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CRITICAL PARAMETERS OF FISSIONABLE MATERIALS SYSTEMS AND

NUCLEAR SAFETY

(A HANDBOOK)

by B. G. Dubovskiy, et al

- USSR -

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CRITICAL PARAMETERS OF FISSIONABLE MATERIALS SYSTEMS AND NUCLEAR SAFETY
(A HANDBOOK)

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Following are translations of selected articles from the Russian-language book Kriticheskiye Parametry Sistem s Delyashchimisya V shchestvami i Yadernaya Bezopasnost' (Spravochnik) / English version above_/, Moscow, Atomic Publishing House, 1966, pages 1-226. The articles are unsigned. The authors of the handbook are B. G. Dubovskiy, A. V. Kamayev, F. M. Kuznetsov, G. M. Vladykov, V. N. Gurin, A. P. Murashov, I. P. Markelov, V. P. Kochergin, A. A. Vaymugin, V. Ya. Sviridenko, L. V. Diyev, V. K. Bogatyrev, V. V. Vavilov, and V. V. Frolov.

CONTENTS

Page

Source Report

BASIC CONCEPTS OF NUCLEAR SAFETY

1. The Concept of Criticality

The nuclei of some heavy elements, capturing neutrons, disintegrate
into several nuclei-fragments; during disintegration, two to three secondary
neutrons are released. This process is called fission. The secondary
neutrons can be captured by other nuclei of these elements, with their
subsequent division and the release of new secondary neutrons. There thus
appears the possibility of the appearance of a self-sustaining fission
chain reaction of atomic nuclei. The fission process is accompanied by
the release of a great quantity of energy and radioactivity.

Utilized for the realization of a fission chain reaction are the
isotopes U^{233} and U^{235}, as well as Pu^{239}, which are called fissionable
isotopes, and the substances containing them are called fissionable
substances.* The part of a system in which fissionable substances are
located and in which a fission chain reaction is realized forms the fuel
core, and the layer of materials surrounding the core forms the reflector.

Not all the secondary neutrons appearing during the fission process
are captured by the fissionable isotopes. A number of them are lost as a
result of escaping from a system (neutron escape); some are captured by the
nuclei of nonfissionable elements which are present in the form of construc-
tion materials, admixtures, etc. Therefore, the appearance of a self-sus-
taining chain reaction requires that the escape of neutrons from a system
and their capture by non fissionable elements not exceed a certain magnitude
/1_7.

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A convenient magnitude employed in analyzing a system with fissionable

* substances = materials

substances is the effective multiplication factor K_{eff}, which is the ratio of the neutrons of the first generation to the number of neutrons of the (i-1) generation. If K_{eff} = 1, then the number of neutrons in a system remains unaltered, i.e., the balance of neutrons is maintained:

<div align="center">Formation = escape + capture</div>

The state of a system during K_{eff} = 1 is called critical, and the values of the parameters characterizing its composition and dimensions with which this state are attained are called critical parameters (the critical mass, critical volume, critical dimensions of the fuel core, critical concentration of the fissionable substance). During $K_{eff} < 1$, a self-sustaining chain reaction is impossible (subcritical state); during $K_{eff} > 1$, the intensity of the chain reaction exponentially increases with the passage of time (supercritical state).

The secondary neutrons released during fission have an average energy of two million electron volts (Mev). A nuclear chain reaction can be caused by fast, intermediate, and thermal neutrons. Neutrons with an energy of over 100 kiloelectron volts (Kev) are called fast neutrons; those with an energy ranging from 100 Kev to thermal, intermediate; neutrons whose kinetic energy is on the order of the critical temperature (kT), i.e., about 0.025 electron volts (Ev), thermal neutrons. In order that a chain reaction occur basically with thermal or intermediate neutrons, it is essential to have in the fuel core a substantial quantity of a moderator, i.e., elements whose collision with nuclei cause neutrons to lose a substantial amount of their energy. The occurrence of a chain reaction with fast neutrons requires the absence of neutron moderators in the active zone.

2. Basic Factors Affecting Criticality

A self-sustaining fission chain reaction is possible only if the quantity of a fissionable substance is equal to the critical mass or greater than it. This quantity depends on a number of factors which are examined below.

Form* of the Fuel Core.

The form of the fuel core exerts a great influence on the escape of neutrons from a system. It is well known that the greater the ratio of the exterior surface area of the fuel core to its volume, the greater is the neutron escape. Consequently, a reduction of the linear dimensions of the fuel core results in an increase of the neutrons escaping from it, since the surface is reduced proportionally to the square, and the volume, proportionally to the cube of the linear dimensions. Of the correct geometric bodies of uniform volume, the sphere possesses the least ratio of exterior surface area to volume. Therefore, systems equipped with a fuel core in the form of a sphere have the least critical mass and critical volume.

For cylindrical fuel cores, a core in the form of an approximately

* Form = Shape

equilateral cylinder has the least critical mass and critical volume, i.e., a cylinder whose height is equal to the diameter; and for a fuel core in the form of a parallelepiped in the form of a cube, for an equilateral cylinder and cube approximate the form of a sphere. With a substantial deviation of the form of the fuel core from the spherical, the critical mass and critical volume sharply increase. For example, for aqueous uranium solutions having an aqueous reflector, when the ratio of the height to the diameter of the fuel core becomes 0, 2, or 4, the critical mass increases by about 1.8 times, compared with a critical mass of an equilateral cylinder. With a corresponding radius of a cylindrical fuel core or the height of a fuel core in the form of a plane layer, the escape of neutrons from a system becomes somewhat greater; with the maximum reduction of the radius or height, a chain reaction becomes impossible, the quantity of fissionable substance in the system notwithstanding. The corresponding radius of the cylindrical fuel core is called the critical radius of an infinite cylinder, and the height of a fuel core in the form of a plane layer is called the critical height of the infinite plane layer.

The Action of Reflectors.

With the availability of a reflector, a part of the neutrons escaping from the fuel core are returned after colliding with the reflector material. Thus, the availability of a reflector leads to a reduction in the escape of neutrons from the fuel core and, consequently, to a reduction in the critical parameters of a system. The escape of neutrons from the fuel core is reduced with an increase in the thickness of a reflector. Despite this, the first layer of a reflector is most effective, the first reflector layer whose thickness is on the order of the length of a fast neutron's migration to the reflector. For an aqueous reflector, this thickness is about 6 centimeters (cm); for a graphite reflector, about 50 cm; and for concrete, about 30 cm. A further increase in the thickness of a reflector leads to an insignificant reduction of the critical parameters.

We shall consider two critical systems whose fuel core have the form of a sphere and whose compositions of the fuel core are uniform. The systems differ in that one is equipped with a reflector and the other is not. Because of the presence of a reflector, the critical radius of the fuel core of the first system will by γ cm less than that of the second. If the reflector is a layer of water of over 20-cm thickness and the fuel core is filled with an enriched uranium solution, the magnitude γ is equal to 3.3 cm. In this situation, the significance of γ is slightly dependent on the concentration of uranium in the solution, provided the change in the solution ranges from 20 to 300 grams per liter. With a change in the curvature of the fuel core surface, the magnitude γ is changed, but with those radii of the curvature most frequently encountered in practice (from 15 cm to infinity), this change can be ignored. With small thicknesses of reflectors, e.g., with an aqueous or a steel reflector of less than one-cm thickness, the magnitude γ is directly proportional to the thickness of the reflector.

The effectiveness of the action of a reflector increases with a reduction in the dimensions of the fuel core, since with small dimensions the role played by neutron escape increases. For metallic U^{235} and plutonium, the best materials for reflectors are beryllium, beryllium oxide, and natural uranium. For example, the critical mass of a system of metallic uranium of 93.5 percent enrichment is equal to 52 kilograms without a reflector and 8.9 kilograms when this system is equipped with a beryllium-oxide reflector. For systems whose fuel core are filled with aqueous solutions of fissionable substances, the best reflector materials are beryllium, beryllium oxide, water, heavy water, and graphite. Heavy water and graphite reflectors are less effective than water reflectors when the reflectors are of five- to ten-cm thickness, but more effective with greater reflector thickness.

The effectiveness of steel and water reflectors is about uniform for systems with solutions of fissionable substances. A reflector evidences maximum effectiveness when it is in direct contact with the fuel core. Even a small distance between the fuel core and the reflector substantially increases the critical dimensions of a system.

At enterprises processing nuclear fuel, the most widely used materials for reflectors are water, steel, and concrete. It is necessary to note that systems without reflectors are rarely encountered in plants, since the technologic apparatuses usually have thick steel walls (0.5-1 cm), all kinds of colling jackets, etc. In addition, the walls and floors of structures housing the technologic equipment also play the role of partial reflectors. Therefore, in prescribing the charge norm for technologic equipment, almost everybody considers that this equipment has a sound aqueous reflector. Since the presence of a reflector reduces the critical dimensions of a system, the task of reducing the effectiveness of reflector action frequently occurs. This problem can be solved by placing, on the border between the fuel core and a reflector, layers of materials having a good capability of capturing neutrons, specifically cadmium or boron.

The Density of a System

The critical parameters of a system are reduced with an increase in the density of the fuel core and reflector. A reduction of the density of an entire system to k time also leads to an increase in the critical dimensions to k time, the critical volume to k^3 time, and the critical mass to k^2 time. This is true provided that, with a reduction in the density of a system, the ratio of the densities of the fuel core and reflector remains constant.

In changing the density of only one fuel core (with the constant density of the reflector), the following relationship exists between the critical mass and density for spheres equipped with aqueous moderators and aqueous reflectors: critical mass (the density of the fuel core)$^{-n}$, since the level n changes from 1.5 to 2, depending on the ratio of the number of hydrogen atoms to the number of atoms of a fissionable isotope (ρ_H/ρ_5 or ρ_H/ρ_9) the fuel core (Figure 1.1).

