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LOS ALAMOS SCIENTIFIC LABORATORY OF THE UNIVERSITY OF CALIFORNIA O LOS ALAMOS NEW MEXICO

REFLECTOR SAVINGS OF MODERATING MATERIALS ON LARGE DIAMETER U(93.2%) SLABS

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REFLECTOR SAVINGS OF MODERATING MATERIALS ON LARGE DIAMETER U(93.2%) SLABS

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ABSTRACT

This report covers critical mass measurements of 15.0" and 21.0" diameter U(93.2%) cylinders unreflected and reflected on one and/or two faces by carbon and the hydrogenous materials water, polyethylene, paraffin, and lucite.

I. Introduction

Neutron multiplications have been measured for 15.0" and 21.0" diameter enriched uranium (Oy) cylinders unreflected and reflected on one and two faces by carbon, water, and the often used water mockup materials - polyethylene, paraffin, and lucite. The critical mass data obtained from these measurements establish reflector savings at two small values of height to diameter ratio and thus guide the estimation of reflector savings for the infinite Oy slab.

On the Comet assembly machine, a 0.019" thick stainless steel diaphragm supported half of the Oy material and reflector while the remaining Oy and reflector were on a 10" high thin-walled aluminum pedestal atop the platen of the lift (Figure 1). The Oy material available consisted of thirty-one 15.0" diameter plates and thirtyone 15.0" ID to 21.0" OD annular rings, all 0.120" thick. The average stack density of the 15.0" diameter Oy cylinders was 17.9 ± 0.2 g/cm³ and that of the 21.0" diameter Oy cylinders was 18.2 ± 0.2 g/cm³. The water reflector was contained in an aluminum can with 1/16" wall.

A Po-Be source of 10^7 neutrons per second was present in all configurations and four BF3 counters in longcounter



UNREFLECTED





geometry monitored the neutron leakage from the assemblies.

A background count, recorded with all sources removed from the remote laboratory, was followed by an unmultiplied count with the source centered on the diaphragm in a small uranium holder. Safe numbers of Oy plates were stacked on the lift and diaphragm, the assembly closed remotely, and multiplied counts recorded. The neutron multiplication is reckoned as the ratio of multiplied to unmultiplied counts with background subtracted from both. A plot of reciprocal multiplication versus Oy mass guided the safe approach to the most reactive configurations (multiplication of ~ 100).

II. Critical Mass Data

Critical masses were determined through extrapolation of the observed reciprocal multiplication, 1/M, versus Oy mass curves to 1/M = 0. The negative reactivity worths of the stainless steel diaphragm were observed as changes in reciprocal multiplication produced by doubling the thickness of the diaphragm. The positive reactivity worths of aluminum support and platen to unreflected and top-face-reflected cylinders were determined from changes in reciprocal multiplication produced by mirror images of

these support components placed on top of the unreflected cylinders. If, during the tracing out of a reciprocal multiplication versus Oy mass curve, it appeared that the addition of another plate of Oy would produce a slightly supercritical configuration, a small air gap was introduced between the lower half of the Oy cylinder and diaphragm, the new multiplication observed, and the displaced curve continued for one more plate.

The neutron multiplication data are listed in Tables I, II, and III and illustrated for a few cases in Figure 2. The inferred critical mass values are listed in Tables IV and V.

The sources of uncertainty of the critical mass values lie in the extrapolations to zero reciprocal multiplication and in the physical specifications of the critical configurations. If one designates the reciprocal multiplication of an n plate assembly by $1/M_n$, it may be verified from Tables I-III that raggedness in the data first becomes apparent in the third differences, $1/M_{n-1} - 3/M_n + 3/M_{n+1} - 1/M_{n+2}$; a 100% uncertainty in third differences amounts, for extrapolations covering a distance of 1/2 Oy plate or less, to ~ \pm 0.15 kg and ~ \pm 0.3 kg Oy, respectively, for 15.0" and 21.0" diameter cylinders. Although the most reactive observed configurations were

