REFERENCE 36

J. T. MIHALCZO, "GRAPHITE AND POLYETHYLENE REFLECTED URANIUM METAL CYLINDERS AND ANNULI," UNION CARBIDE CORPORATION, NUCLEAR DIVISION REPORT Y-DR-81 (APRIL 1972).

Y-DR-81

GRAPHITE AND POLYETHYLENE REFLECTED URANIUM-METAL CYLINDERS AND ANNULI

J. T. Mihalczo

UNION CARBIDE CORPORATION
NUCLEAR DIVISION
OAK RIDGE Y-12 PLANT

operated for the ATOMIC ENERGY COMMISSION under U. S. GOVERNMENT Contract W-7405 eng 26

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Date Issued: April 28, 1972

Document Y-DR-81

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J. T. Mihalczo

Oak Ridge, Tennessee

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J. T. Mihalczo

ABSTRACT

Cylinders and cylindrical annuli of uranium metal (93.15 wt % ^{235}U), with diameters varying from $\sim$7 to $\sim$15 in., were assembled to delayed criticality with graphite reflectors varying in thickness up to 18 in. or with effectively infinite thickness polyethylene on all outer surfaces. In one series of measurements an effectively infinite thickness polyethylene was also placed adjacent to only one flat surface of the cylinders. The multiplication factor calculated by both S_n transport theory and Monte Carlo methods agreed very well with the experimental values. As a result of the high order of S_n and the large number of spatial intervals required, the computing time for the transport theory calculations was a factor of 10 larger than that required for the Monte Carlo calculations with standard deviations of 1%.

INTRODUCTION

Cylinders and cylindrical annuli of uranium metal (93.15 wt % ^{235}U)[a] have been assembled to delayed criticality with various conditions of polyethylene or graphite reflection at the Oak Ridge Critical Experiments Facility (CEF) between March 1963 and November 1967. The cylinder diameters varied from 7 to 15 in. The outside diameters of the annuli varied from 9 to 15 in. and the inside diameters varied from 7 to 13 in. The assemblies constructed were:

1) cylinders with an "infinite" thickness of polyethylene reflector adjacent to only one flat surface,
2) cylinders with graphite of thickness varying from 7 to 15 in. or "infinite" thickness polyethylene reflector adjacent to the surfaces,
3) cylindrical annuli within "infinite" thickness polyethylene reflector and with the central cavity containing polyethylene or void,
4) cylindrical annuli with graphite reflectors of thickness varying from 1 to 18 in. and with graphite or a void within the annuli, and
5) annuli without reflector but with polyethylene or graphite in the center.

These cylindrically symmetric assemblies, in addition to providing information of use in establishing the nuclear criticality safety of uranium processing facilities, have supplied information which was used to verify both the S_n transport theory and Monte Carlo methods of calculation.

EXPERIMENTAL RESULTS

The average density of the uranium metal, which has been previously described,[1] was 18.76 g/cm^3. The careful fabrication of the uranium metal allowed the construction of critical assemblies with average uranium densities between 18.63 and 18.77 g/cm^3. The graphite (National Carbon Type HLM) had an average density of 1.78 g/cm^3. The careful fabrication of the graphite annuli and discs used for reflector resulted

[a] Other uranium isotopes were present in the following amounts: 0.97% ^{234}U, 0.24% ^{236}U, and 5.64% ^{238}U.

1. J. T. Mihalczo, Nucl. Sci. Eng. 20, 60 (1964).

in average reflector densities of between 1.682 and 1.783 g/cm^3. The average density of the polyethylene reflector was 0.916 g/cm^3. These materials did not contain significant impurities and the results of impurity analyses are given in Table 1.

Polyethylene-Uranium Assemblies

The assembly of a 8.996-in.-diam uranium cylinder reflected with polyethylene on all surfaces is shown in Fig. 1. The top and side reflectors were mounted on a 0.5-in.-thick aluminum plate with a 20-in.-diam hole in the center. The side reflector was 24.73 in. outside diameter, 9.09 in. inside diameter, and 13.20 in. high. The top reflector was 24.75 in. in diameter and 10.50 in. in height. The lower part of the assembly, made up of the bottom reflector (9.00 in. i.d. and 11.87 in. high) and the uranium core, was raised remotely into the side reflector by the 24 in. stroke of a piston[2] until the uranium was in contact with the top reflector. The height of the uranium was the variable that was adjusted to attain an assembly with a measureable stable reactor period from which the reactivity could be determined. In most cases the reactivity changes due to perturbations in the fuel height were also measured. This method of assembly was used for cylinders of greater than 6.996 in. diameter and for all reflected annuli with or without polyethylene in the center. For the 6.996-in.-diam cylinder the uranium on the movable part of the assembly was surrounded by a 1-in.-thick polyethylene annulus of equal height. The diametral clearances between the movable part of the assembly and side reflector for the ~7-, ~9-, ~11-, ~13-, and ~15-in.-diam cores were 0.093, 0.087, 0.011, 0.100, and 0.077 in., respectively.

For the uranium cylinders with polyethylene on one flat surface, the uranium, which was mounted on a low mass support stand, was moved remotely against the polyethylene reflector to achieve delayed criticality. For the unreflected uranium annuli with polyethylene in the center, the assembly was split at approximately the half height of the

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2. E. R. Rohrer et al., Neutron Phys. Div. Ann. Prog. Rept. Sept. 1, 1961, ORNL-3193, Oak Ridge National Laboratory, p. 168 (1961).

Table 1. Impurities in the Graphite, Polyethylene, and Uranium.

	Impurity Content (ppm)[a]				
	Graphite[b] For Reflector Thickness of		Polyethylene[c]	Uranium	
Element	≤ 15 in.	> 15 in.		Average	Range[d]
Ag	<1	<1	<1	8	3 - 25
Al	200, 5, 50	50, 10, 50	<5	-	-
B	3, 5, 15	30, 20, 15	<1	-	-
Ba	50, 20, 20	20	<20	<0.01	-
Be	<1	<1	<1	-	-
Bi	<3	<3	<3	164	81 - 311
C	-	-		<10	-
Ca	1000, 200, 300	500, 200, 300	<30	0.1	-
Cb	<10	<10	<10	-	-
Cd	<10	<10	<10	< 1	-
Co	<10	<10	<10	5	2 - 15
Cr	<10	<10	<10	7	4 - 12
Cu	<1, <1, 5	10, <1, <1	<1	25	10 - 40
Fe	500, 150, 200	200, 200, 200	3	-	-
K	10	50, <10, <10	<10	< 0.2	0.2- 0.8
Li	<1	20, 3, 5	1	< 2	-
Mg	15, 10, 100	100, 7, 10	3	3	2 - 3
Mn	<2	<2	<2	56	25 - 89
Mo	<1	<1	<1	< 1	<1 - 1
Na	10	200, <10, <10	<10	27	15 - 50
Ni	<3	<3	<3	100	-
Pb	3	5, <3, 3	<3	-	-
Rb	<10	<10	<10	-	-
Sb	<10	<10	<10	38	10 - 80
Si	70, 200, 500	500, 200, 200	200	-	-
Sn	<3	<3	<3	-	-
Ta	30	30	30	-	-
Ti	<3, <3, 30	100, 100, 100	3	1	-
V	100, 10, 10	20	<3	-	-
W	<30	<30	<30	-	-
Zn	<50	<50	<50	-	-
Zr	<10	<10	<10	-	-

a. These are the results of spectrographic analyses reported as parts (mass) of impurity per million parts of the materials.

b. The results of three analyses are given. If only one value is listed, it was that in all three analyses.

c. Values for one sample.

d. Range of values in 11 samples.

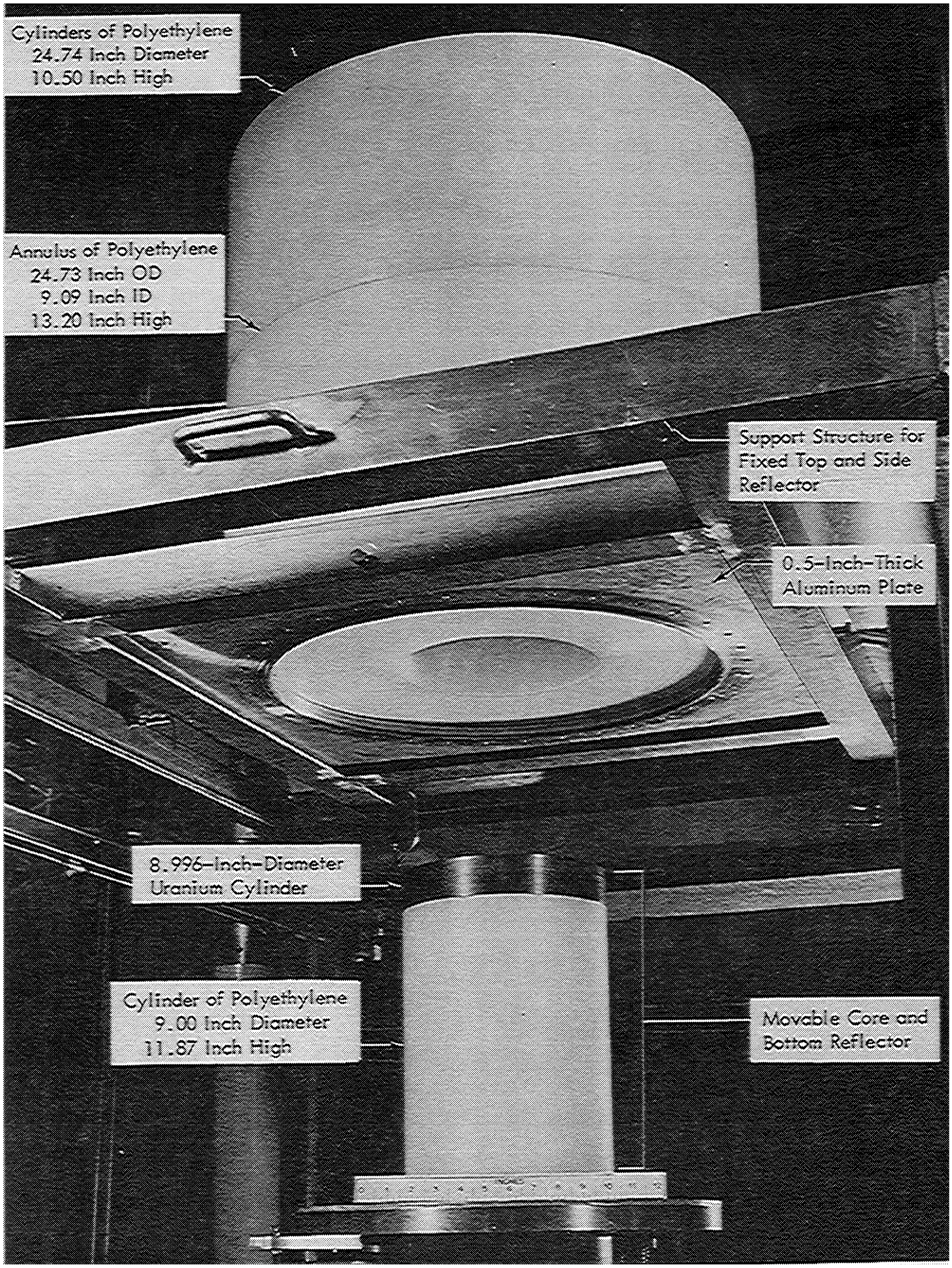

Fig. 1. Typical Polyethylene Reflected Uranium Cylinder in the Disassembled Condition.

uranium. The upper half was supported on a 0.010-in.-thick stainless steel diaphragm and the lower half was supported on a low mass support stand. The assembly procedure consisted of remotely raising the lower section until it was in contact with the upper section.

