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## Critical Mass Irregularity of Steel-Moderated Enriched Uranium Metal Assemblies with Composite Steel-Oil Reflectors

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Critical masses have been determined by experiment and by calculation for enriched-uranium-metal spherical shells moderated internally with a sphere of mild steel of radius 8.01 cm. The shells were reflected with various thicknesses of mild steel followed by an effectively infinite amount of oil. The points representing critical mass as a function of the thickness of the steel reflector are not related by a smooth curve. The irregularity appears to be most severe for a 3-cm-thick steel reflector and is due to the resonance in the neutron elasticscattering cross section of iron.

#### INTRODUCTION

Investigations were undertaken to determine the iron cross-section resonance effects on the critical mass of a reflected and moderated uranium-metal system. The investigations consisted of experiment and two types of calculations. The iron elastic-scattering cross sections cause the irregularity found in the curve obtained from measurements of critical mass as a function of steel thickness in a composite steel-oil reflector.

The system consisted of a spherical region of enriched-uranium metal moderated by an internal mild steel sphere having a radius of 8.01 cm. The assemblies were reflected with various thicknesses of mild steel followed by an effectively infinite thickness of oil. The thickness of the steel reflector was varied from 0.00 to 5.00 cm.

The experimentally determined data are reported along with a brief description of the equipment, components, and procedures. A more detailed description, with less current data, was reported earlier.<sup>1</sup> The experimental and calculational results show that the critical mass is not a smooth function of steel-reflector thickness. An explanation of the cause of this variation is given.

#### EXPERIMENT

The experimental equipment consisted of the fissile assembly, a tank to contain the reflector oil, an oil reservoir, the associated pump, valves, plumbing, and the necessary instrumentation.

A small neutron source was used for all subcritical measurements. Large electrically operated valves connected to the bottom of the reflector tank served as a scram mechanism. The mount for the fissile assembly consisted of an aluminum cylinder 13.6 cm in diameter with a 0.48-cm-thick wall. To fasten the fissile assembly in place, a 0.70-cm-diam mild steel bolt was passed through the assembly and screwed into the mount (Fig. 1).

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<sup>&</sup>lt;sup>1</sup>D. C. COONFIELD, G. TUCK, H. E. CLARK, and B. B. ERNST, "Critical Masses of Steel-Moderated, Enriched Uranium Metal Assemblies with Composite Steel-Oil Reflectors," RFP-1033, The Dow Chemical Company, Rocky Flats Division (1967).



Fig. 1. Aluminum mount and spherical assembly.

As shown in Fig. 2, the experimental assemblies were built of concentric enriched uranium and mild steel shells. Each 0.328-cm-thick shell (steel or enriched uranium) had a 0.714-cmdiam mounting hole at the pole and four 0.317-cmdiam holes near the equator for disassembly tools.

For all of these measurements, the radius of the inner mild steel sphere was 8.01 cm, and the outside uranium region was varied as required to produce the desired experimental mass. The reflector steel thickness was held to a desired value.

Machining tolerance gaps, averaging 0.010 cm between the concentric shells, were filled with petroleum jelly during the assembly. This was a means of having a reasonably uniform amount of hydrogenous material in the assembly throughout the measurements. The specifications for the experimental assembly are given in Table I.

The experimental assembly and its mount were placed in the test tank and incremental amounts of reflector oil were added by remote operation. A test ended when either the assembly reached criticality or was subcritical when surrounded on all sides with at least 19 cm of oil. This provided a composite reflector of steel followed by an essentially infinite amount of oil.

Each subcritical test provided a single inverse multiplication value for a particular fully reflected experimental mass. A series of tests using the same steel reflector thickness and uranium masses up to the largest available subcritical mass provided data which was extrapolated to the fully reflected critical mass. Usually, the value of the extrapolated critical mass was between the masses of the experimental as-



Fig. 2. Typical experimental assembly.

semblies that could be built from the available shells. The largest available subcritical mass provided an absolute lower limit for the critical mass measurement. A test was conducted on the next larger available experimental assembly to determine an absolute upper limit. This assembly was critical when the reflector oil in the tank was <19 cm above the assembly, i.e., less than fully reflected. For the measurements with 4.00- and 5.00-cm-thick steel regions, the neutron multiplication limits on manual assembly did not permit an upper limit to be obtained.