TABLE I

RECIPROCAL MULTIPLICATION, 1/M, VERSUS OY MASS

FOR TOP REFLECTED 15" DIAMETER CYLINDERS

| Reflector/Oy Mass (kg) | 96.65 | 103.12 | 109.64 | 116.12 | |
|---------------------------------|--------|--------|-------------------|---------|--------|
| 3" Polyethylene | 0.168 | 0.102 | 0.0452 0.0612* | 0.0129* | |
| 4" Polyethylene | 0.159 | 0.0942 | 0.0383 0.0612* | 0.0134* | |
| 6" Polyethylene | 0.155 | 0.0917 | 0.0376 0.0564* | 0.0101* | |
| 8" Polyethylene | 0.155 | 0.0917 | 0.0370 0.0554* | 0.0091* | |
| 10" Polyethylene | 0.155 | 0.0907 | 0.0367 0.0547* | 0.0083* | |
| 6" Paraffin | 0.162 | 0.0963 | 0.0417 0.0539* | 0.0077* | |
| 6" Water + Al Tank | 0.166 | 0.104 | 0.0491 | 0.0040 | |
| 6" Lucite | 0.125 | 0.0680 | 0.0191 | | |
| 6" Lucite + Al Tank | 0.131 | 0.0732 | 0.0247 | | |
| 7" Graphite | 0.160 | 0.1087 | 0.0623 | 0.0232 | |
| 8" Graphite | 0.149 | 0.0992 | 0.0551 | 0.0181 | |
| 12" Graphite | 0.142 | 0.0961 | 0.0549 | 0.0191 | |
| 14" Graphite | 0.140 | 0.0945 | 0.0539 | 0.0185 | |
| Reflector/Oy Mass (kg) | 116.19 | 122.71 | 129.19 | 135.70 | 142.10 |
| l" Graphite | 0.213 | 0.147 | 0.0929 | 0.0463 | 0.0081 |
| 2" Graphite | 0.118 | 0.0652 | 0.0215 | | |
| l" Polyethylene | 0.152 | 0.0924 | 0.0425 0.0478* | 0.0060* | |
| 6" Polyethylene + 15 mils Cd | 0.159 | 0.099 | 0.049 | 0.0058 | |
| Reflector/Oy Mass (kg) | 103.05 | 109.52 | 116.04 | 122.51 | |
| 2" Polyethylene | 0.139 | 0.0797 | 0.0295 0.0624* | 0.0175* | |
| 6" Graphite | 0.119 | 0.0722 | 0.0310 0.0442* | 0.0074* | |
| Reflector/Oy Mass (kg) | 142.41 | 148.95 | 155.44 | 161.82 | |
| Bare | 0.139 | 0.0918 | 0.0507 | 0.0149 | |
| Bare + Double Diaphragm | | | | 0.0169 | |
| Bare + Support Image | | 0.0688 | 0.0336 0.0436* | 0.0138* | |

*Air gap introduced between diaphragm and lower half of Oy cylinder.

TABLE II

RECIPROCAL MULTIPLICATION, 1/M, VERSUS OY MASS

FOR TOP REFLECTED 21" DIAMETER CYLINDERS

| Reflector/Oy Mass (kg) | 163.27 | 175.84 | 188.47 | 201.13 | |
|------------------------------|--------|-------------------|-------------------|---------|---------|
| 2" Polyethylene | | 0,203 | 0.106 | 0,0286 | |
| 3" Polyethylene | 0.251 | 0.140 | 0.0518 0.0746* | 0.0036* | |
| 4" Polyethylene | | 0.131 | 0.0411 0.0778* | 0.0062* | |
| 6" Polyethylene | 0.240 | 0.126 | 0.0387 0.0784* | 0.0051* | |
| 8" Polyethylene | 0.238 | 0.126 | 0.0376 0.0766* | 0.0041* | |
| 10" Polyethylene | 0.239 | 0.125 | 0.0371 0.0833* | 0.0098* | |
| 6" Graphite | 0.200 | 0.119 | 0.0510 0.0673* | 0.0102* | |
| 7" Graphite | 0.178 | 0.102 | 0.0392 0.0652* | 0.0106* | |
| 8" Graphite | 0.163 | 0.0898 | 0.0296 0.0624* | 0.0099* | |
| 12" Graphite | 0.133 | 0.0685 | 0.0150 | | |
| 14" Graphite | 0.129 | 0.0654 | 0.0132 | | |
| 6" Paraffin | 0.242 | 0.131 | 0.0432 0.0939* | 0.0205* | |
| 6" Lucite | 0.169 | 0.0749 0.0875* | 0.0135* | | |
| 6" Lucite + Al Tank | 0.178 | 0.0838 0.1018* | 0.0268* | | |
| 6" Water + Al Tank | | 0.137 | 0.0492 0.0820* | 0.0101* | |
| Reflector/Oy Mass (kg) | 213.68 | 226.31 | 238.83 | 251.50 | |
| l" Graphite | 0.221 | 0.135 | 0.0653 0.0746* | 0.0168* | |
| 1" Polyethylene | 0.131 | 0.0580 0.0675* | 0.0080* | | |
| Reflector/Oy Mass (kg) | 201.13 | 213.78 | 226.45 | 239.02 | |
| 2" Graphite | 0.168 | 0.0885 | 0.0242 0.0618* | 0.0071* | |
| 6" Polyethylene + 15 mils Cd | 0.285 | 0.165 | 0.0709 0.1006* | 0.0267* | |
| Reflector/Oy Mass (kg) | 251.93 | 264.63 | 277.26 | 289.81 | |
| Bare | 0.179 | 0.117 | 0.0615 | 0.0147 | (294.2) |
| Bare + Double Diaphragm | | | | 0.0169 | (294.8) |
| Bare + Support Image | | 0.0712 | 0.0285 | | (286.8) |