The reactivity contribution of the support structure, including the diaphragm, was evaluated directly by measuring the effects of appropriate equivalent materials on the reactivity of each assembly. The observed dimensions and reactivities of the assemblies are given in Table 2. The reactivity changes associated with changes in the height of uranium in given radial increments were obtained from measurements in most cases. Height perturbation measurements for the radial increment, 0 to 3.5 in., could not be made for the assemblies with "infinite" polyethylene on all surfaces. In these cases perturbation theory, with forward and adjoint angular fluxes obtained from S_8 transport theory calculations,[3] was used to obtain the reactivity changes per unit height. For the uranium cylinders reflected with polyethylene on only one flat surface, the reactivity effects of the height changes possible with the available uranium parts for the radial increment from 0 to 3.5 in. were too large to obtain directly from stable reactor period measurements. In these cases the values were obtained from the measurements of the reactivity effect of a 1.000 x 1.000 x 0.125 in. piece of uranium as a function of radial position on the unreflected face of the cylinders and the height perturbation measurements for the other radial increments. The dimensions of the assemblies were corrected to those for delayed critical assemblies with uniform heights and without support structure. This conversion employed the reactivity changes associated with changes in the height of the uranium, listed in Table 3, and the reactivity of the experimental assemblies given in Table 2. The critical dimensions are also given in Table 3 and the critical mass of some of these assemblies as a function of outside diameter of the uranium is plotted in Fig. 2.

3. F. R. Mynatt, "A User's Manual for DOT, A Two Dimension Discrete Ordinates Transport Code with Anisotropic Scattering," K-1794, Oak Ridge Computing Technology Center (1968).

Table 2. Dimensions and Reactivities of the Polyethylene-Uranium Assemblies.

Uranium										Polyethylene[b]				
Outside Diameter (in.)	Inside Diameter (in.)	Height (in.)[a] for Radial Increments (in.) of 0-3.5	3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	Average Density (g/cm^3)	Mass (kg)	Height Within the Uranium Annulus (in.)	Reflector Thickness (in.) Top	Bottom	Lateral	Reactivity of the Assembly[c] (cents)
Cylindrical Annuli with Polyethylene in the Center													
14.995	9.002	--	--	5.136	5.150	5.079	18.688	177.057	5.149	0	0	0	33.6
14.995	7.002	--	4.009	4.014	4.021	4.012	18.680	169.687	4.023	0	0	0	4.5
12.996	7.003	--	5.011	4.950	5.026	--	18.701	144.141	5.021	0	0	0	21.2
Cylindrical Annuli with Polyethylene on all Outer Surfaces													
14.995	11.003	--	--	--	3.230	3.220	18.720	80.631	0	10.50	11.87	4.83	13.9
14.995	11.003	--	--	--	3.670	3.665	18.709	91.656	3.633	10.50	11.87	4.83	20.9[d]
14.994	9.002	--	--	2.136	2.131	2.124	18.703	73.709	0	10.50	11.87	4.83	8.9
14.994	9.002	--	--	2.383	2.381	2.374	18.716	82.391	2.382	10.50	11.87	4.83	21.1
14.996	7.003	--	1.664	1.658	1.664	1.664	18.700	70.363	0	10.50	11.87	4.83	30.1
14.996	7.003	--	1.820	1.789	1.822	1.780	18.728	76.330	1.759	10.50	11.87	4.83	24.3[d]
12.996	9.001	--	--	3.169	3.165	--	18.711	67.018	0	10.50	11.87	5.83	8.4
12.996	9.001	--	--	3.571	3.571	--	18.724	75.626	3.567	10.50	11.87	5.83	13.1[d]
12.997	7.003	--	2.100	2.097	2.097	--	18.707	60.548	0	10.50	11.87	5.83	27.9
12.997	7.003	--	2.317	2.316	2.317	--	18.726	66.935	2.315	10.50	11.87	5.83	40.0
10.997	7.003	--	3.166	3.164	--	--	18.703	54.770	0	10.50	11.87	6.86	23.4
10.997	7.003	--	3.419	3.415	--	--	18.700	59.120	3.399	10.50	11.87	6.86	2.0[d]
Cylinders with Polyethylene on One Flat Surface[e]													
14.995	--	2.005	2.001	2.002	2.015	1.999	18.705	108.508	--	6.00	0	0	-95.3
12.996	--	2.191	2.191	2.193	2.195	--	18.703	89.144	--	6.00	0	0	-97.1
10.997	--	2.443	2.441	2.445	--	--	18.692	71.078	--	6.00	0	0	-33.0
8.996	--	2.881	2.879	--	--	--	18.707	56.121	--	6.00	0	0	-50.6
6.996	--	4.009	--	--	--	--	18.717	47.268	--	6.00	0	0	-25.7
Cylinders with Polyethylene on all Surfaces													
14.995	--	1.131	1.134	1.132	1.131	1.125	18.674	61.074	--	10.50	11.87	4.83	3.8
12.996	--	1.253	1.255	1.252	1.256	--	18.699	50.971	--	10.50	11.87	5.83	44.2
10.997	--	1.446	1.417	1.409	--	--	18.665	41.431	--	10.50	11.87	6.85	25.5
8.996	--	1.718[f]	1.694	--	--	--	18.699	33.276	--	10.50	11.87	7.82	- 1.2
6.996	--	2.278[g]	--	--	--	--	18.741	26.893	--	10.50	11.87	8.82	5.8

a. These heights, for a particular assembly, show the unevenness of the upper surface of the uranium. The dimensions of the radial increments given in the column headings are nominal values which are larger than the inside diameters of annuli by less than 0.006 in. and are smaller than the outside diameters of annuli or cylinders by less than 0.004 in.

b. The polyethylene density was 0.916 g/cm^3.

c. The reactivities listed have been corrected for the effects of support structure. The reactivity changes associated with the addition of the diaphragm, clamping ring, and support stand for the 14.995-in.-o.d., 9.002-in.-i.d. annulus with polyethylene in the center were -16.8, 5.4, and 10.0 cents, respectively. Similar values for the second annulus were -18.0, 3.6, and 15.7 cents while those for the third annulus were -17.5, 4.2, and 11.7 cents. The reactivity of the support structure for the assemblies with polyethylene on all outer surfaces was negligible since the support structure was external to the reflector. The reactivity changes associated with the addition of the support structure for the cylinders with polyethylene on one flat surface for the ~15-, ~13-, ~11-, and ~7-in.-diam cylinders were 129, 127, 93, 63, and 38 cents, respectively.

d. These reactivities are not corrected for the effects of increasing the height of the polyethylene within these uranium annuli to that of the annuli. These increases in reactivity were 4.2, 4.1, 0.5, and 1.6 cents, respectively.

e. This polyethylene reflector was 44 in. square and 6.01 in. thick. One flat surface of these cylinders was centrally located and adjacent to the 44-in.-square surface of the polyethylene.

f. A 5.000- x 5.000-in.-square, 0.031-in.-thick piece of uranium was centered on the upper surface of the uranium cylinder with the large flat surface in contact. A 2.500- x 2.500-in.-square, 0.031-in.-thick piece of uranium was located on the 5.000 x 5.000 in. square with its center 1.25 in. from the center of the larger square. The sides of both squares were parallel and the large flat surfaces were in contact. The height of the radial increment 0-3.5 in. included their mass, 300 g, as though the mass were distributed uniformly. The cylinder height and mass without the plates was used to calculate the uranium density.

g. A 5.000- x 5.000-in.-square, 0.031-in.-thick piece of uranium was located on top of this uranium cylinder. The height of the radial increment 0-3.5 in., included the mass of this piece, 240 g, as though it were distributed uniformly. The cylinder height and mass without the plate was used to calculate the uranium density.

Table 3. Critical Dimensions and Uranium Height Perturbation Measurements for Polyethylene Uranium Assemblies.

Uranium									Polyethylene			
Outside Diameter (in.)	Inside Diameter (in.)	Critical Height[a] (in.)	Critical Mass[a] (kg)	Reactivity per Height Change (cents/mil) for Radial Increments[b] (in.) of 0-3.5	3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	Material Within the Uranium Annulus	Reflector Thickness (in.) Top	Bottom	Lateral
Cylindrical Annuli with Polyethylene in the Center												
14.995	9.002	5.095	176.25	--	--	0.39	0.38	0.19	Polyethylene	0	0	0
14.995	7.002	4.011	169.56	--	0.46	0.53	0.43	0.20	Polyethylene	0	0	0
12.996	7.003	4.967	143.34	--	0.44	0.43	0.19	--	Polyethylene	0	0	0
Cylindrical Annuli with Polyethylene on all Outer Surfaces												
14.995	11.003	3.211	80.29	--	--	--	0.45	0.57	Air	10.50	11.87	4.83
14.995	11.003	3.643	91.05	--	--	--	0.45	0.57	Polyethylene	10.50	11.87	4.83
14.994	9.002	2.126	73.59	--	--	0.74	0.74	0.74	Air	10.50	11.87	4.83
14.994	9.002	2.370	82.08	--	--	0.74	0.82	0.59	Polyethylene	10.50	11.87	4.83
14.996	7.003	1.652	69.93	--	0.68	0.96	0.77	0.66	Air	10.50	11.87	4.83
14.996	7.003	1.794	76.03	--	0.70	0.66	0.79	0.68	Polyethylene	10.50	11.87	4.83
12.996	9.001	3.159	66.85	--	--	0.41	0.71	--	Air	10.50	11.87	5.83
12.996	9.001	3.558	75.35	--	--	0.53	0.50	--	Polyethylene	10.50	11.87	5.83
12.997	7.003	2.087	60.23	--	0.84	0.89	0.74	--	Air	10.50	11.87	5.83
12.997	7.003	2.299	66.42	--	0.71	0.78	0.76	--	Polyethylene	10.50	11.87	5.83
10.997	7.003	3.150	54.51	--	0.75	0.80	--	--	Air	10.50	11.87	6.86
10.997	7.003	3.414	59.08	--	0.58	0.54	--	--	Polyethylene	10.50	11.87	6.86
Cylinders with Polyethylene on one Flat Surface[c]												
14.995	--	2.036	110.13[d]	(1.02)[e]	0.75	0.62	0.42	0.20	--	6.00	0	0
12.996	--	2.225	90.47	(1.70)	0.51	0.38	0.30	--	--	6.00	0	0
10.997	--	2.456	71.45	(1.93)	0.37	0.22	--	--	--	6.00	0	0
8.996	--	2.908	56.66	(1.64)	0.24	--	--	--	--	6.00	0	0
6.996	--	4.035	47.57	0.99	--	--	--	--	--	6.00	0	0
Cylinders with Polyethylene on all Surfaces												
14.995	--	1.129	61.02	[1.75][f]	0.86	0.67	0.46	0.33	--	10.50	11.87	4.83
12.996	--	1.243	50.51	[2.09]	0.77	0.70	0.49	--	--	10.50	11.87	5.83
10.997	--	1.427	41.45	[2.39]	0.88	0.65	--	--	--	10.50	11.87	6.85
8.996	--	1.711	33.32	[2.69]	[0.82]					10.50	11.87	7.82
6.996	--	2.276	26.76	[3.90]						10.50	11.87	8.82

a. These values are corrected for the reactivity of the support structure and for the unevenness of the top of the uranium, given in Table 2, using the reactivity per uranium height change tabulated above. The error in the mass is taken as ± 10 or ± 20% of the difference between the assembled mass and the critical mass. If all reactivity changes per unit uranium height were from measurements, then ± 10% error is assumed. These critical masses are for the assemblies with uranium densities as given in Table 2.