The results of these measurements are given in Table II and in Fig. 3. The masses reported are for the fissile assembly with the conditions of the experiment, i.e., there were no corrections made for mounting equipment or the jelly in the gaps. The experimental masses used to determine the absolute lower and upper limits and the oil height at which the upper limit mass was critical are included in Table II.

In arriving at the accuracy of the data stated in Table II, consideration was given to the accuracy of extrapolation of the inverse multiplication data, Specifications for the Experimental Assembly

```
REGION I. Center Moderator
 Mild steel shells (SAE 1018)
   Impurities, wt%
                   0.15 - 0.20
0.60 - 0.91
    Carbon
    Manganese
    Phosphorus
                    0.040 maximum
                    0.050 maximum
    Sulphur
   Individual component density = 7.86 \text{ g/cm}^3
   Average density over region = 7.62 \text{ g/cm}^3
 Petroleum jelly
   Trade name - Petrolatum
   Composition. wt%
    Carbon
                    85
    Hydrogen
                    14.8
   Impurities, ppm
    Aluminum
                    20
    Calcium
                     7
    Copper
                    23
    Iron
                     5
    All others
                    < 5
                                 = 0.816 \text{ g/cm}^3
   Material density
   Average density over region = 0.024 \text{ g/cm}^3
REGION II, Fuel Region
 Enriched uranium metal
   Isotopic content, wt%
       Ū- 1.00
    <sup>235</sup>U - 93.19
    <sup>236</sup>U - 0.40
    <sup>238</sup>U - 5.41
   Individual component density = 18.76 \pm 0.06 \text{ g/cm}^3
   Average density over region = 18.15 \pm 0.05 \text{ g/cm}^3
REGION III, Steel Reflector
 Same as Region I
REGION IV. Oil Reflector
 Trade name - Texaco 522
 Composition, wt%
                    86.8
    Carbon
                    13.1
    Hydrogen
    Sulphur
                     0.2
 Impurities, ppm
                    <1
                                 Iodine
                                                <2
    Barium
                                 Manganese
                                                <2
    Calcium
                    <1
                                 Nickel
                                                <8
    Magnesium
                    <1
                                 Tin
                                                 < 8
    Silicon
                    <1
                                 Vanadium
                                                 <2
     Aluminum
                    <8
                                                <8
     Chromium
                    <8
                                 Boron
    Copper
                    <2
ALUMINUM MOUNT
     Aluminum (6061-T6)
```

TABLE II

| Experimental Data For Steel-Moderated Enriched |
|--|
| Uranium Metal Assemblies With Composite        |
| Steel-Oil Reflectors                           |

| Reflector<br>Steel<br>Thickness<br>(cm)                   | Extrapolated<br>Critical Mass of<br>Fully Reflected<br>Experimental<br>Assembly<br>(kg)  | Lower <sup>®</sup><br>Limit<br>(kg)                                  | Upper <sup>b</sup><br>Limit<br>(kg)                                  | Oil<br>Height <sup>c</sup><br>at<br>Critical<br>(cm)         |
|---|--|--|--|--|
| 0<br>0.67<br>1.33<br>2.00<br>2.33<br>2.67<br>3.00<br>3.33 | $51.8 \pm 1.5$<br>$59.9 \pm 1.5$<br>$65.9 \pm 1.0$<br>$70.5 \pm 1.0$<br>$70.7 \pm 1.5$<br>$72.4 \pm 1.5$<br>$71.0 \pm 1.5^{d}$<br>$72.9 \pm 1.5$ | 49.00<br>57.68<br>62.14<br>66.87<br>66.87<br>71.60<br>66.87<br>71.60 | 53.22<br>62.14<br>66.87<br>71.60<br>76.61<br>76.61<br>71.60<br>76.61 | +4.0<br>+1.2<br>-3.0<br>+1.5<br>-5.5<br>-5.5<br>-1.1<br>-5.0 |
| 4.00<br>5.00  | $72.7 \pm 1.5$<br>$72.5 \pm 1.5$   | 71.60<br>71.60   |  |  |

<sup>a</sup>This is the largest subcritical uranium mass tested.