*Air gap introduced between diaphragm and lower half of Oy cylinder.

TABLE III

RECIPROCAL MULTIPLICATION, 1/M, VERSUS OY MASS

FOR TOP AND BOTTOM REFLECTED CYLINDERS

A) 15" Diameter Cylinders.

| Reflector/Oy Mass (kg) | 64.30 | 70.72 | 77.19 | | |
|---------------------------|-----------|-----------|-------------------|-------------------|-----------------------|
| 2" Polyethylene | 0.158 | 0.0780 | 0.0136 | | |
| 6" Graphite | 0.137 | 0.0770 | 0.0272 | | |
| 7" Graphite | 0.112 | 0.0562 | 0.0093 | | |
| B) 21" Diameter Cylinder: | 5. | | | | |
| Reflector/Oy Mass (kg) | 75.24 | 87.84 | 100.44 | 113.10 | 125.68 |
| 2" Polyethylene | 0.588 | 0.400 | 0.239 | 0.1109 0.1348* | 0.0313* |
| 6" Graphite | 0.265 | 0.159 | 0.0692 0.0983* | 0.0218* | |
| 7" Graphite/Oy Mass (kg) | 0.219 | 0.122 | 0.0405 0.0923* | 0.0206* | |
| C) 21" Diameter Oy Cylind | der Fully | Reflected | with 2" Pol | yethylene | |
| Oy Mass (kg) | 75.43 | 88.12 | 100.65 | 113.29 | |
| 1/M | 0.251 | 0.164 | 0.0925 | 0.0345 0.0381 | (double diaphragm) |

*Air gap introduced between diaphragm and lower half of Oy cylinder.



Fig. 2. Typical reciprocal multiplication, 1/M, vs Oy mass curves. The dotted symbols, often appearing at 1/M < 0, represent values obtained through the artifice of introduced air gaps as illustrated by the two lucite reflector curves.