b. The dimensions of the radial increments given in the column headings are nominal values which are larger than the inside diameters of annuli by less than 0.006 in. and are smaller than the outside diameters of annuli or cylinders by less than 0.004 in.

c. This polyethylene reflector was 44 in. square and 6.01 in. thick. One flat surface of these cylinders was centrally located and adjacent to the 44-in.-square surface of the polyethylene. The uranium height perturbations were made on the unreflected flat surface of these cylinders.

d. This value is 6.5 kg less than a similar measurement reported in Ref. 4 for a 15-in.-diam cylinder with a 15-in.-diam, 6-in.-thick polyethylene reflector adjacent to one flat surface. The difference between the measurements resulted from the lateral extension of the top reflector 14.5 in. beyond the radius of the uranium cylinder in this experiment.

e. The values within parentheses were obtained from measurements of the worth of a 1.000-in.-square, 0.125-in.-thick piece of uranium as a function of radius and the height perturbation measurements for the other radial increments. The error for these values as well as those not in parentheses was ± 5 to ± 10%.

f. The values within brackets were obtained from perturbation theory calculations with the forward and adjoint fluxes from transport theory calculations. Perturbation theory predicted reactivities per uranium height change for the radial increment 3.5 to 4.5 in. within 12% of the measurements for the 10.997-, 12.996-, and 14.995-in.-diam cylinders. The error for these values was assumed to be ± 10 to ± 20%.

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4. G. E. Hansen, D. P. Wood, and B. Pena, "Reflector Savings of Moderating Materials on Large Diameter U(93.2%) Slabs," LAMS-2744, Los Alamos Scientific Laboratory (1960).

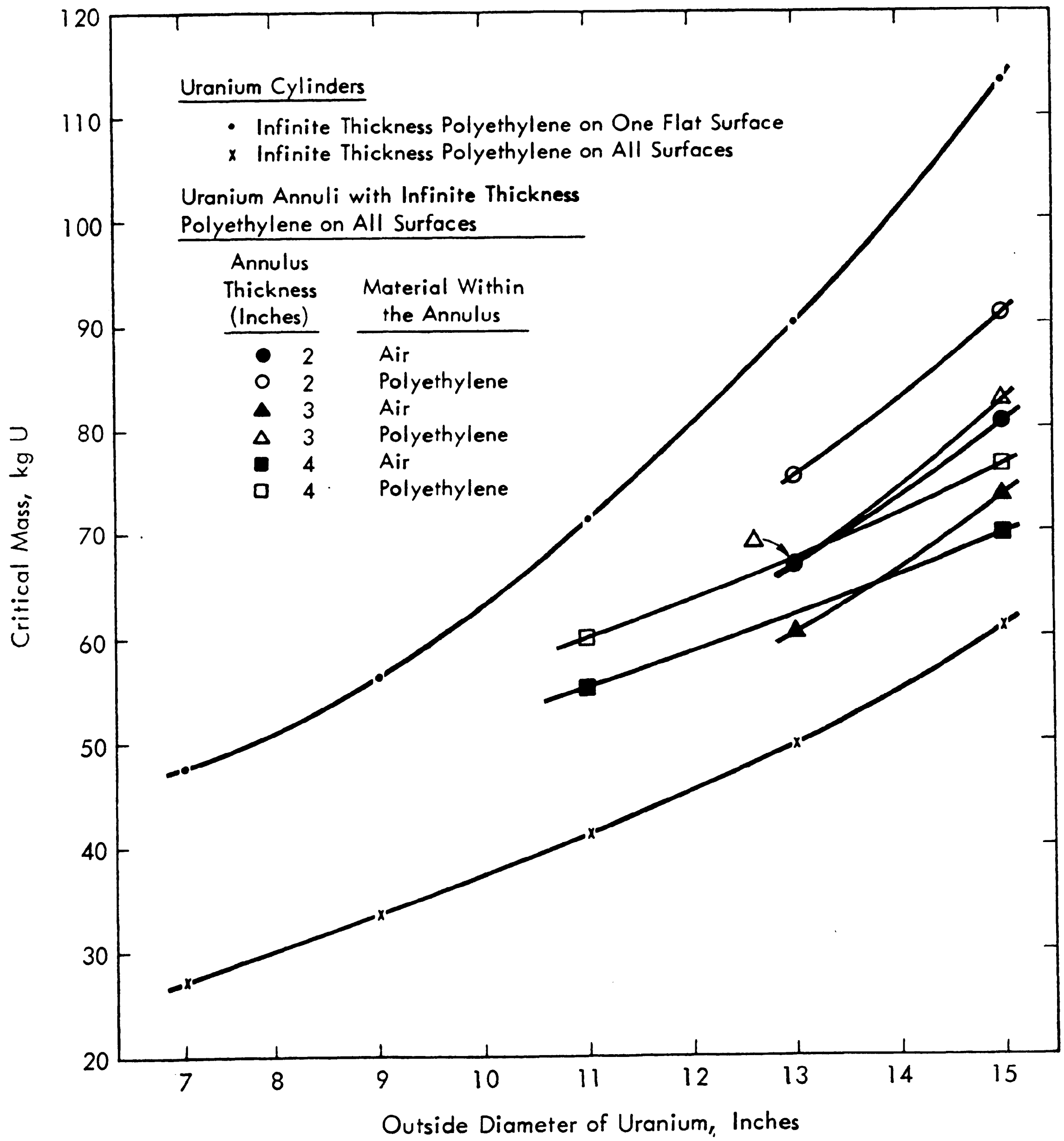


Fig. 2. Critical Mass of Polyethylene-Uranium Assemblies as a Function of Uranium Diameter.

Graphite-Uranium Assemblies

The assembly of a 14.996-in.-o.d., 11.003-in.-i.d. uranium annulus with a 15-in.-thick graphite reflector is shown in Fig. 3. The construction of this assembly and the method of assembly were the same as for the uranium annuli and cylinders with diameters greater than 6.996 in. with polyethylene reflector adjacent to the outer surfaces. This method of assembly was used for all graphite-reflected experiments where the reflector thicknesses were greater than 9 in. For experiments with graphite reflector thicknesses of 1 to 9 in. or with no reflector and graphite within the annulus, the assembly was divided at a horizontal plane. The material above the plane was fixed and supported on a 0.010-in.-thick stainless steel diaphragm and the material below this plane was supported on a low mass support stand which was attached to the piston which could be raised remotely. Delayed criticality was achieved by adjusting the height of the uranium and raising the lower section until contact was made with the upper section. For the annuli with graphite in the center and no reflector and those assemblies with thin reflectors the horizontal plane dividing the assembly passed through the uranium. For the 14.996-in.-diam cylinder with a 7.019-in.-thick reflector this plane was located in the bottom reflector 1 in. below the lower flat surface of the uranium cylinder. For the following annuli the 0.010-in.-thick stainless steel diaphragm was replaced by one 0.015-in.-thick and the plane of division of the assembly in the bottom reflector between 1/2 and 3 in. below the lower surface of the uranium: 10.996 in. o.d., 7.003 in. i.d. with a 9.002-in.-thick reflector; 12.996 in. o.d., 7.003 in. i.d. with a 6.001-in.-thick reflector and with a 8.000-in.-thick reflector with air inside the annulus; and 14.995 in. o.d., 7.003 in. i.d. with 6.997-in.-thick reflector with air inside the annulus. The reactivity effects of the support materials, including the diaphragm, were evaluated directly by measuring the effects of appropriate equivalent materials on the reactivity of each assembly.

The observed dimensions and reactivities of the graphite-reflected uranium annuli with and without graphite in the center are given in Table 4. The reactivity change due to a change in the height of uranium