<sup>b</sup>This uranium mass was critical when the assembly was partially reflected with oil.

<sup>c</sup>This is the oil height at which the upper limit mass was critical. + is above the top of the steel reflector

is below the top of the steel reflector

<sup>d</sup>Average of two measurements.

possible variation of the amount of jelly in the gaps, the repeatability of the measurements, and the absolute limits. The results of two criticalmass measurements on the same configuration gave values differing by 0.4 kg. This was typical of a number of repeated measurements of this type.<sup>2</sup> Independent extrapolations, of the same inverse multiplication data to a critical mass, differed by at most 0.3 kg, which was the extrapolation accuracy. The spread of the repeated measurements plus the extrapolation accuracy was taken as the relative error (±0.7 kg) between two adjacent data points. The error bars shown in Fig. 3 are the relative errors for the experimental data.

The three data points for the 2.67-, 3.00-, and 3.33-cm steel region thicknesses do not fall on a smooth curve. In Table II, note that the 71.60-kg mass was subcritical when fully oil-reflected with 2.67- and 3.33-cm-thick steel reflector and yet critical with the oil below the top of the 3.00-cmthick steel reflector. This irregularity was further investigated with calculational techniques.

#### CALCULATIONS

An attempt to calculate the steel-moderated and reflected assembly critical masses with a

<sup>&</sup>lt;sup>2</sup>G. TUCK, "Enriched Uranium-Metal Measurements, No. 1," RFP-907, The Dow Chemical Company, Rocky Flats Division (1967).



Fig. 3. Critical mass vs thickness of steel in a composite reflector consisting of a steel region followed by an effectively infinite amount of oil.

one-dimensional, multigroup transport code did not confirm the irregularity noted. The critical masses were then calculated and the irregularity verified by the use of a more sophisticated Monte Carlo neutron-transport code.

#### The DTF Calculations

The computer code  $DTF^3$  was used to calculate the critical thicknesses of the fissile material contained in the experimental assemblies described above.

The DTF code is a later development of Carlson's  $S_n$  method<sup>4</sup> which approximates, by difference techniques, a solution to the Boltzmann transport equation. In this approximation to the equation, the energy range from 0 to  $10^7 \text{ eV}$  is divided into 16 energy groups. A cross section is considered constant across each energy group and is illustrated by the macroscopic elastic-scattering cross sections plotted in Fig. 4. Hansen and Roach 16-group neutron cross sections<sup>5</sup> and

the  $S_4$  approximation were used in all cases that are reported here. Although some additional, more accurate calculations ( $S_8$  and  $S_{16}$ ) were done, they failed to show the irregularity.

As input to DTF, the mild steel moderator and reflector were simulated by iron (Fe) with a density of  $7.62 \text{ g/cm}^3$ . In the steel and fissile



Fig. 4. The DTF macroscopic-elastic-scattering cross sections for the iron and petroleum jelly.

<sup>&</sup>lt;sup>3</sup>B. G. CARLSON, W. J. WORLTON, W. GRUBER, and M. SHAPIRO, "DTF Users Manual," UNC Physics/ Math 3321, United Nuclear Corporation, Vol. I (1963), and Vol. II (1964).

<sup>&</sup>lt;sup>4</sup>B. G. CARLSON, in *Methods in Computational Physics, Statistical Physics*, Vol. I, pp. 1-42, BERNI ALDER et al., Eds., Academic Press, New York (1963).

<sup>&</sup>lt;sup>5</sup>G. E. HANSEN and W. H. ROACH, "Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies," LAMS-2543, Los Alamos Scientific Laboratory (1960).

material regions, petroleum jelly was homogenized with the major materials. The oil reflector had a hydrogen-to-carbon atomic ratio of 1.82 and a density of  $0.88 \text{ g/cm}^3$ . The H/C ratio of the calculations is based on information supplied by the oil company. The H/C ratio of the experimental data is based on laboratory analysis. The slight difference in the two is insignificant in terms of calculated critical mass. Since the effect of the petroleum jelly was small, the same H/C ratio was applied to both the oil and the petroleum jelly. The mount was not considered. Table III lists by region, from inner to outer, the materials, densities, and nuclear densities that were utilized in DTF.