TABLE IV

CRITICAL MASSES OF TOP REFLECTED OY CYLINDERS

| | Oy Critical | Mass (kg) ^a |
|----------------------------------------------|---------------------|------------------------|
| Reflector | 15" O. D. | 21" O. D. |
| Bare | 166.5 ± 0.6 | 301.7 ± 0.8 |
| 1" Polyethylene (0.925 g/cm^3) | 137.4 ± 0.5 | 245.1 ± 0.6 |
| 2" Polyethylene (0.925 g/cm ³) | 121.8 ± 0.4 | 213.0 ± 0.6 |
| 3" Polyethylene (0.925 g/cm ³) | 117.2 ± 0.4 | 204.2 ± 0.6 |
| 4" Polyethylene (0.925 g/cm ³) | 116.3 ± 0.4 | 202.2 ± 0.6 |
| 6" Polyethylene (0.925 g/cm ³) | 116.5 ± 0.4 | 201.6 ± 0.6 |
| 8" Polyethylene (0.925 g/cm ³) | 116.3 ± 0.4 | 201.5 ± 0.6 |
| 10" Polyethylene (0.925 g/cm ³) | 116.3 ± 0.4 | 201.3 ± 0.6 |
| 1" Graphite (1.79, 1.73 g/cm ³) | 145.2 ± 0.5 | 260.0 ± 0.7 |
| 2" Graphite (1.79, 1.73 g/cm ³) | 134.4 ± 0.5 | 238.5 ± 0.6 |
| 6" Graphite (1.70, 1.76 g/cm ³) | 123.1 ± 0.4 | 206.3 ± 0.6 |
| 7" Graphite (1.71, 1.76 g/cm ³) | 122.1 ± 0.4 | 204.1 ± 0.6 |
| 8" Graphite (1.72, 1.75 g/cm ³) | 121.3 ± 0.4 | 202.1 ± 0.6 |
| 12" Graphite (1.70, 1.76 g/cm^3) | 121.5 ± 0.4 | 199.2 ± 0.6 |
| 14" Graphite (1.71, 1.76 g/cm ³) | 121.4 ± 0.4 | 198.8 ± 0.6 |
| 6" Lucite (1.18 g/cm ³) | 114.0 ± 0.4 | 195.4 ± 0.6 |
| 6" Paraffin (0.87 g/cm^3) | 117.0 ± 0.4 | 202.3 ± 0.6 |
| 6" Water (1.0 g/cm ³) | 117.5 ± 0.4^{b} | 202.3 ± 0.7^{b} |
| 6" Polyethylene + 15 mils Cd | 138.3 ± 0.5 | 245.1 ± 0.6 |

^aThe listed critical mass values are corrected for incidental reflection (2.0 kg and 7.4 kg, respectively, for 15" and 21" top-reflected cylinders; 2.2 kg and 8.2 kg for bare cylinders) and for the steel diaphragm (-0.4 kg and -0.7 kg, respectively, for the 15" and 21" diameter cylinders as determined by addition of second diaphragm).

^bCorrected for aluminum tank (-0.8 kg and -1.4 kg, respectively, for 15" and 21" cylinders as determined by lucite and lucite + tank critical mass difference).

TABLE V

CRITICAL MASSES OF TOP AND BOTTOM REFLECTED OY CYLINDERS

| | | Oy Critical Mass (kg) ^a | | | |
|----|-----------------------------------------|------------------------------------|------------------------------------|--|--|
| Re | flector | <u>15" O. D.</u> | <u>21" O. D.</u> | | |
| 2" | Polyethylene (0.925 g/cm ³) | 78.3 (73.2) ^b ±0.3 | 126.0 (121.1) ^b ±0.3 | | |
| 6" | Graphite (1.7 g/cm^3) | 80.8 ±0.3 | 111.1 ±0.3 | | |
| 7" | Graphite (1.7 g/cm^3) | 78.2 (55.8) ^b ±0.3 | 106.7 ±0.3 | | |

^aThe listed critical mass values are corrected for the steel diaphragm (-0.4 kg and -0.7 kg, respectively, for the 15" and 21" diameter cylinders).

^bThe parenthesized critical mass values apply to fully reflected cylinders: the values for the 15" diameter Oy cylinders fully reflected by 2" polyethylene and 7" graphite are those reported by G. E. Hansen, H. C. Paxton, and D. P. Wood, Nuclear Sci. and Eng. <u>8</u>, 570(1960). generally less than 1/2 Oy plate distant from critical, reruns of several assemblies indicated uncertainty in the reproducibility of critical mass values consistent with the above numbers. The 1.1% density uncertainty in the stacked Oy plates produces an uncertainty in the unreflected critical mass values of ~ 0.28% and ~ 0.15%, respectively, for 15.0" and 21.0" cylinders. (One may note the density scaling law for unreflected systems which states that critical mass varies inversely as the square of the density applies for scaling fixed shapes. At constant diameter, the mass or equivalently the mass per unit area of squat cylinders is much less sensitive to density changes becoming, of course, independent of density in the limit of zero height/diameter ratio.) Imprecision in ascertaining the reactivity contributions of incidental reflection produce uncertainty in the critical mass values listed in Table IV which apply to simple isolated systems; as implied by Figure 1. assembly support structures are the primary source of incidental reflection. The placement of mirror images of the support structures atop the unreflected assemblies reduced critical masses by 2.0 kg and 7.4 kg Oy, respectively, for the 15.0" and 21.0" diameter cylinders; the critical masses of the bottom-reflected assemblies (where incidental reflection of the