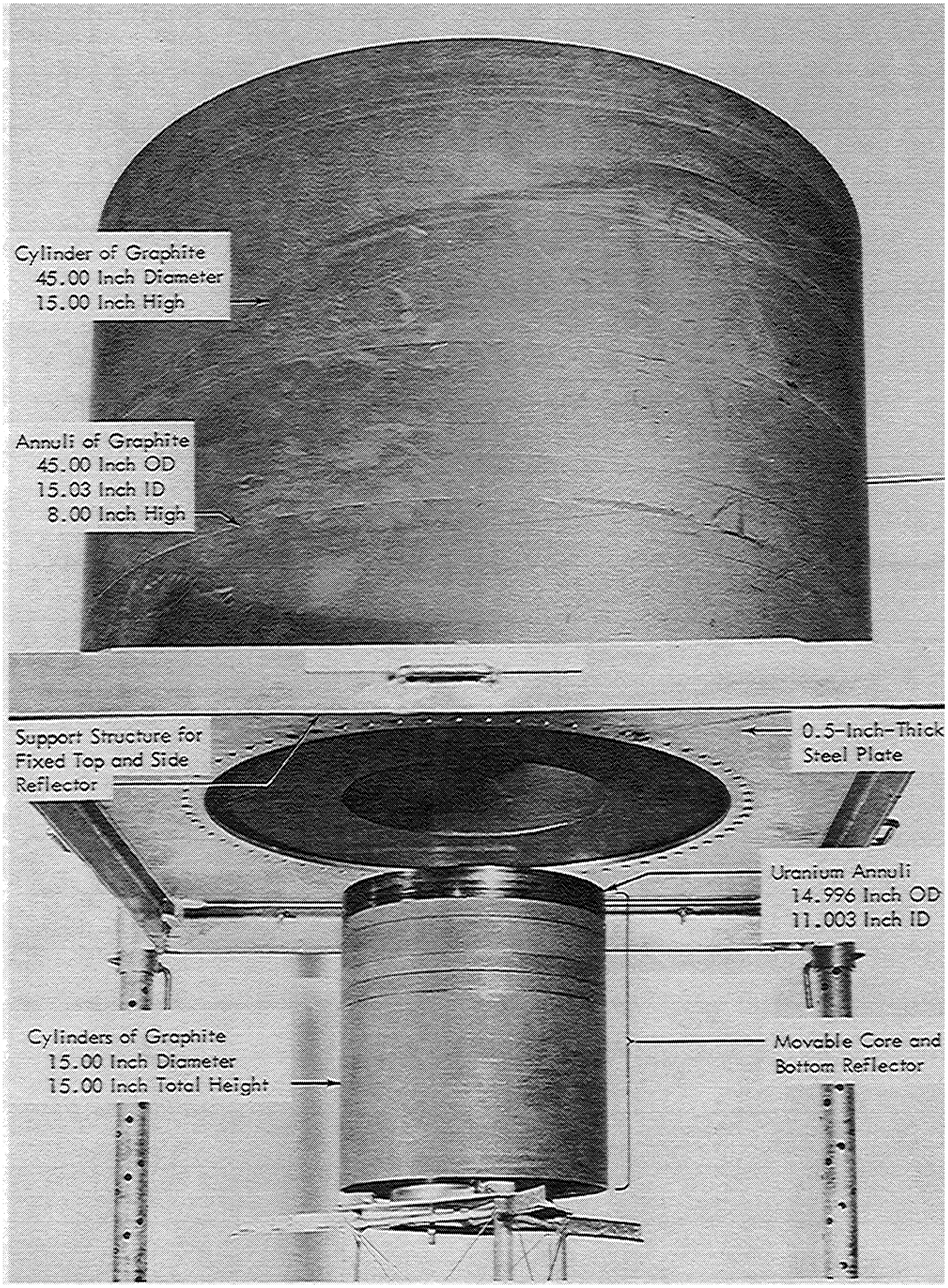


Fig. 3. Typical Graphite Reflected Uranium Annulus in the Disassembled Condition.

Table 4. Dimensions and Reactivities of Graphite-Reflected Uranium Annuli.

Uranium: Height (in.)[a] for Radial Increments (in.) of 3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	Uranium: Mass (kg of U)	Uranium: Density (g/cm³)	Graphite: Density (g/cm³)	Graphite: Reflector Thickness[b] (in.)	Graphite: Height Within the Uranium Annulus (in.)	Reactivity of Assembly[c] (cents)	Reactivity of Support Structure[d] (cents)
Annuli Diameters: o.d. 14.995 in., i.d. 13.002 in.										
--	--	--	5.144	69.048	18.691	1.703	10.012	0	- 5.9	7.2
--	--	--	4.546	61.078	18.709	1.702	10.012	4.505	15.5	6.0
--	--	--	3.639	48.911	18.716	1.759	15.000	0	12.6	2.0
--	--	--	3.422[e]	45.960	18.692	1.760	15.000	3.414	- 8.7	2.0
Annuli Diameters: o.d. 14.995 in., i.d. 11.003 in.										
--	--	6.370	6.246	157.377	18.691	1.677	0.996	6.347	7.2	11.7
--	--	5.328	5.329	133.146	18.706	1.708	1.997	0	19.7	4.2
--	--	4.863	4.860	121.412	18.697	1.702	1.997	4.838	-11.7	4.2
--	--	4.330	4.332	108.123	18.690	1.706	2.997	0	27.2	- 1.4
--	--	4.073	4.075	101.794	18.706	1.704	2.997	4.072	18.0	- 0.5
--	--	3.263	3.261	81.417	18.686	1.720	4.996	0	32.8	11.0
--	--	3.141[f]	3.138	78.367	18.688	1.715	4.996	3.141	15.1	10.6
--	--	2.197[f]	2.197	54.907	18.706	1.692	10.004	0	14.8	7.7
--	--	2.166[f]	2.166	54.147	18.711	1.706	10.004	2.164	19.4	6.8
--	--	1.725[f]	1.696	42.745	18.717	1.760	14.998	0	5.7	2.1
--	--	1.695[f]	1.696	42.412	18.723	1.760	14.998	1.689	- 5.3	3.1

Table 4. (Cont'd)

Uranium: Height (in.)[a] for Radial Increments (in.) of 3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	Uranium: Mass (kg of U)	Uranium: Density (g/cm³)	Graphite: Density (g/cm³)	Graphite: Reflector Thickness[b] (in.)	Graphite: Height Within the Uranium Annulus (in.)	Reactivity of Assembly[c] (cents)	Reactivity of Support Structure[d] (cents)
Annuli Diameters: o.d. 14.995 in., i.d. 9.002 in.										
--	5.317	5.328	5.391	185.290	18.713	1.699	0[g]	5.317	-11.5	9.4
--	3.948	4.016	4.010	138.140	18.682	1.693	0.999	0	21.0	2.1
--	3.951	4.016	3.953	137.501	18.696	1.700	0.999	0[h]	0.7	12.4
--	3.888	3.891	3.887	134.433	18.677	1.692	0.999	3.888	21.7	4.2
--	3.885	3.897	3.894	134.580	18.679	1.691	0.999	3.887[h]	21.0	8.3
--	3.191	3.200	3.192	110.555	18.698	1.704	2.000	0	24.6	- 0.9
--	3.164	3.169	3.169	109.568	18.688	1.701	2.000	3.133	27.9	3.3
--	2.726	2.729	2.725	94.316	18.688	1.704	3.001	0	25.4	- 7.2
--	2.694[f]	2.694	2.698	93.242	18.686	1.702	3.005	0[h]	2.3	- 1.6
--	2.698[f]	2.696	2.700	93.279	18.676	1.704	3.001	2.699	7.1	- 4.9
--	2.694[f]	2.694	2.698	93.242	18.686	1.716	3.006	2.689[h]	13.2	- 0.2
--	2.129	2.143	2.137	73.831	18.667	1.718	4.996	0	-13.4	11.8
--	2.129	2.143	2.137	73.831	18.667	1.717	4.996	2.130	17.1	10.3
--	1.515	1.512	1.507	52.233	18.677	1.704	9.997	0	20.1	7.2
--	1.515	1.512	1.507	52.233	18.677	1.704	9.997	1.498	35.0	11.4
--	1.190[f]	1.191	1.192	41.285	18.724	1.762	14.998	0	11.7	2.6
--	1.190[f]	1.191	1.192	41.285	18.724	1.762	14.998	1.192	16.4	2.6

Table 4. (Cont'd)

Uranium: Height (in.)[a] for Radial Increments (in.) of 3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	Uranium: Mass (kg of U)	Uranium: Density (g/cm³)	Graphite: Density (g/cm³)	Graphite: Reflector Thickness[b] (in.)	Graphite: Height Within the Uranium Annulus (in.)	Reactivity of Assembly[c] (cents)	Reactivity of Support Structure[d] (cents)
Annuli Diameters: o.d. 14.995 in., i.d. 7.003 in.										
4.131	4.072	4.074	4.134	173.633	18.703	1.693	0[g]	4.130	-24.7	9.7
3.071	3.070	3.074	3.070	129.884	18.690	1.689	0.996	0	43.5	10.1
3.071	3.070	3.074	3.070	129.884	18.690	1.690	0.996	3.066	8.7	10.2
2.503	2.478	2.506	2.505	105.699	18.695	1.698	2.002	0	2.3	- 1.9
2.476	2.514	2.506	2.505	105.792	18.686	1.698	2.002	2.506	- 5.4	- 3.1
2.164	2.135	2.162	2.129	90.707	18.682	1.706	3.000	0	1.4	- 5.1
2.131	2.162	2.162	2.161	91.182	18.690	1.705	2.999	2.137	15.7	- 3.6
1.695	1.728	1.699	1.728	72.382	18.662	1.717	4.998	0	3.3	- 4.8
1.695	1.728	1.699	1.728	72.382	18.662	1.717	4.998	1.701	1.7	- 6.0
1.443[f]	1.444	1.445	1.444	61.071	18.689	1.724	6.997	0	17.6	2.7
1.443[f]	1.444	1.445	1.444	61.071	18.689	1.724	6.997	1.446	18.4	3.1
1.193[f]	1.228	1.196	1.227	51.166	18.647	1.704	10.012	0	24.1	11.0
1.193[f]	1.228	1.196	1.227	51.166	18.647	1.704	10.012	1.222	31.3	8.8
0.971[f]	0.939	0.971	0.949	40.406	18.663	1.756	14.997	0	24.9	2.5
0.941[f]	0.971	0.942	0.949	40.133	18.656	1.756	14.997	0.970	- 5.5	2.2
Annuli Diameters: o.d. 12.996 in., i.d. 11.003 in.										
--	--	4.955	--	57.257	18.772	1.704	10.997	0	17.5	5.8
--	--	4.391	--	50.743	18.772	1.700	10.997	4.380	- 7.1	5.4
--	--	3.770	--	43.519	18.752	1.753	15.992	0	30.4	2.2
--	--	3.512	--	40.592	18.776	1.752	15.992	3.511	14.2	2.2

Table 4. (Cont'd)

Uranium						Graphite			Reactivity of	
Height (in.)[a] for Radial Increments (in.) of				Mass	Density	Density	Reflector Thickness[b]	Height Within the Uranium Annulus	Assembly[c]	Support Structure[d]
3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	(kg of U)	(g/cm³)	(g/cm³)	(in.)	(in.)	(cents)	(cents)
Annuli Diameters: o.d. 12.996 in., i.d 9.002 in.										
--	6.139	6.120	--	129.702	18.715	1.691	0.999	6.139	-25.0	8.8
--	6.205	6.094	--	129.960	18.704	1.704	0.998	6.194	13.2	9.5
--	5.202	5.212	--	110.166	18.708	1.700	2.001	0	18.3	2.9
--	4.830	4.773	--	101.515	18.707	1.673	2.001	4.819	16.0	2.4
--	4.260	4.268	--	90.236	18.713	1.706	2.997	0	24.9	4.4
--	4.007	4.018	--	84.911	18.712	1.684	2.996	4.003	-13.6	5.4
--	3.662	3.667	--	77.595	18.724	1.680	3.999	0	28.1	- 1.6
--	3.502	3.513	--	74.293	18.729	1.708	4.000	3.505	4.6	- 1.2
--	1.756	1.755	--	37.181	18.730	1.757	16.001	0	- 4.4	2.0
--	1.756	1.755	--	37.181	18.730	1.757	16.001	1.753	30.9	1.3
Annuli Diameters: o.d. 12.996 in., i.d. 7.003 in.										
5.252	5.255	5.268	--	151.938	18.728	1.695	0[g]	5.255	15.6	3.1
3.884	3.887	3.895	--	112.210	18.703	1.684	1.001	0	-14.4	- 0.4
3.885	3.887	3.895	--	112.138	18.689	1.705	0.999	0[h]	5.9	2.3
3.760	3.821	3.830	--	109.847	18.699	1.703	0.999	3.822	7.3	5.0
3.137	3.141	3.138	--	90.484	18.688	1.698	1.998	0	16.5	1.0
3.137	3.141	3.073	--	89.732	18.688	1.698	1.998	3.141	19.0	3.2
2.695	2.696	2.691	--	77.709	18.701	1.707	2.999	0	22.9	- 3.8
2.637	2.696	2.691	--	77.230	18.693	1.706	2.999	2.697	0	- 1.4
2.377	2.378	2.384	--	68.675	18.705	1.703	3.998	0	8.9	- 6.5
2.417	2.381	2.416	--	69.383	18.705	1.707	3.999	0[h]	42.0	- 5.6
2.385	2.381	2.379	--	68.748	18.716	1.707	3.999	2.388	34.5	- 8.9
1.972	1.971	1.969	--	56.909	18.723	1.718	6.001	0	15.1	- 4.5
1.972	1.971	1.969	--	56.909	18.723	1.711	6.001	1.939	31.6	- 0.4