A convergence criterion of 0.001 was applied in all problems. Uniform fission densities of 1.0 n/  $cm^3$  for each spatial interval were used as input to all problems. Table IV lists the regional dimensions, calculated radii, and number of spatial intervals of all DTF calculations.

Table V lists the steel reflector thicknesses, and the uranium critical masses calculated by

> TABLE III Regions, Materials, Densities, and Nuclear Densities Used in DTF

| Region | Material  | Density<br>(g/cm³) | Nuclear Density<br>(10 <sup>24</sup> atoms/cm <sup>3</sup> ) |  |  |
|--------|---|--------------------|--|--|--|
| I      | Iron (Fe) <sup>a</sup>  | 7.62               | $N_{\rm Fe} = 0.082261$                                      |  |  |
|        | Petroleum jelly<br>$\left(\frac{hydrogen}{carbon} = 1.82\right)^{b}$            | 0.024              | N <sub>C</sub> = 0.0010318<br>N <sub>H</sub> = 0.0018758     |  |  |
| II     | Uranium (93.12%<br><sup>235</sup> U and 6.88%<br><sup>238</sup> U) <sup>c</sup> | 18.1               | $N_{235U} = 0.0432005$<br>$N_{238U} = 0.0031517$             |  |  |
|        | Petroleum jelly<br>$\left(\frac{hydrogen}{carbon} = 1.82\right)^{b}$            | 0.024              | N <sub>C</sub> = 0.0010318<br>N <sub>H</sub> = 0.0018758     |  |  |
| ш      | Iron (Fe)   | 7.62               | $N_{\rm Fe} = 0.082261$                                      |  |  |
|        | Petroleum jelly<br>$\left(\frac{hydrogen}{carbon} = 1.82\right)$                | 0.024              | $N_{\rm C}$ = 0.0010318<br>$N_{\rm H}$ = 0.0018758           |  |  |
| IV     | Oil<br>$\left(\frac{\text{hydrogen}}{\text{carbon}} = 1.82\right)^{\text{b}}$   | 0.88               | $N_{\rm C}$ = 0.0383558<br>$N_{\rm H}$ = 0.0697344           |  |  |

\*Since the impurities of Table I amount to only 1.2 wt% of the mild steel, they were ignored.

<sup>b</sup>The H/C ratio applied in the calculations is based on information supplied by the oil company. The slight difference in the H/C ratio which was from laboratory analysis is insignificant in terms of calculated critical mass. Since the effect of the petroleum jelly was small, the same H/C ratio was used in both the oil and the petroleum ielly.

the petroleum jelly. "The <sup>234</sup>U, <sup>236</sup>U, and <sup>238</sup>U were lumped. The <sup>234</sup>U and <sup>236</sup>U probably should have been lumped with the <sup>235</sup>U, but the results of the calculations would have all shifted in the same ratio in the same direction.

| TABLE IV                   |     |                   |             |  |
|----------------------------|-----|-------------------|-------------|--|
| <b>Regional Dimensions</b> | and | Spatial Intervals | Used in DTF |  |