support structures was shielded out) exceeded those of the top reflected assemblies by 1.8 and 6.6 kg Oy. Although the results of these two experiments are nearly the same, the slightly lower values of the latter would imply incidental reflector savings from more remote objects above the diaphragm amounting to ~ 0.2 kg and ~ 0.8 kg Oy. Including the determination of the reactivity contribution of the stainless steel diaphragm, it is seen for example that five sets of neutron multiplication measurements are involved in establishing the critical mass value and associated probable error for an unreflected cylinder as listed in Table IV or V.

III. Reflector Savings

The reflector savings $\delta(T)$, defined as the decrease in a critical core dimension brought about by addition of reflector with thickness T to the normal bare surface, has often been assumed independent of geometry, in analogy to the relatively accurate end-point theory assumption that a bare extrapolation length is thus independent (thereby having a value given by solution of the half-infinitemedium problem). Although this simple assumption has no theoretical justification, save as a limit law for $T \longrightarrow 0$, early critical data covering a limited number of core

shapes and reflector materials did not seriously challenge its accuracy; its use together with end-point theory permitted quick inferences of critical sizes of bare and reflected spheres, infinite cylinders and slabs from corresponding critical data for a finite cylinder. However, as more critical data for extreme shapes and computations on one-dimensional geometries developed, it became apparent that the simple assumption could lead to gross error in converting critical data for one shape to "critical data" for a substantially different shape.

Departures from this assumption appear in Table VI, which gives reflector savings established by the critical mass data of Tables IV and V. Observed features are: 1) savings for two-face reflection are essentially the same (to ~ 1%) as those for one-face reflection, 2) sensitivity of reflector savings to Oy cylinder diameter is weak except for the thicker non-absorbing graphite reflectors, and 3) savings for the thicker graphite reflectors are governed by radial leakage through the graphite cylinder walls as indicated by saturation at thicknesses \geq cylinder radii and also by markedly larger values for full reflection with respect to one or two face reflection.

TABLE VI

| DFF | i Pr | TOD | C A U | TNCC |
|------|------|-----|-------|------|
| RGF. | | IUN | on v | TUOD |

n

| | Savings (kg Oy/in ²) | | | |
|-----------------------------|---------------------------------------|---------------|-------------|--|
| | | | ∞ OD | |
| Reflector* | <u>15" OD</u> | <u>21" OD</u> | (Estimated) | |
| | 0.305 | 0.104 | 0.100 | |
| 1" polyethylene | 0.165 | 0.164 | 0.166 | |
| 2" polyethylene | 0.253 | 0.257 | 0.261 | |
| 2" polyethylene (2 faces) | 0.250 | 0.254 | 0 260 | |
| 2" polyethylene (full) | 0.251 | 0.257 | 0.200 | |
| 3" polyethylene | 0.279 | 0.282 | 0.286 | |
| 4" polyethylene | 0.284 | 0.288 | 0.293 | |
| 6" polvethvlene | 0.283 | 0.290 | 0.296 | |
| 8" polvethylene | 0.284 | 0.290 | 0.298 | |
| 10" polvethylene | 0.284 | 0.291 | 0.299 | |
| 6" water | 0.277 | 0.288 | 0.297 | |
| 6" paraffin | 0.280 | 0.288 | 0.295 | |
| 6" lucite | 0.297 | 0.308 | 0.317 | |
| 0.015" Cd + 6" polyethylene | 0.159 | 0.164 | 0.169 | |
| l" graphite | 0.116 | 0.121 | 0.125 | |
| 2" graphite | 0.177 | 0.183 | 0.195 | |
| 6" graphite | 0.247 | 0.274 | 0.313 | |
| 6" graphite (2 faces) | 0.245 | 0.273 | 0.311 | |
| 7" graphite | 0.252 | 0.281 | 0.33 | |
| 7" graphite (2 faces) | 0 251 | 0 281 | 0100 | |
| 7" graphite (full) | 0.299 | - | 0.33 | |
| 8" graphite | 0.256 | 0.288 | 0.34 | |
| 12" graphite | 0.256 | 0.296 | 0.39 | |
| 14" granhite | 0 256 | 0 297 | 0 40 | |
| TT Prahutaa | · · · · · · · · · · · · · · · · · · · | | VIIV | |

* Single face reflector unless otherwise noted. Values (± 0.002) apply to 15.0" and 21.0" diameter Oy cylinders with density 17.9 gm/cm³; the unreflected critical masses are, respectively, 0.942 \pm 0.003 and 0.873 \pm 0.003 kg Oy/in². Savings for the graphite reflectors correspond to $\rho(c) = 1.73$ g/cm³; other reflector densities as in Table IV.