Table 4. (Cont'd)

Uranium: Height (in.)[a] for Radial Increments (in.) of 3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	Uranium: Mass (kg of U)	Uranium: Density (g/cm^3)	Graphite: Density (g/cm^3)	Graphite: Reflector Thickness[b] (in.)	Graphite: Height Within the Uranium Annulus (in.)	Reactivity of Assembly[c] (cents)	Reactivity of Support Structure[d] (cents)
Annuli Diameters: o.d. 12.996 in., i.d. 7.003 in. (cont'd)										
1.694	1.724	1.729	--	49.499	18.678	1.726	8.000	0	0.9	- 7.8
1.694	1.724	1.729	--	49.499	18.678	1.726	8.000	1.688	16.2	5.4
1.254	1.256	1.219	--	35.800	18.706	1.756	16.000	0	7.5	1.7
1.224[f]	1.256	1.255[i]	--	35.976	18.706	1.758	16.000	1.251	27.1	1.0
Annuli Diameters: o.d. 10.997 in., i.d. 9.002 in.										
--	4.943	--	--	47.559	18.737	1.697	11.998	0	11.7	4.3
--	4.505	--	--	43.372	18.749	1.701	12.001	4.449	28.9	3.4
--	4.019	--	--	38.607	18.707	1.757	17.004	0	25.9	1.3
--	4.697	--	--	35.518	18.709	1.758	17.007	3.692	-22.3	1.3
Annuli Diameters: o.d. 10.997 in., i.d. 7.003 in.										
6.950	6.953	--	--	120.350	18.710	1.688	1.000	0	-18.7	11.0
6.070	6.075	--	--	105.159	18.715	1.691	1.000	6.085	- 4.4	8.4
5.134	5.137	--	--	88.916	18.712	1.690	2.000	0	23.4	- 1.6
4.755	4.758	--	--	82.400	18.722	1.690	1.999	4.754	16.0	- 2.3
4.261	4.195	--	--	73.106	18.703	1.703	2.999	0	1.0	1.2
4.009	4.007	--	--	69.383	18.710	1.672	3.000	4.008	-20.6	4.3
3.670	3.662	--	--	63.427	18.701	1.688	4.002	0	- 4.4	- 1.6
3.544	3.532	--	--	61.251	18.714	1.709	4.000	3.509	14.4	- 2.2
2.791	2.755	--	--	47.944	18.699	1.685	6.997	0	- 3.2	6.2
2.697	2.726	--	--	46.932	18.695	1.674	6.997	2.700	- 0.3	4.1
2.443	2.444[i]	--	--	42.277	18.704	1.726	8.999	0	-10.3	- 5.2
2.414	2.413	--	--	41.764	18.702	1.720	9.002	2.384	15.3	- 4.0
1.882	1.849	--	--	32.208	18.678	1.758	16.999	0	-10.4	1.0
1.882	1.849	--	--	32.208	18.678	1.758	16.999	1.879	33.7	1.0

Table 4. (Cont'd)

Uranium: Height (in.)[a] for Radial Increments (in.) of 3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	Uranium: Mass (kg of U)	Uranium: Density (g/cm³)	Graphite: Density (g/cm³)	Graphite: Reflector Thickness[b] (in.)	Graphite: Height Within the Uranium Annulus (in.)	Reactivity of Assembly[c] (cents)	Reactivity of Support Structure[d] (cents)
Annuli Diameters: o.d. 8.996 in., i.d. 7.003 in.										
5.225	--	--	--	40.244	18.768	1.707	12.976	0	26.5	2.5
4.695	--	--	--	36.135	18.754	1.707	13.013	4.693	- 2.8	2.2
4.386	--	--	--	33.726	18.737	1.757	18.001	0	- 5.4	1.0
4.069[f]	--	--	--	31.320	18.748	1.757	18.005	4.065	11.0	1.0

a. These heights, for a particular assembly, show the unevenness of the upper surface of the uranium. The dimensions of the radial increments given in the column heading are nominal values which are larger than the inside diameters of annuli by less than 0.006 in. and are smaller than the outside diameters of annuli or cylinders by less than 0.004 in. The actual diameters of the annuli are given in the table except where noted.

b. The thicknesses of the graphite reflector on all outer surfaces were equal.

c. These values were for the assembly as described in this table and were not corrected for the effects of the support structure. The reactivity of the assembly described by the first entry in the table corrected for the effects of the support structure is 13.1 cents.

d. The reactivity change associated with the addition of the support structure.

e. The outside diameter of this annulus was 0.001 in. larger than that given.

f. The inside diameter of this annulus was 0.001 in. smaller than that given.

g. For the annuli without reflector, there were no measurements or calculations of the reactivity changes associated with a uranium height change. These assemblies were not corrected to delayed criticality with a uniform height for all radial increments and are omitted from Table 5.

h. These measurements were repetitions of the measurements given by the previous entry in the table. These assemblies were constructed of different uranium parts and in some cases different graphite parts.

i. The outside diameter of this annulus was 0.001 in. smaller than that given.

in a given radial increment was measured or estimated by interpolation of other measurements; these values are given in Table 5. The dimensions of these assemblies were corrected to those for a delayed critical assembly with a uniform height and without support structure using the reactivities listed in Table 4 and the reactivity changes associated with changes in the height of uranium given in Table 5. The observed dimensions and reactivities of the graphite-reflected uranium cylinders are given in Table 6. The reactivity changes due to changes in the height of uranium in a given radial increment, and the critical dimensions which were obtained by correcting for the reactivity effect of the support structure and the nonuniformity of the uranium height in the assemblies are given in Table 7. The critical mass of some of the cylinders and annuli as a function of dimension are plotted in Figs. 4 through 6.

Table 5. Critical Dimensions and Uranium Height Perturbation Measurements for Graphite-Reflected Uranium Annuli. (C designates graphite.)

Critical Height[a] (in.)	Critical Mass[a] (kg)	Uranium Reactivity per Height Change (cents/mil) for radii (in.) of[b] 3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	Graphite Reflector Thickness (in.)	Material Within Uranium Annulus
Annuli Diameters: o.d. 14.995 in., i.d. 13.002 in.							
5.157	69.22	--	--	--	0.65	10.012	Air
4.542	61.02	--	--	--	1.13	10.012	C
3.631	48.80	--	--	--	1.02	15.000	Air
3.431[c]	46.08	--	--	--	1.43	15.000	C
Annuli Diameters: o.d. 14.995 in., i.d. 11.003 in.							
6.324	157.89	--	--	0.26	0.24	0.996	C
5.313	132.76	--	--	0.54	0.47	1.997	Air
4.879	121.85	--	--	(0.58)[d]	0.50	1.997	C
4.312	107.64	--	--	0.76	0.73	2.997	Air
4.063	101.51	--	--	0.85	0.71	2.997	C
3.248	81.07	--	--	0.75	0.81	4.996	Air
3.138	78.32	--	--	1.26	1.04	4.996	C
2.194[e]	54.84	--	--	1.27	1.28	10.004	Air
2.161[e]	54.03	--	--	1.31	1.34	10.004	C
1.706[e]	42.67	--	--	(1.08)	(1.05)	14.998	Air
1.697[e]	42.44	--	--	1.16	1.08	14.998	C
Annuli Diameters: o.d. 14.995 in., i.d. 9.002 in.							
3.981	137.67	--	--	--	--	0.999	Air
3.968	137.79[f]	--	0.63	0.69	0.55	0.999	Air
3.879	134.11	--	(0.66)	(0.68)	(0.55)	0.999	C
3.887	134.38[f]	--	--	--	--	0.999	C
3.184	110.18	--	0.69	0.91	0.66	2.000	Air
3.156	109.17	--	0.72	0.73	0.67	2.000	C
2.714	93.87	--	0.77	0.99	0.71	3.001	Air
2.694[e]	93.17[f]	--	--	--	--	3.005	Air
2.694[e]	93.12	--	0.79	0.93	0.99	3.001	C
2.691[e]	93.07[f]	--	--	--	--	3.005	C
2.145	74.13	--	0.91	1.02	0.94	4.996	Air
2.134	73.75	--	1.27	1.30	0.86	4.996	C
1.509	52.18	--	1.81	1.95	1.10	9.997	Air
1.507	52.09	--	1.64	1.81	1.34	9.997	C
1.189[e]	41.22	--	1.32	1.79	1.54	14.998	Air
1.188[e]	41.18	--	1.30	1.73	1.56	14.998	C

Table 5 (Cont'd)

Critical Height[a] (in.)	Critical Mass[a] (kg)	Uranium Reactivity per Height Change (cents/mil) for radii (in.) of[b] 3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	Graphite Reflector Thickness (in.)	Material Within Uranium Annulus
	Annuli Diameters: o.d. 14.995 in., i.d. 7.003 in.						
3.058	129.34	0.59	0.79	0.80	0.53	0.996	Air
3.072	129.92	(0.58)	(0.71)	(0.70)	(0.45)	0.996	C
2.495	105.55	0.52	0.95	0.95	0.65	2.002	Air
2.501	105.76	0.73	0.82	0.83	0.55	2.002	C
2.147	90.75	0.76	0.77	1.11	0.80	3.000	Air
2.149	90.86	0.69	0.96	1.08	0.78	2.999	C
1.711	72.28	0.92	1.19	1.18	0.88	4.998	Air
1.713	72.32	0.97	1.33	1.24	1.00	4.998	C
1.442[e]	60.97	1.31	1.52	2.11	1.36	6.997	Air
1.442[e]	60.97	1.25	1.11	1.57	0.90	6.997	C
1.209[e]	51.02	1.10	1.74	1.81	1.19	10.012	Air
1.207[e]	50.93	1.03	1.51	1.58	1.16	10.012	C
0.953[e]	40.24	1.18	1.46	1.42	1.28	14.997	Air
0.953[e]	40.25	0.88	1.51	1.55	1.28	14.997	C
	Annuli Diameters: o.d. 12.996 in., i.d. 11.003 in.						
4.940	57.09	--	--	0.81	--	10.997	Air
4.406	50.92	--	--	0.81	--	10.997	C
3.749	43.28	--	--	1.35	--	15.992	Air
3.504	40.50	--	--	(1.50)	--	15.992	C
	Annuli Diameters: o.d. 12.996 in., i.d. 9.002 in.						
6.194	131.08[f]	--	0.28	0.25	--	0.999	C
6.123	129.50[f]	--	--	--	--	0.998	C
5.193	109.85	--	(0.60)	(0.50)	--	2.001	Air
4.793	101.39	--	(0.65)	(0.57)	--	2.001	C
4.251	89.96	--	0.83	0.74	--	2.997	Air
4.024	85.14	--	0.95	0.84	--	2.996	C
3.645	77.18	--	0.84	0.74	--	3.999	Air
3.505	74.23	--	1.16	0.94	--	4.000	C
1.760	37.28	--	1.23	1.23	--	16.001	Air
1.747	37.01	--	1.50	1.78	--	16.001	C