|                                      |                  | -                       |                         |                                   |
|--------------------------------------|------------------|-------------------------|-------------------------|-----------------------------------|
| Steel-Reflector<br>Thickness<br>(cm) | Region<br>Number | Inner<br>Radius<br>(cm) | Outer<br>Radius<br>(cm) | Number of<br>Spatial<br>Intervals |
| 0.000                                | I                | 0.000<br>5.000          | 5.000<br>8.000          | 12<br>16                          |
|                                      | II<br>III        | 8.000                   | 10.524                  | 15<br>                            |
|                                      | IV               | 10.524<br>13.524        | 13.524<br>37.274        | 13<br>19                          |
| 0.667                                | I                | 0.000                   | 5.000                   | 11<br>14                          |
|                                      | II               | 8.000                   | 10.874                  | 15                                |
|                                      | ĪV               | 11.541                  | 14.541                  | 12<br>21                          |
| 1.000                                | I                | 0.000                   | 5.000                   | 11                                |
|                                      | II               | 8.000                   | 11.002                  | 14                                |
|                                      | III<br>IV        | 12.002                  | 12.002                  | 3<br>12                           |
| 1 999                                | Ţ                | 15.002                  | 41.002                  | 21                                |
| 1.555                                |                  | 5.000                   | 8.000                   | 14                                |
|                                      |                  | 8.000                   | 11.096                  | 15<br>4                           |
|                                      | IV               | 12.429<br>15.429        | 15.429<br>41.096        | 12<br>19                          |
| 1.667                                | I                | 0.000                   | 5.000                   | 11                                |
|                                      | II               | 8.000                   | 11.162                  | 15                                |
|                                      | IV               | 12.495                  | 12.495                  | 13                                |
| 2.000                                | I                | 15.829<br>0.000         | 41.162<br>5.000         | 18<br>11                          |
| 2.000                                | T                | 5.000                   | 8.000                   | 14                                |
|                                      | III              | 11.204                  | 13.204                  | 6                                 |
|                                      | IV               | 13.204<br>16.204        | 16.204 41.204           | 12<br>17                          |
| 2.333                                | I                | 0.000<br>5.000          | 5.000<br>8.000          | 11<br>14                          |
|                                      | II               | 8.000                   | 11.228                  | 15<br>7                           |
|                                      | īv               | 13.561                  | 16.561                  | 12<br>16                          |
| 2.667                                | I                | 0.000                   | 5.000                   | 10                                |
|                                      | п                | 5.000<br>8.000          | 8.000<br>11.238         | 14<br>15                          |
|                                      | III              | 11.238                  | 13.905                  | 8                                 |
|                                      |                  | 16.905                  | 41.238                  | 15                                |
| 3.000                                | I                | 0.000<br>5.000          | 5.000<br>8.000          | 11<br>14                          |
|                                      |                  | 8.000                   | 11.239<br>14 239        | 15<br>9                           |
|                                      | īv               | 14.239<br>17.239        | 17.239<br>41.239        | 12<br>14                          |
| 3.333                                | I                | 0.000                   | 5.000                   | 11                                |
|                                      | п                | 5.000                   | 8.000<br>11.234         | 14<br>15                          |
|                                      |                  | 11.234<br>14.567        | 14.567<br>17.567        | 10<br>12                          |
| 4.600                                |                  | 17.567                  | 41.234                  | 13                                |
| 4.000                                | I                | 0.000<br>5.000          | 5.000<br>8.000          | 11<br>14                          |
|                                      |                  | 8.000<br>11.208         | 11.208                  | 15<br>10                          |
|                                      | IV               | 15.208<br>18.208        | 18.208<br>41.208        | 12<br>13                          |
| 5.000                                | I                | 0.000                   | 5.000                   | 11                                |
|                                      | п                | 5.000<br>8.000          | 8.000                   | 14                                |
|                                      | III<br>IV        | 11.150<br>16.150        | 16.150<br>19.150        | 10<br>12                          |
|                                      |                  | 19.150                  | 41.150                  | 13                                |

where

#### TABLE V

#### Critical Radii, Thicknesses, and Masses of Uranium Calculated by DTF with Various Steel Reflector Thicknesses

|   | Uranium                             |                               |                                     |  |
|---|-------------------------------------|-------------------------------|-------------------------------------|--|
| Steel<br>Reflector<br>Thickness<br>(cm) | Outer<br>Critical<br>Radius<br>(cm) | Critical<br>Thickness<br>(cm) | Critical Mass<br>of Uranium<br>(kg) |  |
| 0.000                                   | 10.524                              | 2.524                         | 48.300                              |  |
| 0.667                                   | 10.874                              | 2.874                         | 58.670                              |  |
| 1.000                                   | 11.002                              | 3.002                         | 62.149                              |  |
| 1.333                                   | 11.096                              | 3.096                         | 64.760                              |  |
| 1.667                                   | 11.162                              | 3.162                         | 66.618                              |  |
| 2.000                                   | 11.203                              | 3.203                         | 67.805                              |  |
| 2.333                                   | 11.228                              | 3.228                         | 68.488                              |  |
| 2.667                                   | 11.238                              | 3.238                         | 68.799                              |  |
| 3.000                                   | 11.239                              | 3.239                         | 68.826                              |  |
| 3.333                                   | 11.234                              | 3.234                         | 68.660                              |  |
| 4.000                                   | 11.208                              | 3.208                         | 67.914                              |  |
| 5.000                                   | 11.150                              | 3.150                         | 66.287                              |  |

DTF. The calculated critical masses are plotted as a function of steel reflector thicknesses in Fig. 3.