Extrapolation to Infinite Slabs

Guidance in the extrapolation to infinite slabs by two-dimensional transport codes is uncertain, at the moment, because of unknown bias and imperfect convergence to one-dimensional limits. The question arises as to whether, in lieu of such guidance, critical masses of spheres or of squat cylinders serve best for estimation of infinite-slab reflector savings. Because extrapolation of the cylinder data to slab geometry must be empirical, the answer depends on the sensitivity of critical height, h, to radial buckling, B_r^2 . Figure 3 compares $h(B_r^2)$ for unreflected squat Oy cylinders computed from end-point theory to that computed from 16 group DS-8 one-dimensional slabs with $DB_r^2 = B_r^2/3\Sigma_{tr}$ absorption; the two included data points confirm the higher accuracy of end-point theory.

If, however, in the DS-8 slab computations, one represents the radial leakage by $B_r^2/3\Sigma_{tr}(E)$ [1+4f/5+...] absorption as suggested by one-group transport theory and with $f_{Oy} = 0.34$, one obtains good agreement with end-point theory (as may be verified by rescaling the abscissa of Figure 3 by the factor 1+4/5f + .. = 1/0.78). For the one or two face reflected squat cylinders, there is no analogous recipe for selecting a constant c for $cB_r^2/3\Sigma_{tr}$ absorption in the slab computations. The choice c(Refl.)



Figure 3. Critical height versus radial buckling for unreflected Oy cylinders ($\rho = 17.9 \text{ gms/cm}^3$)

= c(core), i.e., c = 0.78, yields the DS-8 computed reflected savings $\delta(B_r^2)$ graphed in Figures 4 and 5. One notes that if, in the series expansion $\delta(B_r^2) = \delta_0 + aB_r^2 + b_0^2$ bB_r^4 + ..., the third and higher order terms are negligible for B_r^2 values less than that for the 15" cylinders, then no further guidance from theory is necessary for the 15" and 21" cylinder data to yield δ_0 values; a linear extrapolation of the data suffices. As indicated by Figures 4 and 5, the higher order terms contribute significantly only for the thicker graphite reflectors. Here one notes that, while there is a positive bias of computed over observed reflector savings, the bias decreases with B_{2}^{2} for the one-face 14" graphite reflector but increases with B_r^2 for the two-face 6" graphite reflector; i.e., there is no clear cut bias on the computed slope of $\delta(B_r^2)$. This suggests extrapolating the squat cylinder reflector savings data to slab geometry by paralleling the computed curve. Values of δ_{α} estimated in this fashion are included in Table VI and again under "Est. from Cyl. Data" (column 5) of Table VII.

The latter table summarizes computed and observed or estimated reflector savings for corresponding sphere and slab geometries. The bias in computed critical sizes arising from errors in multigroup cross sections (all



Figure 4. Reflector savings versus radial buckling for Oy cylinders reflected by indicated materials. The guide lines are from DS-8 slab computations with 0.78 DB²_r absorption.



Figure 5. Reflector savings versus radial buckling for Oy cylinders reflected by the indicated materials. The guide lines are from DS-8 slab computations with 0.78 DB_r^2 absorption.

