	$10^{1/2}$	13.2	6.70	28.5	20.2	—	1.70	4.72	0.001	4.46		0.001		0.023							
6	10	13.8	13.3	44.7	25.4	—	117.2	4.63	0.001	4.37		0.001		0.013							
7	$10^{3/2}$	14.0	19.9	48.9	27.8	—	54.7	4.62	0	4.36		0		0.007							
8	10^{2}	14.0	17.8	39.4	25.4	—	49.6							0.004							
9	$10^{5/2}$	14.0	9.24	18.8	19.6	—	15.4							0.002							
10	10^{3}	13.4	5.25	10.5	16.6	—	1.70							0.001							
11	$10^{7/2}$	12.3	2.83	5.90	14.0	—	0.80							0							
12	10^{4}	9.89	1.68	3.99	10.0	—	0.80														
13	$10^{9/2}$	7.66	1.08	3.26	8.53	—	0.60	4.62		4.36	6.00		5.56								
14	10^{5}	6.20	0.584	2.34	6.78	0.033	0.40	4.57		4.32	5.90		5.46								
15	$10^{11/2}$	4.70	0.316	1.69	5.10	0.147	0.22	4.18		3.95	4.87		4.51								
16	10^{6}	3.80	0.184	1.30	4.10	0.591	0.15	3.25		3.07	3.94	0	3.65		0.8629	0.1371		0.8205	0.1795		
17	$10^{13/2}$	3.00	0.129	1.21	3.27	1.261	0.06	2.09		1.97	2.43	−0.002	1.93		0.9120	0.0880		0.8028	0.1972	0	
18	10^{7}	2.50	0.114	1.37	2.80	0.478	0.02	1.49		1.41	2.59	−0.474	1.12		0.6370	0.3630		0.4354	0.3566	0.2080	