#### The Monte Carlo Calculations

The Oak Ridge Random Research Reactor Routine<sup>6</sup> computer code (O5R) was used as a second computational method for calculating the critical masses.

O5R allows a highly detailed neutron crosssection description as input. In the cross-section treatment, the energy range from  $1.5 \times 10^7$  to 0.1 eV is divided into 25 supergroups whose boundaries are successive powers of two in speed squared. Internally, O5R uses neutron speed squared, rather than the neutron energy. Each supergroup is divided into some power of two (0 to 6) equally wide subgroups. Cross-section data are constant over each subgroup. Three cross-section structures were applied in the investigation. In the first case, 64 subgroups per supergroup were utilized. In the second case, 1 subgroup per supergroup was used. Thirdly, 64 subgroups per supergroup were applied everywhere except between the energies of  $2.0 \times 10^4$  to  $1.0 \times 10^5$  eV in the inner and outer steel regions. Across this energy range, the scattering cross

section for iron from DTF was substituted. The total cross sections were adjusted accordingly.

All neutrons with energies below 0.1 eV were processed as a one-velocity thermal group. The neutrons were processed in batches of 200. Observation of the O5R output reported here has shown that the batch neutron multiplications are independent of source effects after the first two batches. For this reason, the first two batches were omitted in calculating the average neutron multiplication. The average neutron multiplication  $(\bar{k}_{eff})$  for each problem was calculated by forming the arithmetic average of the remaining batches; i.e.,

$$\overline{k}_{eff} = \frac{\sum_{i=3}^{n} k_{eff}^{i}}{n-2}$$

where n is the number of batches calculated for the problem, and  $k_{eff}^{i}$  is the neutron multiplication for the *i*'th batch.

The statistical error (i.e., the standard deviation,  $\sigma$ ) for each problem was calculated by

$$\sigma = \left(\frac{\beta - \overline{k}_{eff}^2}{n - 3}\right)^{1/2} ,$$

$$\beta = \frac{\sum_{i=3}^{n} (k_{eff}^{i})^2}{n-2}$$

Excluding the thermal group and the cases mentioned later, all O5R calculations were done with the Howerton<sup>7</sup> neutron cross-section data. For the one-velocity thermal treatment, the Hansen and Roach<sup>5</sup> thermal-group (group 16) data were utilized.

The model, materials, and densities of O5R were the same as those of DTF except that petroleum jelly was not homogenized in the fissile material. The reduced uranium density was used to compensate for the air gaps and holes in the shells. The mount was not considered in either O5R or DTF. For each problem, Table VI lists the steel-reflector thickness, fissile material thickness, and corresponding mass of uranium in Region II, the calculated  $\overline{k}_{eff}$ , the calculated standard deviations, and the number of batches for calculating  $\overline{k}_{eff}$ .

For each iron-reflector thickness, Table VII lists the calculated critical mass and the calculated error for the critical mass in terms of one standard deviation ( $\sigma$ ). Since O5R calculates only  $\overline{k}_{eff}$  and the corresponding  $\sigma$  for an input mass,

<sup>&</sup>lt;sup>6</sup>D. C. IRVING, R. M. FREESTONE, Jr., and F. B. K. KAM, "O5R, A General Purpose Monte Carlo Neutron Transport Code," ORNL-3622, Oak Ridge National Laboratory (1965).

<sup>&</sup>lt;sup>7</sup>NORMAN L. PRUVOST, Private Communication, Lawrence Radiation Laboratory (December 18, 1965).