Figure 1.1

Indicator of the level n for determining the critical mass of uranium and plutonium in a spherical fuel core equipped with an aqueous moderator and a reflector
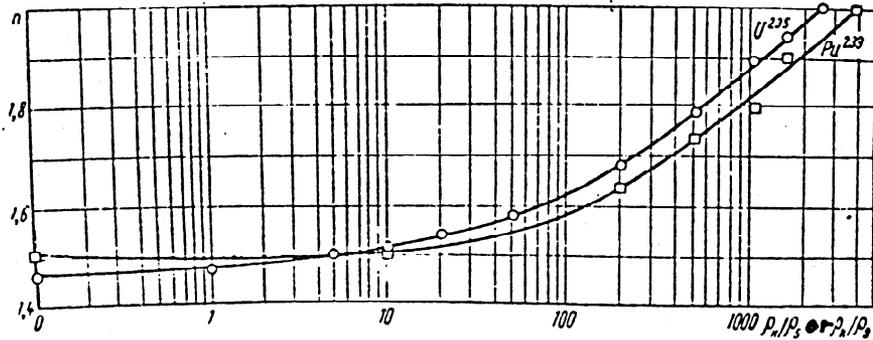
The Action of the Moderator.

If there are light-element nuclei in the fuel core, fissionable neutrons are slowed down upon colliding with these nuclei, i.e., they gradually lose their energy. The energy of neutrons is reduced most rapidly upon their collision with the nuclei of the lightest elements; consequently, the energy depends on the content of such light elements as hydrogen, beryllium, and carbon in the fuel core. With a sufficient, relatively small concentration of a fissionable substance and a small absorption cross section of neutrons in the moderator, all the neutrons are practically captured following the attainment of a thermal balance with the environment. A further reduction in the concentration of the fissionable substance does not result in a reduction of the average energy of neutrons, but only in an increase in the relative capture of the neutrons of fissionable isotopes.

The neutron absorption cross section by the nuclei of a fissionable isotope and the subsequent process of the fission of this nucleus are largely dependent on the energy of neutrons. In general, the less the energy of neutrons, the greater is the fission cross section -- -- which leads to a substantial reduction in the critical mass of a system. For example, for uranium-aqueous systems the critical mass, depending on the hydrogen content of the fuel core, is changed from 22 kilograms (metal) to 0.8 kilograms (solutions), despite the substantial reduction of the density of the fissionable substance in the fuel core.

The Action of the Absorbers

For moderator-equipped systems, the introduction of absorbers into fuel cores increases the critical dimensions. Absorbers can be introduced

both by homogeneous means in the form of solutions and powders and by heterogeneous means in the form of various kinds of absorbing insertions (rods, plates, worms, etc.). In almost every fissionable substance fed for processing, there are elements that capture neutrons. For example, U^{238} can be considered to be an absorber of thermal neutrons. For thoroughly diluted solutions, the capture of neutrons by the hydrogen of the environment becomes substantial. However, elements possessing a great capability of capturing neutrons -- elements such as cadmium, boron, etc. -- are employed for substantially increasing critical parameters.

The action of a homogeneously introduced absorber is more effective than that of the heterogeneously introduced variety, given equal weights. This is connected with the fact that a so-called blocking effect exists in a heterogeneous absorber. This phenomenon is the result of the most effective operation of only the external layers of the absorber, for the internal layers are partially or completely shielded by the external layers. With the homogeneous introduction of an absorber, there is no blocking effect. But in this situation the technologic method of processing nuclear fuel is frequently complicated, since in the terminal stages of processing it is essential to separate the absorbing element from the fissionable substance.

The effectiveness of the action of heterogeneous absorbers in initial approximation is proportional to the external surface area of an absorber. Thus, the effectiveness of a cylindrical absorbing rod grows with an increase of the radius of the rod in accordance with the linear law. The effectiveness of a rod centrally located in a cylindrical system is reduced with an increase in the radius of the fuel core. For a system without a reflector, the effectiveness of a single, central rod in initial approximation is inversely proportional to the square of the radius of the fuel core. When only a single absorbing rod is in an apparatus, it is advisable to place it in the center, since this is the location at which it is most effective. In placing several rods in an apparatus, it is essential to take into account the fact that the effectiveness of each of them is less than that of a single, central rod and that this effectiveness will depend on the number of rods and their spacing from one another.

It is essential to remember that the effectiveness of an absorber changes in accordance with the average energy of neutrons in the fuel core and, consequently, depends on the quantity of the moderator in the fuel core. Cadmium, for example, is effective only if the average energy of the neutrons does not exceed 0.4 Ev. With the energy of neutrons exceeding this level, the effectiveness of cadmium declines sharply. The effectiveness of boron is proportional to $\frac{1}{\sqrt{E_n}}$, where E_n is the average energy of the neutrons in the fuel core.

Absorbers can be utilized for ensuring nuclear safety only after their effectiveness has been experimentally verified. Especially advisable is the

introduction of heterogeneous absorbers for the purpose of increasing
the values of minimal critical parameters (minimal volume and radius of an
infinite cylinder). For example, the introduction of a boron rod of 58-
millimeter (mm) diameter increases the minimal critical radius of an
infinite cylinder equipped with an aqueous reflector for nitric-acid
solutions of uranium from 8 to 10 cm.

During the dilution of a fuel core in real systems, there occurs
simultaneously a change in the moderator and absorber and a change in density.
In order to get an idea of the action of these factors in the aggregate, we
will consider how the critical parameters of a spherical fuel core of PuO_2
will change with the addition of water to it. In this situation, we will
consider that the system is always homogeneous. When a small quantity of
water is added, both the critical mass and the critical volume are increased.
At this stage, the effect of the change in the density of the plutonium
predominates, and an additional moderator exerts little influence. However,
when the ratio of the number of hydrogen atoms to the number of plutonium
atoms exceeds four, the action of the moderator begins to be reflected, and
the critical mass is reduced although the critical volume continues to be
increased. With $(\rho_H/\rho_{Pu}) = 10 - 20$, the moderator also causes a reduction

in volume. Another addition of water reduces the critical mass until it
reaches the minimum. Subsequently, an addition of water causes an increase
of the critical mass, for the capture of neutrons by hydrogen then begins
to be reflected; and with $(\rho_H/\rho_{Pu}) > 2,800$, the critical mass and critical
volume become infinite, for in this situation the proportion of neutrons
captured by the hydrogen is so great that the factor K_{eff} becomes less
than a unit, thereby making the chain reaction impossible.

The Heterogeneity of a System.

The critical masses of systems in which fissionable substances are
found in the form of rods, blocks, etc. (heterogeneous systems) differ from
the critical masses of homogeneous systems. The critical masses of hetero-
geneous systems with enriched uranium of over five percent exceed the critical
masses of homogeneous systems. Conversely, with enriched uranium of less
than five percent, the critical masses are less in heterogeneous systems than
in the homogeneous variety.

Interaction.

With the presence of a number of containers having fissionable sub-
stances, it is essential to consider the conditions of the criticality of the
entire system of containers as a whole, and not each individual container.
This is connected with the fact that there exists neutron interaction among
the containers, for a part of the neutrons escaping from one container lands
in another and causes fission there. This leads to a situation in which the
value K_{eff} of the system of containers will be greater than the K_{eff} of the
separate containers which make up the system. The factor K_{eff} of a whole system
grows with an increase in the number of containers and a reduction of the distance
between them.

3. Some Properties of Fissionable Substances

During the processing of fissionable materials at plants, one encounters great quantities of various compounds and mixtures, in the composition of which are fissionable materials (plutonium, uranium variously enriched by the isotope U^{235} and U^{233}). Most frequently encountered are fissionable materials in the form of the following products: metals, chemical compounds (UO_2, U_3O_8, UC, UF_4, UF_6, PuO_2, etc.), aqueous solutions and hydrogen-containing mixtures, and waterless mixtures with a large quantity of fissionable isotopes.

We will briefly discuss some of the characteristics of these products.

Metals.

Metallic compounds have a great density of fissionable material; therefore the critical dimensions of such systems are very small. For example, the critical volume of a sphere of plutonium with the presence of an aqueous reflector totals 0.3 liter. It is necessary to note that the critical mass of metallic uranium increases sharply with a reduction of enrichment, and with an enrichment below five percent it reverts to infinity.

Table 1.1

Critical Parameters of Fissionable Substances in the Form of Metals

Critical parameters	U^{235}	U^{233}	Pu^{239}
Mass of a sphere, kilograms	22.8	7.5	5.6
Diameter of an infinite cylinder, centimeters	7.8	4.8	4.3
Thickness of the infinite plate, centimeters	1.5	0.7	0.6

Table 1.1 presents the basic critical parameters of systems equipped with the usual aqueous reflector. The fuel cores of these systems consist of metallic U^{235}, U^{233}, and Pu^{239} (the density of the uranium is 18.8 g/cm^3, of the plutonium 19.6 g/cm^3 _[3]_.

Chemical Compounds.

It is characteristic of these compounds that their density (3-13 g/cm^3) is substantially less than that of metallic uranium and, therefore, all their critical parameters are greater than the corresponding parameters for metals. The presence in these compounds of an insignificant quantity of other elements (oxygen, fluorine, carbon, etc.) does not lead to a moderation of the neutrons and, consequently, does not reduce the critical mass. But the presence in them of even small quantities of hydrogen-containing substances (water, HF, paraffin,

glycerin) can sharply reduce critical parameters. This is especially true of uranium compounds of small enrichment.

Aqueous Solutions and Hydrogen-Containing Mixtures.

Solutions and hydrogen-containing mixtures of fissionable substances are frequently encountered in chemical plants processing nuclear fuel. There is the greatest amount of experimental and rating data on solutions and hydrogen-containing mixtures.

The magnitude of a critical mass largely depends on the concentration of hydrogen in these compounds. The concentration of hydrogen is usually expressed in the form of the ratio of the number of hydrogen atoms to the number of the atoms of the fissionable substance. This magnitude can be changed, ranging from zero for metals or dry, waterless, salt to several thousands for diluted aqueous solutions.

In lowering the concentration of the fissionable substance, a sharp reduction of the critical mass initially occurs. The critical mass then passes through the minimum (the minimal critical mass), following which it begins to increase. With a definite, very low concentration of the fissionable substance (minimal critical concentration), the critical mass increases to infinity. This means that during such a concentration the capture of neutrons of the fissionable isotopes is so great that the chain reaction becomes impossible. The minimal critical concentration for solutions corresponds to the ratio of the number of hydrogen atoms to the number of atoms of the fissionable substance, amounting to 2,800 for plutonium systems and 2,200 for uranium systems (U^{235}).