Table 5 (Cont'd)

Critical Height[a] (in.)	Critical Mass[a] (kg)	Uranium Reactivity per Height Change (cents/mil) for radii (in.) of[b] 3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	Graphite Reflector Thickness (in.)	Material Within Uranium Annulus
Annuli Diameters: o.d. 12.996 in., i.d. 7.003 in.							
3.898	112.47	..........1.82..........			--	1.001	Air
3.888	112.18[f]				--	0.999	Air
3.801	109.64	0.58	0.63	0.43	--	0.999	C
3.133	90.31	0.79	--	0.78	--	1.998	Air
3.114	89.76	(0.80)	(0.82)	(0.62)	--	1.998	C
2.686	77.47	1.09	(1.10)	0.93	--	2.999	Air
2.674	77.11	0.91	1.01	0.78	--	2.999	C
2.374	68.50	--	--	--	--	3.998	Air
2.385	68.82[f]	0.94	1.34	0.74	--	3.999	Air
2.368	68.35	0.90	1.29	0.91	--	3.999	C
1.965	56.74	1.22	1.85	1.01	--	6.001	Air
1.961	56.63	0.93	1.34	0.96	--	6.001	C
1.712	49.34	1.32	1.24	1.12	--	8.000	Air
1.712	49.32	1.30	1.28	1.07	--	8.000	C
1.239	35.74	..........4.15..........			--	16.000	Air
1.240[h]	35.77	..........3.85..........			--	16.000	C
Annuli Diameters: o.d. 10.997 in., i.d. 9.002 in.							
4.934	47.47	--	0.79	--	--	11.998	Air
4.478	43.12	--	0.96	--	--	12.001	C
4.005	38.47	--	1.70	--	--	17.004	Air
3.712	35.67	--	1.54	--	--	17.007	C
Annuli Diameters: o.d. 10.997 in., i.d. 7.003 in.							
7.006	121.29	0.27	0.24	--	--	1.000	Air
6.096	105.56	0.35	0.28	--	--	1.000	C
5.115	88.55	0.62	0.58	--	--	2.000	Air
4.743	82.16	0.69	0.64	--	--	1.999	C
4.230	73.21	0.84	0.74	--	--	2.999	Air
4.024	69.67	0.84	0.71	--	--	3.000	C
3.669	63.49	1.17	0.91	--	--	4.002	Air
3.530	61.13	0.80	0.85	--	--	4.000	C
2.778	48.05	1.17	1.20	--	--	6.997	Air
2.714	46.95	1.12	1.13	--	--	6.997	C
2.446[i]	42.32	0.74	0.75	--	--	8.999	Air
2.408	41.67	1.52	1.43	--	--	9.002	C
1.871	32.33	1.43	1.26	--	--	16.999	Air
1.857	32.09	1.87	1.82	--	--	16.999	C

Table 5 (Cont'd)

Uranium							Graphite	Material
Critical Height[a] (in.)	Critical Mass[a] (kg)	Reactivity per Height Change (cents/mil) for radii (in.) of[b]					Reflector Thickness (in.)	Within Uranium Annulus
		3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5			
Annuli Diameters: o.d. 8.996 in., i.d. 7.003 in.								
5.197	40.03	0.86	--	--	--		12.976	Air
4.705	36.21	0.75	--	--	--		13.013	C
4.391	33.77	1.23	--	--	--		18.001	Air
4.055[e]	31.21	0.87	--	--	--		18.005	C

a. These values were corrected for the reactivity of the support structure and for the unevenness of the top of the uranium, given in Table 4. using the reactivity per uranium height change tabulated above. The error in the mass is taken as ±10 or ±20% of the difference between the assembled mass and the critical mass. If all reactivity changes per unit uranium height were from measurements, then ±10% error is assumed. These critical masses are for the assemblies with uranium densities as given in Table 4.

b. The dimension of the radial increments given in the column headings are nominal values which are larger than the inside diameters of annuli by less than 0.006 in. and are smaller than the outside diameter of annuli or cylinders by less than 0.004 in. The actual inside and outside diameters of the annuli are given in the table except where noted.

c. The outside diameter of this annulus was 0.001 in. smaller than that given.

d. The values in parentheses are estimates based on other measurements and are known to be ±10 to ±20%.

e. The inside diameter of this annuli was 0.001 in. smaller than that given.

f. These results are from repetition of the measurements of the previous entry in the table with different uranium or graphite parts.

g. The values indicated in this manner are for the radial increment from 3.5 to 6.5 in.

h. The diameters of this annulus were 0.001 in. smaller than those given.

i. The outside diameter of this annulus was 0.001 in. smaller than that given.

Table 6. Dimensions and Reactivities of Graphite Reflected Uranium Cylinders.

	Uranium							Graphite		Reactivity	Reactivity
Diameter (in.)	Height (in.)[a] for Radial Increments of (in.) 0-3.5	3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	Average Density (g/cm^3)	Mass (kg)	Average Density (g/cm^3)	Reflector Thickness[b] (in.)	of the Assembly[c] (cents)	of Support Structure (cents)
14.997	1.066	1.068	1.042	1.038	1.065	18.662	57.036	1.711	7.019	11.8[d]	18.0[e]
14.995	0.874	0.878	0.882	0.879	0.877	18.660	47.402	1.710	10.007	9.6	14.3
14.997	0.749	0.687	0.691	0.689	0.685	18.743	38.04[illegible]	1.756	14.997	-4.6[d]	2.8
12.995	0.939	1.005	1.005	1.006	--	18.691	40.060	1.711	11.013	15.0	9.5
12.996	0.812	0.850	0.845	0.845	--	18.692	33.983	1.760	15.996	13.3	4.0
10.996	1.127	1.130	1.130	--	--	18.704	32.856	1.709	12.015	1.5[d]	6.3
8.997	1.381[f]	1.414	--	--	--	18.637	27.187	1.708	13.013	25.9[d]	3.0
8.997	1.256	1.288	--	--	--	18.774	24.813	1.757	18.004	1.6	1.0
6.996	1.897[g]	--	--	--	--	18.740	21.919	1.705	14.008	8.7	1.9

a. These heights, for a particular assembly, show the unevenness in the upper surface of the uranium. The dimensions of the radial increments given in the column heading are nominal values which are larger than the inside diameter of annuli by less than 0.006 in. and are smaller than outside diameters of annuli or cylinders by less than 0.004 in.

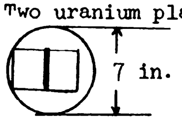
b. The thicknesses of the graphite reflector on all surfaces were equal.

c. These reactivities were not corrected for the effect of the support structure. The reactivity of the first entry in the table corrected for the effect of the support structure is -6.2 cents.

d. These values have been corrected for the effect of small differences in the reflector thicknesses from those given in this table. These corrections were -5.3, -4.6, +2, and +6.5 cents, respectively. For the other entries this effect was negligible (< 0.5 cents).

e. A 0.010-in.-thick stainless steel diaphragm divided this assembly 1.0 in. below the bottom of the uranium cylinder. The reactivities of the diaphragm and the support structure were -5.1 and 23.0 cents, respectively.

f. Two uranium plates 2.500 x 2.500 x 0.031 in. were located on top of this uranium cylinder as shown in the following sketch.



Their mass, 120 g, was included in the height given as though it were uniformly distributed. The cylinder height without the plates, 1.394 in., and the mass excluding that of the plates was used to calculate the uranium density.

g. A 5.000 x 5.00-in.-square, 0.063-in.-thick piece of uranium was located on top of this uranium cylinder and its mass was included in the height as though it were uniformly distributed. The cylinder height without the plate, 1.816 in., and the mass excluding that of the plate was used to calculate the uranium density.

Table 7. Critical Dimensions and Uranium Height Perturbations for Graphite-Reflected Uranium Cylinders.

Uranium								Graphite Reflector Thickness (in.)
Diameter (in.)	Critical Height[a] (in.)	Critical Mass[a] (kg)	Reactivity per Height Change (cents/mil) for Radial Increments (in.) of[b]					
			0-3.5	3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	
14.996	1.059	57.19	(2.28)[c]	1.22	1.18	0.97	0.80	7.019
14.955	0.878	47.42	(2.30)	1.14	1.39	1.14	0.88	10.009
14.997	0.709	38.48	(2.52)	(1.30)	(1.36)	(1.34)	(1.10)	14.997
12.995	0.982	39.88	2.54[d]	1.67	1.55	1.35	--	11.013
12.996	0.831	33.78	(2.55)	1.30	1.35	1.15	--	15.996
10.996	1.130	32.89	(2.97)	1.60	1.52	--	--	12.015
8.997	1.387	26.92	(3.16)	1.26	--	--	--	13.013
8.997	1.264	24.71	(4.18)	(1.67)	--	--	--	18.004
6.996	1.895	22.37	4.07[d]	--	--	--	--	14.008

a. These values were corrected for the reactivity of the support structure and for the unevenness of the top of the uranium, given in Table 6, using the reactivity per uranium height change tabulated above. The error in the mass is taken as ±10 or ±20% of the difference between the assembled mass and the critical mass. If all reactivity changes per unit uranium height were from measurements, then ±10% error is assumed. These critical masses are for the assemblies with uranium densities as given in Table 6.

b. The dimensions of the radial increments given in the column heading are nominal values which are larger than the inside diameter of annuli by less than 0.006 in. and are smaller than the outside diameter of annuli or cylinders by less than 0.004 in.

c. Values in parentheses are from perturbation theory calculations using forward and adjoint angular fluxes from S_4 transport theory calculations. Other values were measured directly unless noted.

d. From measurements of the worth of a 2.500 x 2.500 x 0.031 in. piece of uranium as a function of radius.