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#### TABLE VI

Uranium Radii, Masses, Neutron Multiplication Factors, and Standard Deviations From O5R

| Cross-Section<br>Structure | Steel Reflector<br>Thickness<br>(cm) | Uranium Thickness<br>(cm) | Uranium Mass<br>(kg) | <i>k</i> eff | σ     | Number of<br>Batches |
|----------------------------|--------------------------------------|---------------------------|----------------------|--------------|-------|----------------------|
| 64 subgroups per           | 0.000                                | 2.580                     | 51.369               | 0.991        | 0.010 | 9                    |
| supergroup                 | 0.000                                | 2.600                     | 51.591               | 1.011        | 0.007 | 8                    |
|                            | 1.333                                | 3.132                     | 65.902               | 0.994        | 0.012 | 7                    |
|                            | 1.333                                | 3.142                     | 66.190               | 1.012        | 0.011 | 8                    |
|                            | 2.667                                | 3.350                     | 72.184               | 0.998        | 0.008 | 6                    |
|                            | 2.667                                | 3.357                     | 72.390               | 1.003        | 0.008 | 8                    |
|                            | 3.000                                | 3.304                     | 70.840               | 0.995        | 0.010 | 6                    |
|                            | 3.000                                | 3.310                     | 71.013               | 1.002        | 0.013 | 7                    |
|                            | 3.333                                | 3.360                     | 72.478               | 0.996        | 0.012 | 8                    |
|                            | 3.333                                | 3.367                     | 72.698               | 1.002        | 0.015 | 6                    |
|                            | 4.000                                | 3.360                     | 72.482               | 0.997        | 0.009 | 8                    |
|                            | 4.000                                | 3.367                     | 72.676               | 1.003        | 0.010 | 8                    |
| 1 subgroup per             | 2.667                                | 3.190                     | 67.553               | 0.986        | 0.014 | 9                    |
| supergroup                 | 2.667                                | 3.200                     | 67.839               | 1.009        | 0.014 | 9                    |
|                            | 3.000                                | 3.210                     | 68.125               | 0.980        | 0.016 | 9                    |
|                            | 3.000                                | 3.230                     | 68.699               | 1.002        | 0.010 | 9                    |
|                            | 3.333                                | 3.210                     | 68.125               | 0.988        | 0.007 | 9                    |
|                            | 3.333                                | 3.220                     | 68.412               | 1.016        | 0.011 | 10                   |
| 64 subgroups per           | 2.667                                | 3.310                     | 71.013               | 0.989        | 0.011 | 10                   |
| supergroup<br>modified     | 2.667                                | 3.325                     | 71.306               | 1.000        | 0.008 | 9                    |

#### TABLE VII

Critical Uranium Masses and Errors Calculated from O5R Data

| Cross-<br>Section<br>Structure                     | Steel<br>Reflector<br>Thickness<br>(cm)            | Uranium<br>Critical<br>Mass<br>(kg)                      | Error<br>(kg                                       |
|--|--|--|--|
| 64<br>subgroups<br>per super-<br>group             | 0.000<br>1.333<br>2.667<br>3.000<br>3.333<br>4.000 | 51.471<br>66.004<br>72.257<br>70.959<br>72.631<br>72.578 | 0.092<br>0.184<br>0.299<br>0.277<br>0.483<br>0.337 |
| 1<br>subgroup<br>per super-<br>group               | 2.667<br>3.000<br>3.333                            | 67.727<br>68.647<br>68.105                               | 0.188<br>0.375<br>0.103                            |
| 64<br>subgroups<br>per super-<br>group<br>modified | 2.667  | 71.307   | 0.277  |

two  $\overline{k}_{eff}$ 's and  $\sigma$ 's were necessary to interpolate a critical mass. The assumption was made that  $\overline{k}_{eff}$  varies linearly with small changes in mass at

masses close to critical; i.e.,  $\overline{k}_{eff}$  close to 1.0. For an input mass  $(M_2)$ , O5R was used to calculate a  $\overline{k}_{eff}$   $(k_2)$  slightly above 1.0 and the corresponding  $\sigma_2$ . For a smaller mass  $(M_1)$ , O5R was used to calculate another  $\overline{k}_{eff}$   $(k_1)$  slightly below 1.0 and the corresponding  $\sigma_1$ . For the purpose of critical mass interpolation only, the calculated changes in  $\overline{k}_{eff}$  with respect to the changes in mass  $(\Delta k/\Delta M)$ were calculated.