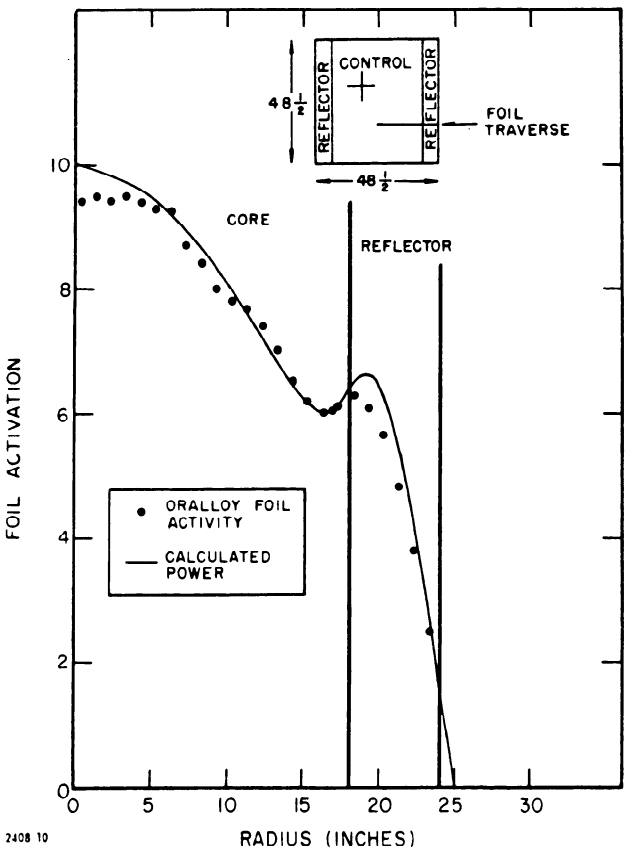


Figure 10. Oralloy foil activation for carbon-reflected system U5

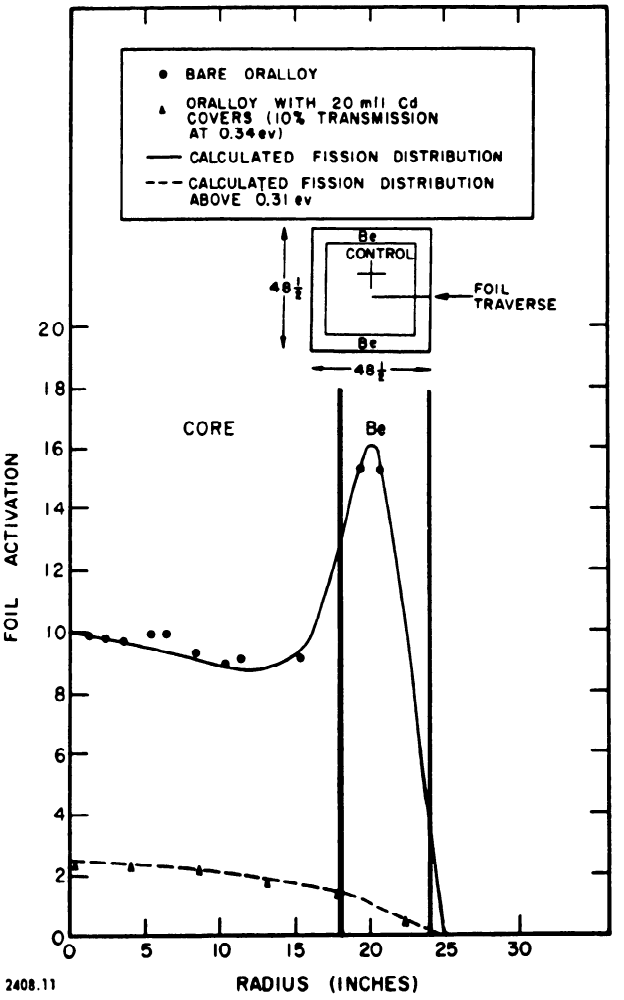


Figure 11. Oralloy foil activation and Cd difference for Be-reflected system U9

Results

The results of the diffusion calculations for the UCRL experiments are given in Table 6. The results of the transport calculations are also shown for the one-dimensional systems. In the case of the bare systems the bucklings are calculated. For the reflected systems the value of the calculated reactivity is given. For purposes of orientation, the relation between reactivity and critical mass is approximately:

$$\frac{\partial k}{k} = \frac{1}{12}\frac{\partial M}{M}$$

for these systems.

For the bare systems the diffusion and transport calculations differ for the small, low C/U systems but do converge as the systems get larger, as would be expected. The constants that have been used are apparently more appropriate for the diffusion calculations. It is of interest that the angular distribution of the neutron velocity vectors leads to effects of reasonable magnitude for systems of this size. Both the diffusion and transport calculations for the reflected systems are in good agreement with the data. The diffusion and transport calculation should

Table 6. Results of Calculations for UCRL System[a]

Expt. no.	*C* U^{235}	*Measured critical buckling, cm⁻² ± 0.1*	*Calculated critical buckling, S_4 cm⁻²*	*Calculated critical buckling Zoom, cm⁻²*	*Calculated reactivity Zoom or Angie*	*Calculated reactivity S_4*
U1	301	23.07×10^{-4}	24.8×10^{-4}	23.3×10^{-4}	—	—
U2	603	22.26×10^{-4}	23.3×10^{-4}	22.2×10^{-4}	—	—
U3	1206	21.01×10^{-4}	21.4×10^{-4}	20.8×10^{-4}	—	—
U4	2355	19.42×10^{-4}	19.3×10^{-4}	19.0×10^{-4}	—	—
U5	1210	—	—	—	1.006	0.999
U6	1210	—	—	—	0.995	—
U7	2360	—	—	—	0.993	0.988
U8	2360	—	—	—	0.994	1.002
U9	2360	—	—	—	0.989	—

[a] See Tables 1 and 5 for further description of the systems.

agree for these systems because the C/U is above 1200 where the methods are converging and the reflected systems are not as sensitive to differences in the external boundaries. The carbon-to-U^{235} ratio is plotted vs the buckling (B^2) in Fig. 9 for the experiments and calculations. The activation of oralloy foils for experiments U5 and U9 are shown in Figs. 10 and 11. The results of the diffusion calculations are presented for comparision. For the Be-reflected system, Fig. 11, the results are also shown for oralloy foils covered with 20-mil Cd. The cut-off energy for the Cd (10% transmission) is 0.34 ev.