The critical volume of the sphere, the radius of the infinite cylinder, and the height of the infinite plane layer also largely depend on the concentration of hydrogen. Like the minimal critical mass, there exist the minimal critical volume of a sphere, the radius of the infinite cylinder, and the height of the infinite plane layer in definite ratios of the number of hydrogen atoms to the number of atoms of the fissionable substance. It is essential to note that these ratios are substantially less than those observed during the minimal critical mass of a sphere. For example, the minimum of a critical mass for solutions of U^{235} with a fuel core in the form of a sphere is noted in the ratio of the number of atoms of hydrogen to the number of atoms of U^{235}, which is 500, and the minimum of the critical volume in a ratio approximating 50.

Table 1.2 presents the critical parameters of systems containing solutions of fissionable substances, with the presence of an infinite aqueous reflector /3/.

With a cylindrical form of fuel core, the critical mass and critical volume depend not only on the concentration of hydrogen in the fuel core, but also on the diameter of the fuel core. Each diameter

has its own values of critical mass and critical volume, values corresponding to definite concentrations of hydrogen in the .uel core. For example, for solutions of UO_2F_2 of 93 percent enrichment in a cylindrical fuel core made of stainless steel of 20.3-cm diameter and equipped with a complete aqueous reflector, the minimal critical mass of U^{235} is about 1.4 kilograms, and for a fuel core of 16.5-cm diameter it is 3.2 kilograms.

Table 1.2

Critical Parameters of Solutions of Fissionable Substances

Critical parameters (minimal magnitudes)	U^{235}	U^{233}	Pu^{239}
Critical mass, kilograms	0.82	0.59	0.51
Diameter of infinite cylinder, centimeters	13.7	11.2	12.4
Thickness of infinite plate, centimeters	4.3	3.0	3.3
Critical volume, liters	6.3	3.3	4.5

The critical parameters of systems containing aqueous solutions of uranium are increased with a reduction in the enrichment of uranium. In an aqueous solution of uranium with an enrichment of less than one percent, the nuclear chain reaction is generally impossible. Similarly, the critical parameters of aqueous solutions of plutonium increase with the presence of U^{238} in the solution. The presence of nitrogen in nitrate solutions, which are frequently used during chemical processing, increases the critical parameters of solution systems, especially for solutions of uranium of low enrichment. The critical parameters of hydrogen-containing systems also depend on the density of the fissionable substance. An increase in its density leads to a reduction of the critical parameters.

Waterless Mixtures with a Large Content of Nonfissionable Elements.

The waterless mixtures containing a large quantity of nonfissionable substances -- -- mixtures which are encountered in enterprises -- -- have a very diverse chemical composition. Among them are mixtures which contain elements in large amounts, elements that are good moderators (especially carbon). At the present time, there are very little experimental and rating data on the critical paramenters of mixtures containing large quantities of nonfissionable elements. The critical parameters of such mixtures are usually greater than those of hydrogen-containing systems. For example, the minimal critical mass of a U^{235} mixture with graphite and with a graphite reflector amounts to about 1.5 kilograms, and for aqueous solutions of U^{235} with an aqueous reflector 0.8 kilogram.

Since nonfissionable materials are usually contained in mixtures at

enterprises — — materials whose nuclear characteristics (moderation, capture of neutrons) are worse than those of carbon, the critical parameters of such mixtures will be greater than the parameters of mixtures of nonfissionable substances containing carbon. Therefore, the latter can frequently serve as the minimal range in evaluating critical parameters of waterless mixtures containing large amounts of nonfissionable elements.

4. General Principles for Providing Nuclear Safety

The opportunity for the onset of a fission chain reaction during the processing of fissionable substances depends, firstly, on the amount and kind of these substances found in the technologic equipment and, secondly, on the dimensions and geometric shape of the equipment.

As was stated previously, in reducing the dimensions of a fuel core, the escape of neutrons increases; and given certain values of these dimensions (minimal critical dimensions), the escape becomes so great that a chain reaction is impossible. Therefore, if the dimensions of the technologic apparatuses are less than the minimal critical dimensions, the chain reaction cannot appear in them, the quality of the fissionable substances in these apparatuses notwithstanding. Thus, one of the ways to provide nuclear safety is the use of equipment having safe dimensions. Since several technologic apparatuses are usually located in a single structure, thereby presenting conditions for interaction among them, and since deviations from assigned dimensions in favor of their increase are possible in manufacturing and operating these apparatuses, the dimensions of safe equipment must always be less than the minimal critical values. It is reasonable to assume the following reserve factors in designing and manufacturing the equipment: the volume of an apparatus must not exceed the minimal critical volume by 0.8, and the diameter of a safe cylinder and the thickness of a safe plate (the height of the layer) must not exceed 0.92 of the minimal critical diameter of the infinite cylinder or the height of the infinite plane layer. For example, during the processing of aqueous solutions of highly enriched uranium, safe equipment must be made: volumetric capacity should not exceed 5 liters, pipes with a diameter of no more than 12.6 cm should be installed, and the thickness of the plate should not exceed 4 cm. It is most advisable to use equipment having safe dimensions for a continuous technologic process.

It is clear that the best way to provide nuclear safety is the use of geometrically safe equipment, but in practice this is rarely attained. It frequently appears that it is impossible or very uneconomical to subject this equipment to technologic surgery. In such circumstances, nuclear safety is ensured by limiting the quantity of fissionable substance in each apparatus. For example, if, at a plant processing solutions of highly enriched uranium, the quantity of fissionable substance in each apparatus is always less than the minimal critical mass for solutions of U^{235}, the chain reaction cannot occur (under conditions excluding interaction).

It is necessary to note that with this type of limitation opportunities for the onset of a chain reaction are greater, compared with the use of equipment having geometrically safe dimensions. This is connected with the fact that errors of servicing personnel, the disruption of the technologic process, and mistakes in the selection and analysis of tests can result in a situation whereby an amount of fissionable substance adequate to start a chain reaction is collected in one of the apparatuses.

The limitation of the amount of fissionable substance in conjunction with a partial limitation of the volume or dimensions of apparatuses yields good results. For example, the minimal critical mass of aqueous solutions of highly enriched uranium, with the diameters of the apparatuses not exceeding 17.8 cm, increases by almost two times the minimal critical mass for spherical apparatuses. Consequently, the employment of apparatuses of 17.8-cm diameter permits a substantial increase in the charge norm in them, compared with the charge norms in apparatuses of arbitrary form.

A modification of the limitations on the quantity of a fissionable substance, as well as the increase of the dimensions of safe apparatuses, can be obtained by introducing heterogeneous absorbers into the apparatuses. In this situation, additional monitoring is required to see that the absorbers are always in the equipment and that they do not change their absorbing properties with the passage of time.

In some cases during the processing of aqueous solutions of fissionable substances, nuclear safety can be ensured by carrying out the technologic process in such a way that the concentration of a fissionable substance will not exceed the minimal critical concentration. As was stated previously, minimal critical concentration is characterized by a definite ratio of the number of atoms of hydrogen to the number of atoms of the fissionable substance. Specifically, for plutonium solutions this ratio must be over 2,800 ($C = 9.4$ g/l), and for solutions of highly enriched uranium, 2,200 ($C = 12$ g/l). With this procedure of providing nuclear safety, it is essential to have in mind that, when the fissionable substance separates into a residue during the process, the ratio of the number of atoms of hydrogen to the number of atoms of the fissionable substance in the residue must also be less than the above-mentioned values.

It is essential to note that nuclear safety cannot be ensured in the requisite manner without strict administrative control, i.e., the adminis- tration must see to it that the requirements of nuclear safety are observed. Appropriate administrative control must also be exercised at enterprises and over all the equipment having geometrically safe dimensions. It is not possible to guarantee that there will be no escape of neutrons from such apparatuses; hence, precautionary measures must be taken so that a solution will not fall into a container with large dimensions and will not cause a chain reaction in it.

An especially important role is played by administrative control at enterprises in which nuclear safety is provided by limiting the quantity or concentration of a fissionable substance. In this situation, all measures must be taken to avoid a disruption of the technologic process or mistakes by servicing personnel, mistakes which could cause a chain reaction. Specifically, the limitation of the mass and concentration must be such that no less than two simultaneous and unconnected disruptions of the technologic process or mistakes of servicing personnel could cause a chain reaction. We will consider, for example, a chemical apparatus in which a fissionable substance is placed in a solution, and the charge of the apparatus is controlled by measuring the concentration and volume of the solution. Despite this, there is the possibility that mistakes will be made by servicing personnel, that analytical errors will occur, and that the technologic process will be disrupted. All these can lead to an increase in the concentration and volume of the solution in the apparatus. Therefore, the charge norm of the given apparatus must be prescribed in such a manner that only the simultaneous increase in controlled magnitudes (volume and concentration) to maximally possible values could result in an accident.

Proceeding on this basis, the values of the parameters responsive to the conditions of nuclear safety are established by dividing the corresponding critical values of the parameters into a certain reserve factor. The significance of the reserve factor as applied to various kinds of equipment and the work conditions of personnel is usually determined on the basis of the experience gained in designing and operating enterprises of the atomic industry.

METHODS OF DESIGNING HOMOGENEOUS REACTORS

Introduction

Problems connected with the theory and methods of designing nuclear reactors have been illuminated in the works of Soviet and foreign scholars $/ 1$, $2 - 28$, etc.$/$. Nuclear reactors are divided into homogeneous and heterogeneous reactors in accordance with fuel arrangement. This division also depends on what field of energy -- fast, intermediate, and thermal -- accounts for the basic part of fission. In designing reactors of unidimensional configuration, the most effective method is the spherical harmonic $/ 27 /$. In computations of the critical mass, it is necessary to be limited in the majority of cases by the P_1-approximation and only for fast and intermediate reactors is it essential to solve the problem in the higher approximations of the spherical-harmonic method, e.g., in the P_3-approximation. For equations of a reactor in the P_1- and P_3-approximations, with the utilization of modern computer techniques, the method of finite-difference linear or matrix factorization is very effective $/ 27 /$.