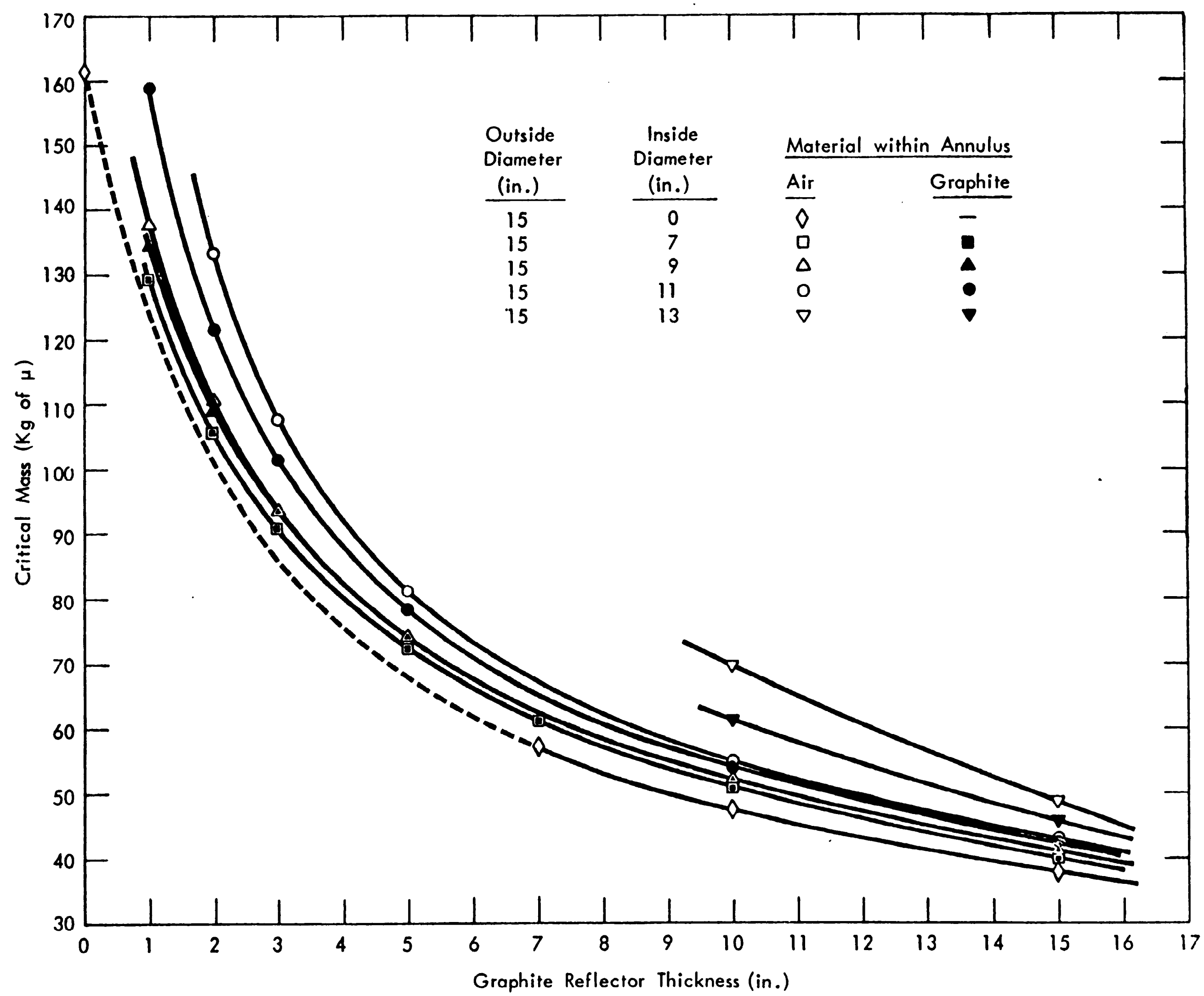


Fig. 4. Critical Mass of 15 in. o.d. Uranium Annuli and Cylinders as a Function of Graphite Reflector Thickness.

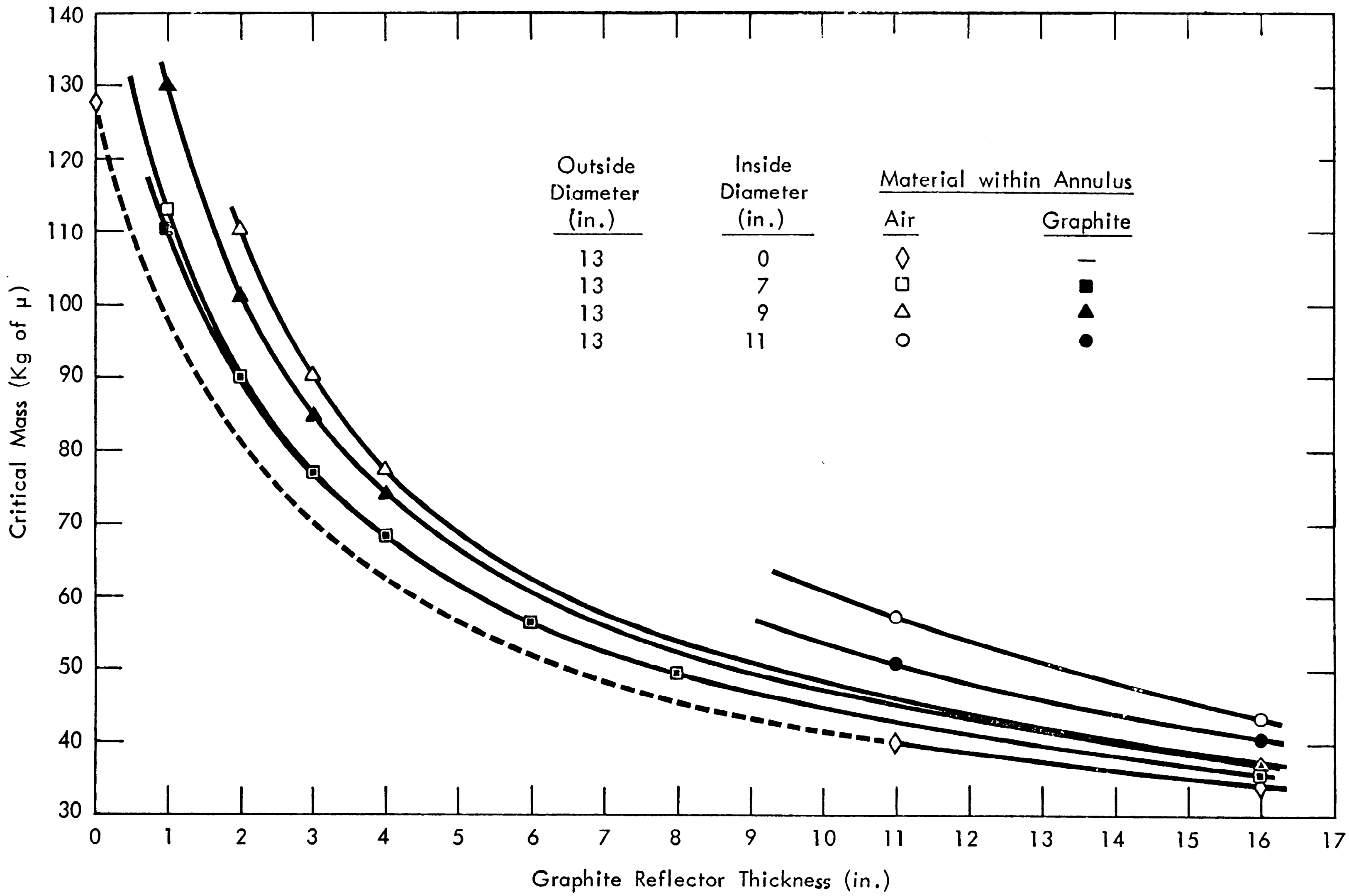


Fig. 5. Critical Mass of 13-in.-o.d. Uranium Annuli and Cylinders as a Function of Graphite Reflector Thickness.

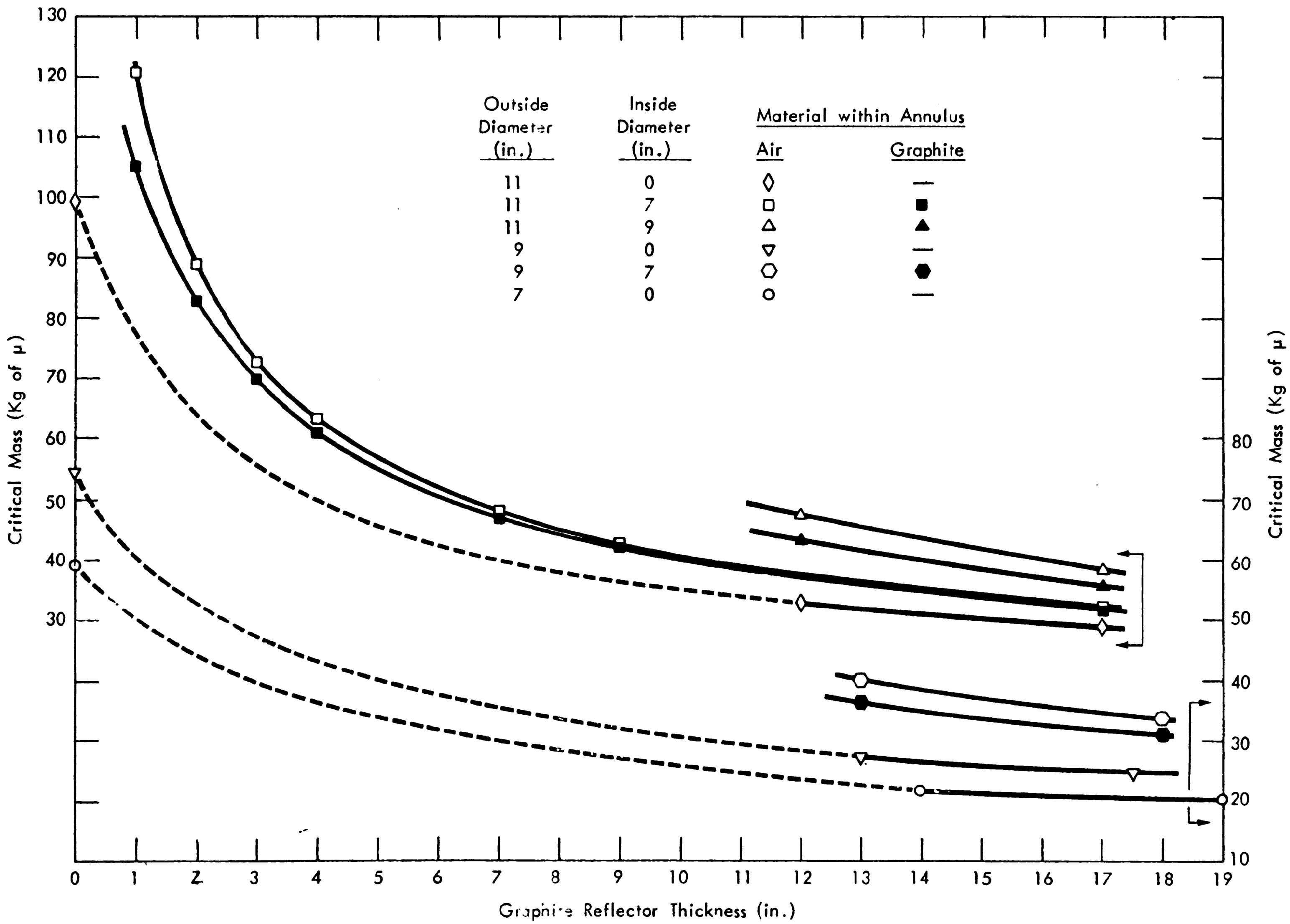


Fig. 6. Critical Mass of 11-, 9-, and 7-in.-o.d. Uranium Annuli and Cylinders as a Function of Graphite Reflector Thickness.