TABLE VII

REFLECTOR SAVINGS (KG OY/IN²) FOR SPHERES AND SLABS

| Reflector | Sphere $\simeq 18.7$ | , ρ(Οy) g/cm ³ | | Slab | |
|------------------------------------|----------------------|------------------------------|-------------|------------------------|------------------------|
| | obs.* | DS-4 | DS-8** | Est. from Cyl. Data | Est. from Sph. Data |
| 1" C | 0.115 | 0.113 | 0.118 | 0.125 | 0.120 |
| 2" C | 0.163 | 0.173 | 0.192 | 0.195 | 0.182 |
| 4'' C | 0.219 | 0.237 | | | |
| 6" C | 0.260 | 0.274 | 0.315 | 0.313 | 0.301 |
| | | | 0.320(2F) | 0.31 ₁ (2F) | 0.306 |
| 8'' C | 0.277 | 0.293 | 0.352 | 0.34 | 0.336 |
| 14" C | | | 0.408 | 0.40 | |
| | | | 0.398(2F) | | |
| 17" C | 0.319 | 0.340 | | | |
| 1" CH ₂ | 0.149 | 0.146 | 0.166 | 0.166 | 0.169 |
| 2'' CH ₂ | | | 0.267(2F) | 0.258(2F) | |
| 6" CH ₂ | | | 0.305 | 0.296 | |
| 0.015" Cd + 6" CH ₂ | | | 0.176 | 0.169 | |
| 6'' Н ₂ О | 0.233 | 0.242 | 0.309 | 0.297 | 0.300 |
| 0.015" CD + 6" H ₂ O | 0.135 | 0.137 | | | |
| *Uncertain LAMS-2415 | ties ar • | e gener | ally ±0.005 | kg/in ² . D | ata are from |

****One face reflection unless marked 2F(ace).**

from LAMS-2543) is certainly expected to be similar in the different geometries. Regarding bias in reflector savings arising from DSN approximations, we surmise that the bias in DS-8 computed slab reflector savings is of the same sign as and comparable magnitude (~ 1%) to DS-4 computed sphere reflector savings. A plausible estimate of slab reflector savings from sphere data is then obtained from $[\delta_{est}, -\delta_{DS-8}]_{slab} = [\delta_{obs}, -\delta_{DS-4}]_{sphere};$ resulting values are included in the last column of Table VII. Except for the slabs reflected on one face by 2"C and 6"C, the numerical agreement of the two estimations is sufficiently good as to make irrelevant the point as to which yields the better values (however the reflector savings data for the squat cylinders are generally more precise than those for the spheres). For the slab reflected on one face by 2"C, the value of 0.182 kg/in² estimated from sphere data is too low since it has already been surpassed with the 21" cylinder; it is believed that estimates for both 2"C and 6"C reflected slabs are better made from the cylinder data.

^{*}We have found $\delta_8/\delta_4 = 0.98$ for Oy slabs reflected by various hydrogenous reflectors and $\delta_8/\delta_4 = 0.99$ for Oy spheres reflected by thick uranium.

Extrapolation of the unreflected cylinder data is clearer since biases in the end-point theory guide line of Figure 3 are better known. The nature of these biases is as follows: a) computations by Bell and Carlson (Proc. of 2nd Gen. Conf., Vol. XVI, p 535-49 indicate slightly larger extrapolation lengths for slabs than for spheres, e.g., for Oy ($f \stackrel{\sim}{-} 0.34$), end-point theory overestimates critical slab thickness by ~ 0.001 kg/in² both absolutely and relative to a given critical sphere radius^{*}, and b) Frankel and Nelson, in LA-53, show that extrapolation lengths are diminished proportionately to the transverse curvature or buckling of the flux, and their computations, given in LA-53A, indicate that the extrapolation length for a critical Oy cube is ~ 0.013 kg/in² less than the end-point theory value^{**}. Allowing for these biases,

^{*(}The Godiva critical radius thus implies for the critical slab thickness 0.805 ± 0.007 kg/in², the uncertainty arising principally from the uncertainty in the sphere extrapolation length $\lambda = 0.248 \pm 0.007$ kg/in² which in turn arises principally in a ~ 5% uncertainty in Oy collision cross-sections.)

^{*(}Five unreflected Oy cylinders of not extreme shape reported by Paxton in LAMS-2415 all had extrapolation lengths less than for the sphere and by as much as ~0.010 kg/in² for height to diameter ratios near unity).

the lower curve of figure 3 shifts to the dotted line and the 15" and 21" cylinder data yield $0.806 \pm 0.003 \text{ kg/in}^2$ for the critical thickness of an unreflected Oy slab. This value together with the value $1.054 \pm 0.002 \text{ kg/in}^2$ for the Godiva critical radius yields a bare sphere extrapolation length of $0.247 \pm 0.004 \text{ kg/in}^2$.