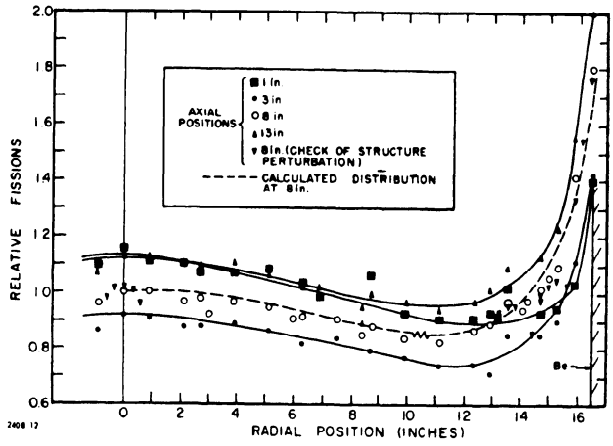
The results for the two-dimensional calculations for the cylinders are shown in Table 7. The agreement is not as good as obtained in the above systems. However, it should be remembered that greater approximations are involved in reducing the data to calculable systems for these experiments. In particular the effect of the corners and edges for these assemblies can be as large as 5% in reactivity. Approximately the same ratio between reactivity and critical mass applies to these systems as given above. System L5 is one dimensional and has also been calculated by the S_4 method. The reactivity obtained is less than that given by the diffusion method.

Radial fission distributions for Assembly L3 are given in Fig. 12. The calculated fission distribution 8 in. from the core end is also given. The radial distribution has been measured where the core and reflector are $\frac{1}{2}$ in. larger than the average as used in the calculation. Therefore $\frac{1}{2}$ in. has been added to the radii for the calculated system beyond 10 in. in radius. This was done to place the core-reflector interface for the calculated system at the same position as that for the measured system. The lost $\frac{1}{2}$ in. is represented on the plot by the jagged line.

Since the graphite-reflected parallelepipeds given in Table 4 are three-dimensional systems, they require reduction to simpler systems before the multigroup calculations may be used. These systems may be reduced to reflected spheres by means of constant buckling relations and then the multigroup calculations applied. Since the results obtained are more a matter of the ingenuity applied in the reduction than the accuracy of the multigroup calculation, they will not be given here.

CONCLUSIONS

The results of the diffusion calculations of the critical mass of simple, enriched-uranium, graphite-moderated systems, both with and without reflectors, are within about 10% of experimental values. This can be considered to be excellent agreement in this region of buckling. As indicated previously, 10% in critical mass is about 1% in reactivity. More precise calculations would involve many improvements. The results are very sensitive to the constants for the energy groups from 1 to 10 Mev. This region should be covered by more groups with more attention given to the angular distribution of scattering. At the other end of the energy scale, crystalline effects



[Figure 12. Radial oralloy foil activation for Assembly L3

should be taken into account. Transfer coefficients could be improved by iterations upon the computed energy spectrum of the flux. Other improvements may be easily envisioned. Transport calculations with more carefully chosen constants would presumably also improve the agreement.

In the case of more practical systems such as the pseudo-cylinders, the application of straightforward methods of calculation is less successful. In such cases the ingenuity of the calculator must be applied to estimate the effects of the perturbations applied to the ideal systems to make them approximate actual reactor systems. In such cases critical experiments will always be a necessity.

Table 7. Results of Calculations for LASL Cylinders [a]

Experiment no.	*C/U²³⁵*	*Calculated reactivity Angie*
L1	124	1.024
L2	1020	1.021
L3	412	1.059
L4	412	1.050
L5	412	1.045
		1.003 (transport)
L6	380	1.072

[a] See Table 4 for further description of the systems.

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