For obtaining a satisfactory accuracy in designs of critical mass reactors, multigroup approximation with the subsequent application of high-speed computers has appeared essential in many cases (fast and intermediate reactors).

This article will devote special attention to the methods of designing hydrogen-containing reactors as the most complex in respect to computation, for the model of continuous moderation (age approximation) that provides a simple mathematical algorithm is not applicable to the moderation of hydrogen. In solving the tasks connected with designing uranium-aqueous reactors, two approaches were basically observed. The first -- -- the approximation -- --

is based on a knowledge of the basic parameters of the fuel core of a reactor and the square of the length of the moderation of neutrons. They are characteristically determined, as a rule, as a result of processing a large amount of experimental material and then theoretically generalizing it into a concrete class of tasks. The critical mass or the effective multiplication factor K_{eff} is determined by a speciallly formulated equation of criticality.

The second approach is based on the application of sequentially exact hypotheses and mathematical algorithms, which permit the obtaining of all the requisite information on uranium-aqueous reactors without drawing on additional information, save for elementary physical constants. It is precisely this approach that is taken in this article.

1. Multigroup Methods of Solving Reactor Equations

a) Basic Equations of a Reactor

Basic for the physical design of nuclear reactors is the problem of the critical parameters and the spatial-energy distribution of neutrons. We will subsequently assume that the reactor is in a stationary state and that there are no independent sources of neutrons.

We will write Boltzmann's kinetic equation in the field of moderation in the following manner [27]:

$$\Omega\nabla\varphi + \Sigma\varphi = \int_{u-r}^{u} du' \int d\Omega' \Sigma_s(u')\varphi(r', \Omega, u') \times$$

$$\times f(\mu_0, u-u') + \frac{1}{4\pi}\int_{-\infty}^{u} du' \Sigma_{in}\varphi_0(r', u')w(u, u') + \frac{\chi(u)}{4\pi}Q(r). \qquad (3.1)$$

Here, Σ_s and Σ_{in} are the cross-sections of elastic and inelastic diffusion, respectively; $f(\mu_0, u-u')$ and $w(u, u')$ is the indicatrix of elastic and inelastic diffusion; $\mu_0 = (\Omega, \Omega')$ is the cosine of the angle of diffusion; $u = \ln\frac{E_0}{E}$ is the lethargy; $r = 2\ln\frac{M+1}{M-1}$ is the maximum logarithmic loss of

energy upon the collision of a neutron with a nucleus of the mass M;

represent the sources of fission; $\chi(u)$ is the fission spectrum;

$$\varphi_0(r', u') = \int d\Omega' \varphi(r', \Omega', u').$$

The limiting specification is characterized by the absence of a neutron flux from within:
$$\varphi(r, \Omega, u) = 0$$
on the surface S with $(\Omega, n) < 0$.

The nontrivial solution of the equation (3.1) can be found by the source iteration method $[27]$.

b) Multigroup Equations of a Reactor in P_1-Approximation

In solving a large complex of neutron-physical problems, it is possible to be limited by the P_1-approximation, the essence of which is that, in separating the neutron flux by spherical functions, the first two members impose limits:

$$\varphi(r, \Omega, u) = \frac{1}{4\pi}[\varphi_0(r, u) + 3\Omega\varphi_1(r, u)],$$

where

$$\varphi_0(r, u) = \int d\Omega \varphi(r, \Omega, u);$$

$$\varphi_1(r, u) = \int d\Omega \Omega \varphi(r, \Omega, u).$$

The appropriate limiting specifications in the P_1-approximation are obtained from Marshak's ratio $[27, 28]$.

The integral operators describing the diffusion of heavy nuclei ($M \gg 1$) can be simplified by applying transport age approximations

$$\left.\begin{array}{l} \Sigma_s\varphi(r, u') = \Sigma_s\varphi(r, u) - (u - u')\frac{\partial}{\partial u}[\Sigma_s\varphi_0(r, u)]; \\[2mm] [\Sigma_s\varphi_1(r, u') = \Sigma_s\varphi_1(r, u). \end{array}\right\} \tag{3.3}$$

We will assume, as previously in $[27]$, that $\varphi_0(r, u)$ and $\varphi_1(r, u)$ is in the form of the product of two functions, one of which takes into account the basic change in energy; the other, within the limits of a group, is only slightly dependent on energy. The transition to multigroup equations is effected by integrating the system of equations in P_1-approximation for lethargy within the limits of the group $U_{j-1} \div U_j$). The system of multigroup equations in the field of moderation assumes the following form:

$$\left.\begin{array}{l} \nabla\varphi_1^j + \Sigma_{\gamma n}^j\varphi_0^j = \sum_{l=1}^{j-1}\sum_{in}^{l\to j}\varphi_0^l + \alpha_0^j\sum_{l=1}^{j-1}\beta_0^l\varphi_0^l + \left(\frac{\xi\Sigma_s'}{\Delta u}\right)^{j-1}\varphi_0^{j-1} + \chi^j Q(r); \\[5mm] \frac{1}{3}\nabla\varphi_0^j + \Sigma_1^j\varphi_1^j = \alpha_1^j\sum_{l=1}^{j-1}\beta_1^l\varphi_1^l. \end{array}\right\} \tag{3.4}$$

The moderation of hydrogen in the equation (3.4) is taken into account separately. Group constants are determined in the form of fractional-linear functionals $\underline{/}$ 27 $\underline{/}$.

The presentation of group constants in the form of fractional-linear functionals permits, in the capacity of the weight function $\varphi_o(u)$ and $\varphi_1(u)$ the application of their approximate values (see, e.g., $\underline{/}$ 27, 29 $\underline{/}$).

It is essential to supplement the system of equations (3.4) with a system of multigroup equations in the field of thermalization $\underline{/}$ 27 $\underline{/}$. However, for the many problems connected with the design of reactors, it is possible to consider the field of thermalization as a single group with several effective constants. For the computation of effective one-group constants of a thermal group, it is possible in some cases to apply the spectrum of thermal neutrons N (x), which is consonant with Wilkins' equation $\underline{/}$ 30 $\underline{/}$.*

$$xN''(x)+(2x^2-1)N'(x)+(4x-\Delta)N(x)=0. \qquad (3.5)$$

Here $\Delta = \dfrac{4\Sigma_a^o}{\xi\Sigma_{st}}\sqrt{\dfrac{T_o}{T}}$; $x= \dfrac{v}{\sqrt{2kT}}$; Σ_α^o is the absorption cross-section, with E = 0.025 Ev; v is the velocity of a neutron; T and T_0 are the environmental temperature and the room temperature in absolute units; k is Boltzmann's constant.

The equation (3.5) can also be applied for determining the spectrum of thermal neutrons in water, provided that there is assigned to hydrogen the effective mass (M $\approx$ 6) for the purpose of calculating its bond in a molecule of water. Finally, the system of multigroup equations assumes the following form $\underline{/}$ 27 $\underline{/}$:

$$\left.\begin{aligned}
&\nabla\varphi_1^j+\Sigma_{\gamma_3}^j\varphi_0^j=\sum_{l=1}^{j-1}\sum_{ln}^{l\to j}\varphi_0^l+\Big(\frac{\xi\Sigma_s^{'j-1}}{\Delta u}\Big)^{j-1}\varphi_0^{j-1}+x^jQ\,(\mathrm{r});\\
&\tfrac{1}{3}\nabla\varphi_0^j+\Sigma_1^j\varphi_1^j=\alpha_1^j\sum_{l=1}^{j-1}\beta_1^l\varphi_1^l;\\
&\Sigma_{\gamma_3}^j=\begin{cases}\Sigma_{\gamma_3}^j & (j=1,\,2,\,\dots,\,m);\\[2pt]\Sigma_{aT} & (j=m+1);\end{cases}\\
&\Sigma_1^j=\begin{cases}\Sigma_1^j & (j=1,\,2,\,\dots,\,m);\\[2pt]\Sigma_{tr,\,T} & (j=m+1).\end{cases}
\end{aligned}\right\} \qquad (3.6)$$

For calculating resonance absorption and fission, the probability of averting resonance absorption of individual isotopes is expressed in the

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* An analogous equation was obtained by B. I. Davydov in 1937.

following manner:
$$\varphi = C^{-\frac{\rho}{|\xi_i|} J_{\text{s}\phi\phi}(T)},$$
(3.7)

where C is the number of atoms of the absorber in a unit of volume; is the moderating means of the environment; $\xi \Sigma_s$ is the effective resonance integral; it is determined by the equation

$$J_{\text{s}\phi\phi} = J \frac{\eta(\xi, h)}{\sqrt{1+h}}.$$
(3.8)

where J is the complete resonance integral for the given level of energy; T is the temperature of the environment; $h = \Sigma_r / \Sigma_s$ where Σ_r is the absorption

cross-section of maximum resonance; Σ_s is the section of potential diffusion; $\xi = \Gamma/\Delta$, where Γ is the half-width of resonance; $\Delta = 2\sqrt{\frac{kTE}{M}}$.

The function $\eta(\xi, h)$, which has been tabulated in another work $[31]$, takes into account the Doppler enlargement of the resonance lines under the influence of the thermal motion of the nuclei of the moderator.

Resonance absorption within the limits of the group in age approximation to the density of the moderator can be presented in the following form:

$$\Delta \Sigma_c^i = \frac{\sum_k \varrho J_{\text{s}\phi\phi}(u_k)}{\Delta u_j},$$

where k is the number of the level.

Resonance fission with U^{235} in age approximation is expressed as follows:

$$\Delta(\nu \Sigma_f^j) = \frac{1}{\Delta u_j} \sum_k \frac{\varrho J_{\text{s}\phi\phi}^{235}(u_k)}{1+\alpha_k},$$

where $\frac{1}{1+\alpha_k}$ is the probability of fission at the level of k; $\alpha_k = \frac{\Sigma_c}{\Sigma_f}$.

c) Moments $\overline{r^n}$. <u>The Length of Neutron Moderation</u>

We will find the spatial-angular moments of the function of distributing neutrons in an infinite environment with a flat isotopic source at z = 0 via the equation
$$\Phi_{ln}(u) = \int_{-\infty}^{\infty} dz z^n \int d\Omega P_l(\mu) \varphi(z, u, \mu),$$
(3.9)

where $\varphi(z, u, \mu)$ satisfies the kinetic equation.