CRITICALITY CALCULATIONS

Transport theory and Monte Carlo calculations were performed to determine the adequacy of these methods in predicting the neutron multiplication factor for assemblies of this type. The multiplication factors computed by both methods are compared with those from the experiments.

Transport Theory Calculations

The neutron multiplication factors of some of the experiments, calculated by the S_n method with the 16-group cross sections of Hansen and Roach[5] for the ^{235}U and ^{238}U isotopes of uranium and for carbon and hydrogen, are compared with the experimental values in Table 8. The two-dimensional transport theory codes, TDC,[6] DDK,[7] and DOT[3] were used in the calculations. The spatial interval near the boundary of the thermalizing reflector and the uranium was small compared to the mean free path of thermal neutrons in the materials and adjacent intervals differed by less than a factor of two. This resulted in the proper treatment of the neutrons returning from the reflector. Coarse spatial intervals near this boundary led to overestimates of the neutron multiplication factor by as much as 2%. The agreement between the calculated and experimental results was good except for the annulus with the 17.004-in.-thick graphite reflector. Excluding this case the average difference between the calculated and experimental multiplication factor for the graphite-reflected assemblies was +0.07%. The average discrepancy for the polyethylene-reflected assemblies was -0.12%.

5. Gordon E. Hansen and William H. Roach, "Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies," LAMS-2533, Los Alamos Scientific Laboratory (1961).
6. B. G. Carlson, C. E. Lee, and J. Worlton, "The DSN and TDC Neutron Transport Codes," LAMS-2346, Los Alamos Scientific Laboratory (1960).
7. B. G. Carlson and J. Worlton, Los Alamos Scientific Laboratory, Private Communication (1968).

Table 8. Comparison of Calculated and Measured Multiplication Factors for Graphite and Polyethylene Reflected Uranium Metal Cylinders and Annuli. (C and CH_2 designate graphite and polyethylene, respectively.)

Uranium Dimensions[a] (in.)		Material	Reflector		Multiplication Factor		Order	Convergence	
Outside Diameter	Inside Diameter	Within Annulus	Material	Thickness (in.)	Measured[b]	Calculated	of S_n	Criteria ϵ	Computer Code
				Transport Theory Calculation					
14.996	9.002	C	--	--	0.9992	0.9980	8	10^{-5}	TDC
14.996	7.003	C	--	--	0.9982	0.9963	8	10^{-5}	TDC
12.996	7.003	C	--	--	1.0011	1.0014[c]	8	10^{-5}	TDC
12.996	9.002	C	C	0.998	1.0009	1.0033	8	10^{-5}	TDC
14.996	11.003	C	C	2.997	1.0012	1.0083[d]	6	2×10^{-4}	DOT
10.996	7.003	Air	C	6.997	1.0000	0.9975	8	10^{-4}	DDK
10.996	7.003	C	C	6.997	0.9998	0.9989	8	10^{-4}	DDK
14.996	7.003	Air	C	10.012	1.0016	1.0042	8	10^{-4}	DDK
8.996	7.003	Air	C	12.976	1.0018	1.0024	8	10^{-4}	DDK
10.996	9.002	Air	C	17.004	1.0018	1.0220[e]	6	2×10^{-4}	DOT
14.996	0	--	CH_2	≥ 4.83	1.0006	0.9939	8	2×10^{-4}	DOT
13.996	0	--	CH_2	≥ 5.83	1.0030	1.0045	8	2×10^{-4}	DOT
10.996	0	--	CH_2	≥ 6.85	1.0017	1.0051	8	2×10^{-4}	DOT
8.996	0	--	CH_2	≥ 7.82	1.0000	1.0033	8	2×10^{-4}	DOT
6.996	0	--	CH_2	≥ 8.82	1.0004	0.9933	8	2×10^{-4}	DOT
				Monte Carlo Calculations					
14.996	11.003	C	C	2.997	1.0012	1.010 ± 0.006[f]			
14.996	11.003	Air	C	2.997	1.0018	1.018 ± 0.005[f]			
14.996	9.002	Air	C	3.001	1.0024	1.005 ± 0.005[f]			
12.996	9.002	Air	C	2.997	1.0017	1.019 ± 0.006[f]			
10.996	7.003	C	C	3.000	0.9998	1.006 ± 0.007[f]			
14.996	11.003	C	C	14.998	0.9996	0.997 ± 0.008[f]			
14.996	9.002	C	C	14.998	1.0011	0.997 ± 0.007[f]			
12.996	7.003	C	C	16.000	1.0018	1.004 ± 0.007[f]			
10.996	9.002	Air	C	17.004	1.0018	0.993 ± 0.011[g]			
8.996	0	--	C	18.004	1.0008	1.003 ± 0.0097[f]			
8.996	7.003	C	C	18.005	1.0008	1.002 ± 0.0064[f]			
8.996	7.003	Air	C	18.001	0.9996	1.005 ± 0.0094[f]			
14.995	11.003	Air	CH_2	≥ 4.83	1.0010	1.006 ± 0.006[h]			
14.995	11.003	CH_2	CH_2	≥ 4.83	1.0014	1.003 ± 0.006[h]			
14.996	7.003	Air	CH_2	≥ 4.83	1.0020	1.012 ± 0.008[h]			
14.996	7.003	CH_2	CH_2	≥ 4.83	1.0016	1.007 ± 0.005[h]			
14.995	0	--	CH_2	≥ 4.83	1.0006	1.006 ± 0.007[h]			
10.997	7.003	Air	CH_2	≥ 6.86	1.0001	0.995 ± 0.005[h]			

a. Further descriptions of these assemblies are given in Tables 2, 4, and 6.

b. A value of β_{eff} 0.0068 was used to convert the reactivities given in Tables 2, 4, and 6 to neutron multiplication factor.

c. Multiplication factors from S_2, S_4, and S_6 calculations were 0.9683, 0.9988, and 1.0013.

d. The multiplication factor from an S_4 calculation with $\epsilon = 5 \times 10^{-4}$ was 1.0000. The computation time on the IBM 360-75 for the S_6 calculation was about one hour.

e. The multiplication factor for an S_4 calculation was 0.0006 smaller. The computation time for the IBM 360-75 for the S_6 calculation was 1.5 hours.

f. The results of the first three batches of 100 batches of 200 neutrons were discarded for the neutron multiplication factor calculation. The calculation time on the IBM 360-91 for the assemblies with ~3-in.-thick graphite reflector varied from 1.0 to 1.2 minutes while that for the other annuli were 4.7, 4.6, 4.5, 2.6, 4.5, and 2.6 minutes, respectively.

g. The results of the first three batches of 100 batches of 200 neutrons were discarded for the neutron multiplication factor calculation. The calculation time on the IBM 360-75 was 8.1 minutes.

h. The first three batches of 62 batches of 250 neutrons were neglected for the neutron multiplication factor calculation.

Monte Carlo Calculations

Since the time required for an S_6 transport theory calculation was prohibitive ($\sim$1.5 hr) on the IBM 360-75, Monte Carlo calculations were performed for some assemblies using the KENO[8] code and the Hansen-Roach cross sections to determine if a reasonable estimate of the multiplication factor ($\sim \pm 1\%$ standard deviation) could be obtained in a shorter computing time. The KENO code used was a multigroup Monte Carlo method with isotropic scattering and with the neutron slowing down treated by a transfer matrix. In these calculations the source neutrons were put in the uranium uniformly for the first iterations. The resulting fission distribution was used for the next iteration. Subsequent iterations used the distributions from previous iterations. The results of the first few iterations were not used in the calculation of the multiplication factor. The neutron weight below which neutrons were killed by Russian Roulette was a function of the position in the reflector for the graphite-reflected assemblies, while for the polyethylene-reflected assemblies it was not. This function was inversely proportional to the adjoint flux, obtained from S_n transport theory calculations for a delayed critical uranium-metal sphere surrounded by an 18-in.-thick graphite reflector. This function was used for all graphite reflectors and was truncated for reflector thicknesses less than 18 in. Typical times for the calculations of 100 batches of 200 neutrons on the IBM 360-91 were 4 and 1 min for annuli with 15- and 3-in.-thick graphite reflectors, respectively. For the 10.996-in.-o.d., 9.002-in.-i.d. uranium annulus with 17.004-in.-thick graphite reflector, the Monte Carlo calculation satisfactorily predicted the multiplication in a factor of $\sim$12 less computing time. This large reduction in calculation time results from the complication of the transport theory

8. G. E. Whitesides and N. F. Cross, "KENO-A Multigroup Monte Carlo Criticality Program," CTC-5, Computing Technology Center, Oak Ridge Gaseous Diffusion Plant (1969).

calculation, i.e., the large number of spatial intervals, high order of S_n and the tight convergence criteria required to produce a reliable value of the multiplication factor. The average neutron multiplication factor from the Monte Carlo calculations was 0.3% larger than the experimental.

CONCLUSIONS

These experiments in addition to providing information of use in establishing the nuclear safety of uranium fabrication facilities have verified the ability of calculation methods to predict the criticality of polyethylene-uranium or graphite-uranium assemblies. In particular the transport theory code DOT in the S_8 approximation with the 16-group cross sections of Hansen and Roach has predicted the multiplication factor for the polyethylene-uranium and graphite uranium assemblies very well. However, the calculation time was long because of the complication of the transport theory calculations, i.e., the high order of S_n, the large number of spatial intervals particularly near the boundary of the uranium and the thermalizing reflector, and the tight convergence criteria, all required to obtain a reliable value of the multiplication factor. Monte Carlo calculations using the KENO code and the same cross sections have also satisfactorily predicted the multiplication factor in a factor of at least 10 less computing time.

ACKNOWLEDGEMENTS

The work of the staff of the Oak Ridge Critical Experiments Facility is acknowledged, in particular the work of J. J. Lynn and J. R. Taylor in performance of the experiments and handling of the data. Appreciation is expressed to J. R. Knight* and G. E. Whitesides* of the Oak Ridge Computing Technology Center for their work in transport theory and Monte Carlo calculations.

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*Now at Oak Ridge National Laboratory.