The computed critical mass  $(M_c)$  in kgs was calculated by

$$M_c = M_2 - \frac{k_2 - 1.0}{\Delta k / \Delta M}$$

,

where

$$\frac{\Delta k}{\Delta M} = \frac{k_2 - k_1}{M_2 - M_1}$$

To determine the calculated critical-mass error, the two standard deviations involved were combined to calculate a pooled standard deviation. The variances were defined to be the squares of the standard deviations. Using the number of batches for each calculated  $\bar{k}_{eff}$  as weights, the two variances were combined into a single estimate. The pooled variance ( $\sigma^2$ ) was given by:

$$\sigma^{2} = \frac{(n_{1} - 1) \sigma_{1}^{2} + (n_{2} - 1)\sigma_{2}^{2}}{(n_{1} + n_{2} - 2)}$$

for  $n_1$  batches used to calculate  $k_1$  and  $n_2$  batches used to calculate  $k_2$ .

The calculated error in critical mass  $(\Delta M_c)$  for one standard deviation was calculated by

$$\Delta M_c = \frac{\sigma^2}{\Delta k / \Delta M}$$

Discussion

For comparison purposes, Fig. 3 shows the O5R, DTF, and experimental critical masses and errors.

The O5R (64 subgroups per supergroup) results show the irregularity at the same steel-reflector thicknesses as the experimental data. DTF and O5R with 1 subgroup/supergroup and 64 subgroups/supergroup modified did not show the irregularity.

Figures 4, 5, and 6 show a comparison of the O5R (64 subgroups per supergroup), the O5R (1 subgroup per supergroup), and the DTF iron elastic scattering cross sections, in  $\text{cm}^{-1}$ , as a function of energy, in eV, as they were applied in the codes.

Figures 4 and 5 show that the averaging techniques over the broad energy ranges in the group structure of DTF misses all the structure of the cross-section set. Figures 5 and 6 show that the 1 subgroup per supergroup cross sections miss much of the fine cross-section structure. On the other hand, the averaging of cross sections over much smaller energy increments as done in the



Fig. 5. The O5R (64 subgroups per supergroup) macroscopic-elastic-scattering cross sections for the iron and petroleum jelly.



Fig. 6. The O5R (1 subgroup per supergroup) macroscopic-elastic-scattering cross sections for the iron and petroleum jelly.

O5R (64 subgroups per supergroup) calculations reproduces much of the structure of the cross section.

#### SUMMARY AND CONCLUSIONS

Critical masses have been experimentally and calculationally determined for enriched uraniummetal spherical assemblies. These assemblies were moderated internally with a mild steel sphere and reflected by various thicknesses of mild steel followed by an effectively infinite amount of oil. As shown in Fig. 3, the critical masses are not a smooth function of steel reflector thickness.

The calculations were done by both the DTF and O5R computer codes. With the use of the input parameters described in this report, the critical masses calculated by DTF were within 8.6% of the experimental masses. The critical masses calculated by O5R using 64 subgroups per supergroup were within 0.7% of the experimental masses. The three O5R (1 subgroup per supergroup) calculated critical masses were within 6.5% of the experimental critical masses. The O5R (64 subgroups per supergroup modified) calculated critical mass was within 1.5% of the experimental mass. The DTF code results did not show the irregularity. It was shown by the O5R code using the higher resolution cross sections.

The error bars in Fig. 3 for the O5R data points are for one standard deviation. For the experimental points, the error bars represent the relative accuracy in measurement between adjacent points. It is highly unlikely that the two independent methods (experimental and calculational) for obtaining the critical masses would produce two sets of data points, each point of which is in error in the same direction and by approximately the same amount. Therefore, the structure shown by the curve in Fig. 3 probably exists.

In the attempt to find the cause of the irregularity, the resonance structure in the iron cross sections was investigated. The results of the one subgroup per supergroup O5R calculations show that the irregularity vanishes, and the O5R results approach those of DTF. The differences in the calculated (1 subgroup per supergroup) and experimental masses are explained by the importance of the cross-section structure, since the one subgroup per supergroup cross section, which removed much of the resonance structure, was insufficient to show the irregularity. By replacing the iron-scattering cross section over the  $2.0 \times 10^4$ to  $1.0 \times 10^5$  eV energy range with the DTF (group 6) scattering cross section, the O5R (64 subgroups per supergroup modified) calculated critical mass is 21% closer to the one subgroup per supergroup calculation. Such a large mass reduction with the cross section change over a small energy range implies that the curve irregularity is caused by the iron-elastic-scattering cross-section resonance structure.

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