We determine the value of the magnitude z^n as follows:
$$\overline{z^n}(u) = \frac{\Phi_{0n}(u)}{\Phi_{00}(u)}.$$
(3.10)

In case of a point isotopic source at r = 0, the equation is

$$\overline{r^n} = (n+1)\overline{z^n}.$$
(3.11)

The length of neutron moderation determines the average distance of a

neutron in the process of decelerating from the point of generation to the point at which a neutron becomes "thermal." According to Fermi's age theory, the length of neutron moderation $\sqrt{\tau}$ is linked with the secondary moment by the following relationship:

$$\tau = \frac{1}{6}\bar{r}^2 = \frac{1}{2}\bar{z}^2 = \frac{\Phi_{v2}}{2\Phi_{00}}.\qquad(3.12)$$

Another work $\underline{/\,32\,\underline{/}}$ contains an equation for squaring the length of neutron moderation, with two modifications which take into account the first and last lengths.

It is also possible to determine the length of moderation in a multigroup approximation by applying Marshak's method $\underline{/\,33\,\underline{/}}$.

The solution of the respective system of equations is found by the method of sequential approximations. For example, the equation $\tau^j = \dfrac{\Phi^j}{2\Phi_{00}^j}$ enables one to determine the length of neutron moderation in various homogeneous environments on the basis of the effects of deceleration (elastic and inelastic) and multiplication. A comparison of the theoretical and experimental values of the squares of the lengths of moderation provides one with the opportunity of checking various systems of multigroup constants.

The multigroup system of constants, which was applied in the calculations, was constructed on the basis of data on cross sections $\underline{/\,34,\ 35\,\underline{/}}$. This system of constants is basically presented in the work of G. I. Marchuk $\underline{/\,27\,\underline{/}}$.

Observed is a good concurrence of experimental and theoretical values of the squares of the lengths of moderation in light water up to indium resonance -- lengths designed for multigroup approximations. For example, the work of L. N. Yurova, et al $\underline{/\,36\,\underline{/}}$ presents the value $\tau_{ex} = 27.1 \pm 0.9 \ cm^2$ (the experimental error determined by the dimensions of the source is minimized) and the computation of the square of the length of deceleration in water -- a computation made in the eighteenth group approximation, $\tau_{calc} = 27.5 \ cm^2$. Such a concurrence provides one with the opportunity of making reliable computations for determining the critical parameters of a reactor in P_1-approximation.

A system of constants for designing a fast-intermediate reactor in P_3-approximation can be verified by comparing the numerical values r^n at n = 2, 4, 6 with those of an experiment. The results of squaring the lengths of neutron moderation are discussed in number 3 of this article.

d) Computation of Kinetic Effects in a One-Group Approximation

The critical masses obtained with a one-group computation in P_1-approximation

for fast and intermediate reactors appear too high, compared with those of an exact computation. The highest overestimation of the critical masses is obtained for systems with an aqueous moderator. Because of this, it is essential to refine the results of computations at the expense of kinetic effects. An P3-approximation is an essential refinement of an P1-approximation in computations of a critical mass. However, with a large number of groups, computation work sharply increases; therefore, it is essential to be precise when working within the framework of the one-group method.

The goal of the one-group method is to obtain constants that result in values analogous to those obtained through the application of the usual multigroup theory for the critical parameters of a reactor.

The problem of obtaining one-group constants with the aid of the solution of the multigroup problem in P_1-approximation is considered in detail by G. I. Marchuk $\underline{/\ 27\ /}$.

With the assistance of one-group constants, it is possible to produce a refinement of the critical masses of nuclear reactors at the expense of kinetic effects by solving a one-group kinetic equation, which can be written in the following manner:

$$\Omega\nabla\varphi + \overline{\Sigma}_{tr}\varphi = \frac{(\overline{\Sigma}_{tr}-\overline{\Sigma}_a+\overline{\nu\Sigma}_f)}{4\pi} \int \varphi \, d\Omega. \qquad (3.13)$$

The kinetic equation (3.13) can be solved by the method of spherical harmonic, e.g., in P3-approximation.

The results of computing critical parameters with the aid of the one-group theory will be discussed in number 3 of this article.

2. **Methods of Computing the Critical Masses of Nuclear Reactors**

a) __The Recalculation of the Critical Masses of Spherical Reactors__
with Infinite Reflectors
from Single Enrichments to Others

In problems concerned with nuclear safety, the approximate methods of designing reactors are of practical significance. In the work of G. I. Marchuk and V. P. Kochergin $\underline{/\ 37\ /}$, a method has been developed for recalculating the critical masses of spherical, infinitely reflected reactors, from single enrichments to others.

We will consider a critical reactor in which the fuel core of the radius R is surrounded with an infinite reflector. The solution of a one-group

diffusion equation for the neutron flux to the reflector will be as follows
$\boxed{1\!\!\!\!/}$:

$$\varphi(r) = C\frac{e^{-\lambda r}}{r}.\qquad\qquad(3.14)$$

The effect of the reflector on reducing the critical dimensions of the
reactor is characterized by the following effective addition:

$$\delta = R_{\vartheta} - R,\qquad\qquad(3.15)$$

where R_{ϑ} is the extrapolated radius of an unreflected reactor.

We will determine the effective addition in a way analogous to that
applied to determine the length of the linear extrapolation of the neutron
flux to the reflector on the edge of the fuel core of the reactor:

$$\delta = -\left.\frac{\varphi}{\frac{d\varphi}{dr}}\right|_{r=R} = \frac{1}{\lambda + \frac{1}{R}}.\qquad\qquad(3.16)$$

Via the equations of the expressions (3.15) and (3.16), the value λ can be
determined:

$$\lambda = \frac{1}{R_{\vartheta}-R} - \frac{1}{R}.\qquad\qquad(3.17)$$

If the magnitude λ is known, we determine on the basis of the equation (3.17)
that

$$R = \frac{\lambda R_{\vartheta} - 2 + \sqrt{4+(\lambda R_{\vartheta})^2}}{2\lambda}.\qquad\qquad(3.18)$$

The cadmium ratio for U^{235} is a convenient magnitude characterizing
the neutron energy spectrum in an unreflected reactor:

$$R_{Cd} = \frac{\int_{u=15,5}^{\infty} \nu\Sigma_f\varphi(u)\,du}{\int_{-\infty}^{u=15,5} \nu\Sigma_f\varphi(u)\,du}.\qquad\qquad(3.19)$$

The magnitude of λ as a function of the cadmium ratio R_{Cd} is determined
with the aid of the critical parameters R_{ϑ} and R, which are obtained by solving
the multigroup diffusion equations with assigned densities of nuclear fuel and
the moderator, as well as with assigned enrichment of uranium by the isotope U^{235}.

The critical dimension of an infinitely reflected reactor during other
enrichments and with the same densities of fuel and moderator is computed in
accordance with the equation (3.18), with the use of a graph for the function λ.
However, it is first necessary to make a multigroup computation of unreflected
reactors for determining the magnitude R_{ϑ} and the cadmium ratio.

A graph of the function λ for aqueously reflected systems is presented
in Figure 3.1. A comparison of the critical dimensions of the fuel core
of these systems, which are determined with the aid of multigroup computations
and by the equation (3.18) in the broad range of the change in ratios $\alpha = \theta_H/\theta_F$ is
presented in Table 3.1. The asterisk denotes a radius found in accordance
with the equation (3.18). The deviations in the critical mass rest within
the limits of accuracy, with which is made a multigroup computation. These
deviations do not basically exceed ten percent.

We will observe that, in computing the critical dimensions of the

systems UO_2-PuO_2-H_2O ($\gamma_{UO_2} = \gamma_{PuO_2} = 6$ g/cm³) in accordance with the equation (3.18), it is possible to apply the function λ presented in Figure 3.1. This is also confirmed by Table 3.1, in which the uranium is natural. This reflects the fact that the graph of the function λ does not change when plutonium replaces uranium, provided the compound in which the former is present has the same density. Thus, the equation (3.18) makes possible the computation of the critical mass of any combination of fissionable elements in a given compound, with the presence of the graph of the function λ .

Table 3.1

The Critical Dimensions of Uranium-Water and Uranium-Plutonium-Water Systems

| | UO — H₂O | | | | | | UO₂ — PuO₂ — H₂O | | | | | |
| | P = 35% | | | P = 5% | | | β = 0 | | | β = 10 | | |
a	R_3	R	R*	R_3	R	R*	R	R	R*	R_D	R	R*
2000	87,0	78,0	78,2	—	—	—	34,6	28,0	27,7	36,9	30,0	29,6
1200	34,6	28,0	28,0	44,4	36,8	36,8	25,4	19,0	18,7	27,8	21,0	20,8
800	27,3	21,0	20,7	34,5	27,3	27,0	—	—	—	—	—	—
600	—	—	—	—	—	—	20,8	14,5	14,2	23,6	16,5	16,3
400	22,4	15,7	15,7	30,3	22,2	22,2	—	—	—	—	—	—
200	—	—	—	—	—	—	18,4	11,8	11,6	23,9	15,5	15,5
120	20,6	13,2	13,4	39,3	27,3	28,2	18,0	11,2	11,0	26,5	17,0	17,3
60	—	—	—	68,2	52,9	52,8	—	—	—	—	—	—
30	24,3	15,0	15,1	—	—	—	18,7	11,1	11,0	52,2	35,0	36,8
10	32,5	20,0	20,4	—	—	—	19,8	11,5	11,4	—	—	—
3	41,4	25,0	25,0	—	—	—	20,8	11,6	12,0	154,0	120,0	116

Note: For the systems UO_2 - H_2O $\alpha = R_H/\rho_U$, and for the systems UO_2 - PuO_2 - H_2O $\alpha = R_H/\rho_U$, $\beta = \rho_U/\rho_{Pu}$, P is the proportion of enrichment with isotope U^{235}.

Figure 3.1

The Function $\lambda = \dfrac{1}{R_3 - R} - \dfrac{1}{R}$ for spherical reactors consisting of the mixtures UO_2 - H_2O. The density of UO_2 is 6 g/cm³ (R_{Cd} - cadmium ratio).
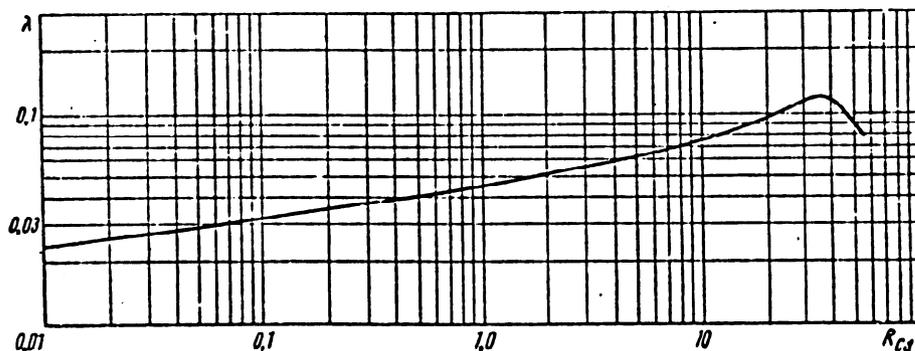
To determine the critical masses of spherical, infinitely reflected reactors having other densities of fuel and moderator, it is necessary to make a computation of the critical parameters with the aid of multigroup diffusion equations, with some kind of enrichment of fuel for finding the function λ.

Unreflected reactors are similar, and the critical dimensions are easily recalculated if a change occurs in the densities of the fuel and the moderator. If, for example, $\alpha = \ell_{mod}/\ell_{fuel}$ is fixed in a system consisting of two components, the number of atoms of fuel in one cm^3 will be:

$$\varrho_{rop} = \cfrac{1}{\cfrac{1}{\varrho_{fuel}^a} + \cfrac{\alpha}{\varrho_{mod}^a}}, \qquad (3.20)$$

where $\varrho^a = \dfrac{N\gamma}{A}$ (N = the Avogadro number; γ is the density, g/cm^3; A is the atomic weight).

In changing the values of $\gamma_{fuel}, \gamma_{mod}$, the magnitudes ℓ_{fuel}, ℓ_{mod} will also be changed in a uniform ratio; therefore, in effecting the transition from one system to another, the macroscopic cross section must be multiplied by $\delta = \dfrac{\ell_{fuel}^a}{\ell_{fuel}^a}$. In order not to disrupt the multigroup diffusion eqs., it is

essential to divide the linear dimensions by the similitude factor δ. Thus,

$$R_s^{(2)} = \frac{R_s^{(1)}}{\delta}. \qquad (3.21)$$

The curve connections between the critical charge and the critical volume, upon changing the magnitudes $\gamma_{fuel}, \gamma_{mod}$ in the same ratio, is carried over a parallel manner without changing the form, for the similitude factor in this case is the same for all the α. If the values $\gamma_{fuel}, \gamma_{mod}$ are changed not in a uniform ratio and, consequently, the simulitude factor depends on α, the curve charges from the volume, apart from the conversion, are still deformed.

b) Recalculation Based on the Laplace Transform

The methods of recalculating the critical mass of nuclear reactors from one form of reactor to another -- methods based on the Laplace transform - usually prevail in cases in which the spectrum of reactors with uniform composition of the fuel core either are not converted (unreflected reactor) or are insignificantly converted, basically via the reflector $/26/$.

We will consider the approximation with which a finite homogeneous reactor is replaced by an infinite homogeneous environment, and the finite dimensions of a reactor are taken into account through the assumption that the neutron flux is equal to zero on some fixed surface. To these requirements respond the well-known diffusion approximation with a zero limiting condition for flux to the extrapolated limit.

The first basic theorem of the theory of reactors is formulated in the following manner:

"At the function $\overline{\Phi}(x, E)$ describing the stationary distribution of neutrons in a critical unreflected reactor, the variables x, E are divided:

$$\Phi(x, E) = \varphi(E)\psi(x).$$

The spatial distribution $\psi(x)$ is the basic solution of the wave equation

$$\Delta\psi(x) + \varkappa^2\psi(x) = 0, \tag{3.22}$$

i.e., a solution which is absolute for the entire reactor and equal to zero on the extrapolated limit."

The position of the extrapolated limit determines both the Laplacian $\varkappa^2$ and $\psi(x)$; in this situation, the value $\psi(x)$ is determined with accuracy up to the constant absolute multiplier. The effective multiplication factor K_{eff} depends on the properties of the materials composing the fuel core of a reactor, and its form and dimensions are dependent only through $\varkappa^2$. Consequently, K_{eff} has the same value for the two homogeneous reactors constructed from the same materials, but having different dimensions and form if they have the uniform value $\varkappa^2$, which is determined by their extrapolated limits.

For an extrapolated surface of an arbitrary form, it is impossible to obtain a solution of the wave equation in explicit form. But for some cases, the solutions are known and have a very simple form.

Rectangular Parallelepiped with Sides a, b, c.

Three coordinate planes are chosen so that they will coincide with the three planes of the effective surface; the three borders will then coincide with the three coordinate axes. The basic solution is via the following equations

$$\psi_0 = \sin\left(\frac{\pi x}{a+2\lambda}\right)\sin\left(\frac{\pi y}{b+2\lambda}\right)\sin\left(\frac{\pi z}{c+2\lambda}\right), \tag{3.23}$$

where λ is the length of extrapolation.

The lowest Laplacian value to which corresponds the eigenfunction is

$$\varkappa_0^2 = \left(\frac{\pi}{a+2\lambda}\right)^2 + \left(\frac{\pi}{b+2\lambda}\right)^2 + \left(\frac{\pi}{c+2\lambda}\right)^2. \tag{3.24}$$

Cylinder with H Height and R Radius.

We will assume that the axis of the cylinder coincides with the axis z, and the plane of the foundation with the plane (xy). The lowest value $\varkappa^2$ is equal to

$$\varkappa_0^2 = \left(\frac{2,405}{R+\lambda}\right)^2 + \left(\frac{\pi}{H+2\lambda}\right)^2; \tag{3.25}$$

the corresponding eigenfunction

$$\psi_0 = \sin\left(\frac{\pi z}{H+2\lambda}\right)J_0\left(\frac{2,405r}{R+\lambda}\right). \tag{3.26}$$

The Sphere of R Radius.

In this case, the solution of a spherically symmetrical problem yields the eigenfunction $\psi_0 = \frac{1}{r}\sin(\varkappa_0 r)$, where

$$\varkappa_0^2 = \left(\frac{\pi}{R+\lambda}\right)^2.$$

It is necessary to note that the extrapolated limit must be a completely determined surface, i.e., it must have the same position for neutrons of any energy. This limits the accuracy of the method underlying the Laplacian transform, since the average length of free range is $1/\Sigma$ and, consequently, the length of extrapolation $\lambda = \frac{2}{3\Sigma}$ also depends on the energy in a diffusion approximation.

Despite the fact that the method of the Laplacian transform is less accurate than the diffusion theory, it has been widely applied in rough computations of a critical mass.

Through the saving of a reflector, a reactor can be converted into an equivalent reactor having several effective dimensions. This reactor's Laplacian can be determined for a sphere, for example, by the relationship

$$\varkappa^2 = \left(\frac{\pi}{R+\gamma+\lambda}\right)^2, \qquad (3.27)$$

since R is the real radius of the reflected reactor, γ is reflector economy, and λ is the length of extrapolation.

The application of the Laplacian transform to a reflected reactor results in an additional, fundamental inaccuracy. This is connected with the fact that the neutron spectrums in reactors with uniform compositions of the fuel core and with and without reflectors differ from one another in accordance with the size of the fuel core.

The Laplacian transform method, which utilizes experimental data on the lengths of extrapolation and reflector economy, is widely used in converting one form of reactor into another.

3. Critical Parameters of Hydrogen-Containing Systems

a) A Comparison of the Computations of Critical Parameters of Aqueous Salt Solutions of UO_2 $(NO_3)_2$ with Those of an Experiment

The methods of computing the critical parameters of homogeneous nuclear reactors in P_1- and P_3-approximations are applied to the miscalculations of experiments conducted with aqueous salt solutions of UO_2 $(NO_3)_2$ enriched in the amount of 90, 10, and 5 percent with uranium [235].

Experiments have revealed the discernible influence of nitric acid contained in aqueous solutions of uranyl nitrate on the critical mass of homogeneous reactors. The work of B. G. Dubovskiy, et al., [11] has shown

that this influence is the result of two factors. Firstly, the addition of nitric acid to an aqueous solution of salt of $UO^2(NO_3)_2$ leads to a reduction (with a constant concentration of uranium in the solution) in the total plane of hydrogen nuclei. This condition increases the escape of neutrons from a reactor and leads to the growth of the critical mass. Secondly, the addition of nitric acid increases the critical mass by introducing additional nuclei of nitrogen, which has a discernible absorption cross section (in the thermal field, and changes according to the $1./v$-law.

Figure 3.2

The relationship between the critical radius R of an aqueously reflected sphere and the concentration of uranium in the aqueous salt solution of $UO_2(NO_3)_2$ with different ratios of Q_N/Q_u. The enrichment of uranium is 90 percent. The computation is of a one-group P_3-approximation. A = Critical radius of the sphere, in cm; B = Concentration of uranium, g/l.
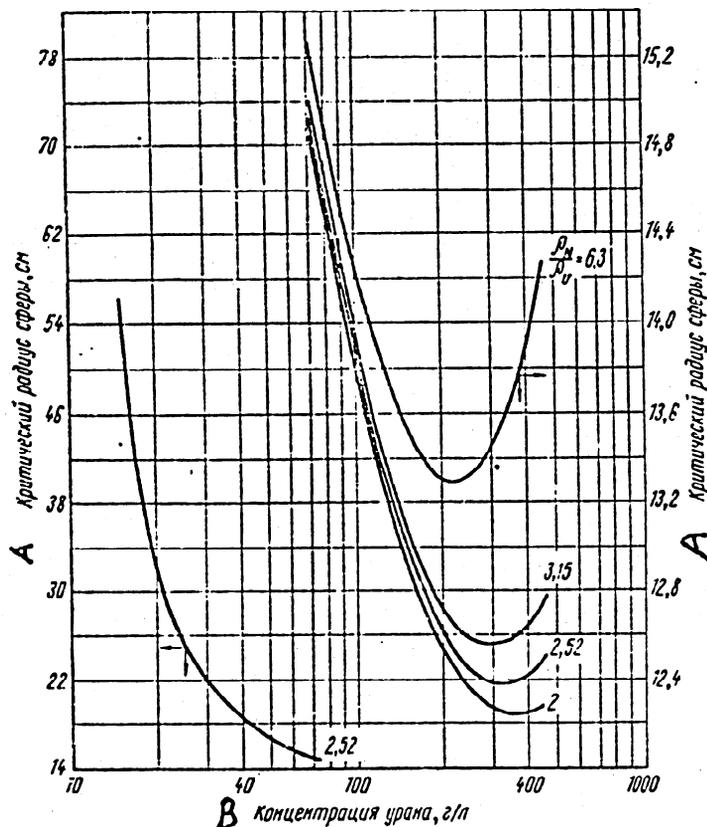
We will consider the results of the computations of the critical radii of spherical homogeneous reactors in accordance with the concentration of uranium in a solution with various ratios of nitrogen and uranium

nuclei: $\beta_N = \dfrac{\rho_N}{\rho_U}$. The spherical reactor has an infinite reflector
of water at $20°$ C. Figure 3.2 presents the results of computations
for uranium of 90 percent enrichment in a P_3 one-group approximation.
The lower curve corresponds to $\beta_N = 2$, i.e., there is no nitric acid.
This curve indicates the minimal limit for the critical radius of the
sphere.

The critical radius of the sphere increases commensurately with
the addition of nitric acid ($\beta_N = 2.52$; 3.15; 6.3); with a three-fold
increase of β_N, the minimal critical radius of the sphere becomes larger
by seven to eight percent. An increase of nitric-acid content in the
solution, with a given concentration of uranium, leads to a reduction
in the ratio ρ_N / ρ_S, which characterizes the moderating pro-
perties of the sphere. Since the minimal critical radius corresponds
fully to a definite value of ρ_N / ρ_S, an increase of β_N consequently leads
to a reduction in the values of the concentrations corresponding to the
minimal radii of the spheres. With the changing of β_N from 2 to 6.3, the
uranium concentration corresponding to the minimal critical size is reduced
from 350 to 215 g/l. Beginning with a uranium concentration of about 40 g/l,
the effect of nitrogen on the critical size is practically reduced to zero.

Experiments $\boxed{11}$, aimed at determining the critical masses of
solutions of $UO_2 (NO_3)_2$ enriched in the amount of 90 percent, have been
conducted on assemblies having the form of cylinders, rectangular parallele-
pipeds, and spheres equipped with an aqueous reflector. Found experimentally
has been the relationship of the economy of the steel reflector through
the thickness t:
$$\gamma = 0.9t \tag{3.28}$$
which is made with $t \leqslant 1$ cm for all the forms of assemblies in the
field of U^{235} concentrations of up to 200 g/l. The lengths of the
extrapolation for spheres and parallelepipeds within the range of
experimental errors do not depend on the concentration of U^{235} in the
solution when concentrations range from 67-210 g/l and are equal to
2.7 ± 0.15 and 2.65 ± 0.15 cm, respectively $\boxed{11}$. The savings of steel
and aqueous reflectors have been determined in the work of B. G. Dubovskiy
et al $\boxed{10}$. The economy of the aqueous reflector is constant within the
limits of experimental errors in the range of uranium concentrations of up
to 135 g/l and is equal to 3.3 ± 0.3 for spheres and parallelepipeds $\boxed{10}$.

Compared in Figure 3.3 are the experimental and computed values
of the critical radii of the spheres and the size-shape factors of aqueous
solutions of $UO_2 (NO_3)_2$ enriched in the amount of 90 percent. Corrections
taking into account the effect of the steel bodies of the assemblies have
been made in the experimental results. The computation of the critical
radii of the spheres has been made in the P_3 multigroup approximation;
the multigroup system of constants has been selected in accordance with
work previously performed $\boxed{27, 34, 38}$.

Figure 3.3

Critical parameters of aqueous salt solutions of $UO_2 (NO_3)_2$ in relationship to concentrations of U^{235}; $\rho_N/\beta_{u} = 2.5$. The enrichment of uranium is 90 percent. The material parameter: 1 is the computation according to the formula $\varkappa^2 = \left(\dfrac{\pi}{R_3 + \lambda_{p h}}\right)^2$; 2 is the radius of an unreflected sphere (a computation in a multigroup P_3-approximation); 3 is the same but with a water reflector; experiment conducted with parallelepipeds = (⊖), reflected spheres = (⊗), and unreflected spheres = (O). A = Critical radius of the sphere in cm; B = Material parameter in cm^{-2}; and

C = Concentration of U^{235} in g/l.
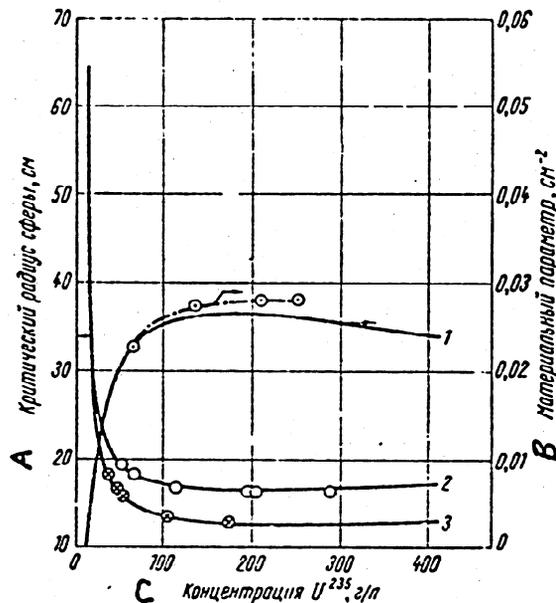
The material parameter of the solutions is calculated according to the equation

$$\varkappa^2 = \left(\frac{\pi}{R_3 + \lambda_{p h}}\right)^2 , \tag{3.29}$$

since R_3 is the critical radius of an unreflected sphere in P_3-multigroup approximation; λ_{cf} is the length of extrapolation for a sphere in P_1-approximation.

The greatest concurrence with the experiment is observed in the computation of the critical radius of an aqueously reflected sphere (± 1.5 percent); the worst concurrence with the experiment was noted in the computation of the critical radius of an unreflected sphere (± 2.5 percent). With U^{235} concentrations of less than 100 g/l in the solution, there was complete concurrence between the computation and the experiment.

The material parameters $\varkappa^2$ obtained by the computation method are different from those experimentally measured previously /11/(see Figure 3.3). The maximum difference in the parameters $\varkappa^2$ is 8 percent and occurs when concentrations are about 200 g/l.

Experiments with a solution of UO_2 $(NO_3)_2$ of five and ten percent uranium enrichment have been conducted in a cylindrical configuration /11_7. In obtaining the critical radii of spheres and geometric parameters of solutions of low enrichment from experimental data, the length of extrapolation was assumed to be 2.6 cm, and the effective addition of the aqueous reflector, 6 cm. The computation made in the multigroup P3-approximation concurred satisfactorily with the experiment (Figure 3.4).

Figure 3.4

The critical parameters of aqueous solutions of UO_2 $(NO_3)_2$. The relationship ρ_N/ρ_U = 2.5. The enrichment of uranium is five and ten percent.

The material parameter is according to the equation $\varkappa^2 = \left(\dfrac{\pi}{R + \lambda_{sp\perp}}\right)^2$

: 10 (1) and 5 percent (2). The radius of the sphere is in multigroup P3-approximation 5 (3) and 10 percent (4). a is unreflected; b is reflected. O, Ө, ⊙ represent experiments on cylinders. A = Critical radius of the sphere in cm; B = Material parameter in cm^{-2}; C = U^{235} concentration in g/l.
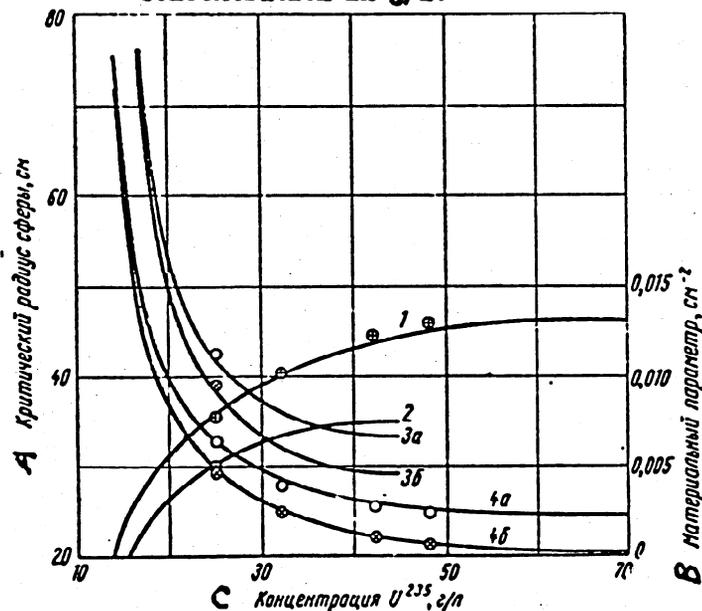
The comparison presented of experimental and computation results for solutions of uranium of 90 percent enrichment indicate that the multigroup method of designing a reactor with an aqueous moderator -- a method presented in this article -- and the system of constants selected for the computation properly takes into account the moderation of hydrogen. Computation and experimental concurrence was observed to be good in the investigated field of concentrations of uranium of low enrichment. This means that the resonance effects in the capture and multiplication of

neutrons were accurately taken into account.

The results of computations in P_3-multigroup and P_3-one-group approximations have been compared in another work $/11/$; it demonstrated that the maximum difference amounted to 2.5-3.C percent and was increased with the growth of the concentration of uranium in the solution.

The square of the length of neutron moderation of the fission spectrum of a salt solution of UO_2 $(NO_3)_2$ up to indium resonance is computed in a multigroup Pl-approximation (Figure 3.5). The concentration of uranium in the solution is changed from 0 to 500 g/l; enrichments of 10, 36, and 90 percent are provided for.

Figure 3.5

The relationship between the square of the length of neutron moderation of the fission spectrum in aqueous salt solutions of UO_2 $(NO_3)_2$ and the concentration of uranium in the solution $\frac{\theta_N}{\sim} = 2.5$. Enrichment of uranium is in percent: 10 (1); 36 (2); 90 (3). $A \stackrel{\theta_U}{\sim}$. Square of the length of neutron moderation in cm^2; B = Concentration of uranium in g/l.
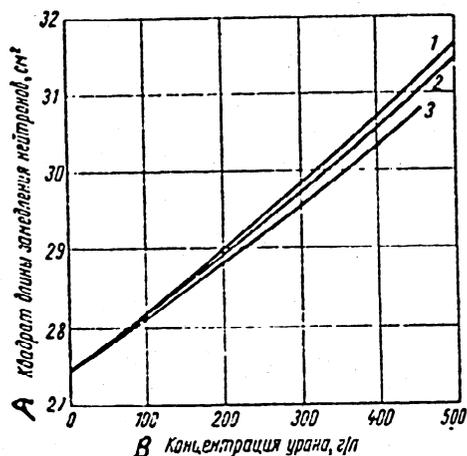
The computation of the value t includes the effects of elastic and inelastic diffusions, with a consideration of anisotropy, moderation of the deceleration process, and the effect of multiplication on the fast neutrons in U^{238}. Breeding of U^{235} is not considered.

The upswing of the curves with an increase in the concentration of uranium in the solution is mainly determined by the reduction in the concentration of hydrogen nuclei and the multiplication of fast neutrons in U^{238}. The effect of inelastic diffusion of uranium competes with these neutrons. With the same concentration of uranium in the solution, the value t grows with an increase of U^{238} content through the multiplication of fast neutrons in this substance.

b) Critical Parameters of Mechanical Mixtures of Plutonium and Uranium with Water

Presented below are the data on the critical parameters of mechanical mixtures of uranium-plutonium-water with uranium of various enrichments of U^{235}. The data were obtained on the basis of numerical computations made in accordance with the method of spherical harmonic in P_1- and P_3 approximations.*The basic equations for the method and a brief description of it were presented at the beginning of this article. The investigation of the critical parameters was conducted for a unidimensional configuration: the sphere, the infinite cylinder, and the infinite plate. The spheres have been computed with the inclusion of the kinetic effect in P_3-one-group approximation. For cylinders and plates, kinetic corrections for diffusion approximation were not taken into account; computations were made in the rougher P_1-multigroup approximation (18 groups). Ordinary water was used in the capacity of a moderator and reflector. The common systemization of mixtures -- a systemization adopted in the computations -- permits a consideration of such character-istic, frequent cases as mixtures of uranium of various enrichments (without plutonium) and water, uranium of diverse enrichments without water, aqueous mixtures largely containing plutonium, etc. Computations were made for the following enrichments of uranium (in percent): 0.71, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2, 5, 6.5, 10, and 23. Mixtures of plutonium and uranium with water -- mixtures having higher enrichments of uranium -- are of less practical interest. Some of the most important, frequent cases of such mixtures, e.g., mixtures of highly enriched uranium and water, have been fully discussed in other paragraphs of this handbook.

All the results are presented in the form of graphs which are grouped according to the enrichments of uranium. Presented for each enrichment is the set of the most characteristic critical connections: the critical mass of a mixture, the critical volume of a mixture, the critical radius of an infinite cylinder, and the critical half-thickness of the infinite plate. For studying the critical parameters of the triternary mixtures of uranium-plutonium-water, it is convenient to consider the following magnitudes:

ρ_H is the number of hydrogen atoms in one cm^3 of a mixture;

ρ_5 is the number of U^{235} isotopes in one cm^3 of a mixture;

ρ_q is the number of atoms of the Pu^{239} isotope in a mixture;

$\beta_H = \rho_H/\rho_5$ is the ratio of the number of hydrogen atoms to the number of isotope U^{235} atoms in a mixture;

$\beta_H^* = \rho_H/(\rho_5 + \rho_q)$ is the ratio of the number of hydrogen atoms to the total number of atoms of the fissionable isotopes in a mixture;

$\alpha = \rho_q/\rho_5$ is the ratio of the number of atoms of an isotope of Pu^{239} to the number of atoms of an isotope of U^{235} in

* All the computations were made in the Mathematics Department of the FEI (Feliks Edmundovich Dzerzhinskiy Institut -- the Feliks Edmundovich Dzerzhinskiy Institute) on a machine of the BESM-2 type.

in a mixturo;

C_0 is the total weight concentration of fissionable isotopeo in a mixture, kg/l;

p is the enrichment of uranium by the isotope U^{235}, percent.

In calculating the number of atoms of the individual components in one cm^3 of a mixture, it was assumed that the volume of the mixture was equal to the volumen of the individual components. The densities of uranium, plutonium (α-phase), and water in a mixture were in g/cm^3: 18.7, 19.6, 1.0, respectively.

It is convenient to characterize the relative content of plutonium in a mixture with the parameter α. Computations were made for the following values of the parameter : 0, 0.1, 0.3, 0.5, 1, 3 , 5, 10, and 50. The minimal value of α, which is equal to zero, corresponds to a simple mixture of uranium and water, without plutonium. The greater values of the parameter of this series (5, 10, 50) primarily characterize mixtures of plutonium with the resonance absorber U^{238}. When the definite magnitude of α and the content of one of the fissionable isotopes in a mixture are known, it is an easy matter to calculate the proportion of the other fissionable isotope by the following equation:

$$m_{Pu239} = m_{U235} \cdot \alpha \cdot \frac{239}{235} .$$

(3.30)

Analogously, with simple mixtures containing one fissionable isotope, the magnitude β_H^* is considered as the moderation parameter of the tricomponent environments. It is unequivocally connected to the total weight concentration of C_0 of the fissionable isotopes of the uranium and plutonium in a mixture. This relationship is expressed by the equation

$$\beta_H^* \approx \frac{238}{9} \left[\frac{1}{C_0} - \frac{1}{\gamma_{Pu}} \cdot \frac{\alpha}{1+\alpha} - \frac{100}{\gamma_U P (\%)} \cdot \frac{1}{1+\alpha} \right] .$$

(3.31)

(Graphically, this relationship is represented by the sets cf monotypic curves in Figures 3.10, 3.15, 3.20 - 3.35; each curve corresponds to one value of the parameter α).

In some cases, the results of calculations of critical parameters have been compared with certain, experimentally critical parameters of solutions and mixtures which are encountered in practice. The difference between a computation and an experiment is largely and obviously explained by the fact that, with uniform parameters of moderation of β_H^*, ideal mechanical mixtures have a greater concentration of fissionable isotopes, compared with mixtures and solutions investigated experimentally. This difference in density must result in lower critical parameters for ideal mixtures. It is especially great for a slightly diluted system moderator. If this effect is taken into account, it then appears that the computed and experimental results differ by five to ten percent in the critical mass and critical volume of systems in the form of a sphere.

The above-mentioned evaluation is not exhaustive. Because of the lack of experimental data on tricomponent systems at the present time, it has not been possible to make, for these systems, a detailed comparison of calculation and experiment. A qualitative consideration of the critical relationships obtained in computations provides evidence that the presence of plutonium in a mixture has been accurately taken into account.

Computations have been made for a broad range of concentrations of the C_0 of fissionable isotopes: from the slightly diluted moderator of fast systems to thermal neutron systems. The relationships between the critical parameters (critical mass, critical volume, critical radius of a cylinder, and critical half-thickness of a plate) and the parameter of moderation of β_H^* have a number of characteristic features which are more or less maintained for all the enrichments of uranium. It is convenient to consider them primarily for mixtures of uranium and water, without plutonium. With small dilutions of uranium via the moderator, the critical mass of the isotope U^{235} is increased; with several dilutions, the maximum is reached. Following this, the critical mass in the intermediate field of the neutron energy spectrum in the fuel core is reduced and becomes minimal, with the parameters of moderation corresponding to those of thermal neutron reactors. With subsequent dilutions of uranium via the moderator, there is observed a monotonic increase in the critical mass of the fissionable isotope. Demanding attention is the fact that the relative magnitude of the maximum of the critical mass in a fast field of the neutron energy spectrum is comparatively small with a ten-percent enrichment of uranium; the relative magnitude increases substantially with a reduction of enrichment, assuming in this range of the spectrum a practically infinite value with a 6.5-percent enrichment. This maximum is either slightly expressed or is completely absent when the enrichments of uranium exceed 20 percent; for enrichments below 6.5 percent, criticality on uranium-water mixtures is not attained in the considered range of moderations.

The minimum of the critical mass of uranium in the range of moderations corresponding to thermal neutron reactors shifts somewhat to the side of great dilutions of the moderator system upon a reduction of enrichment.

All that has been stated above relative to the general character of the relationship between the critical mass and the moderation parameter is to a large extent characteristic of other critical parameters: the critical volume, cylinder radius, and the half-thickness plate. These parameters have minimal values with moderations corresponding to intermediate neutron reactors. With a reduction of uranium, this minimum is shifted, as was so in respect to the critical mass, to the range of great dilutions of the uranium of a moderator. The nature of the change in the data of the parameters in the range of slightly diluted systems

for all enrichments via the moderator is qualitatively similar to the behavior of the critical mass of uranium in this range.

Considered above were the critical parameters of a uranium-plutonium-water system with α = o for various enrichments of uranium. The addition of plutonium to a mixture corresponds qualitatively to an increase in the proportion of isotope U^{235} in uranium. Therefore, the curves in Figures 3.6-3.35 with indices   differing from zero have, with equivalent enrichments, the same character as the corresponding curves for systems without plutonium.

Figures 3.36-3.47 indicate the results of computations for the mechanical mixtures UO_2 + H_2O, with three diverse planes of uranic oxide Yuo_2 : 3, 6, and 8.45 g/cm^3. The critical parameters of these mixtures are calculated for spherical and unidimensional cylindrical shapes of fuel cores with enrichments of uranium by the isotope U^{235} — — enrichments ranging from 1.5 to 90 percent. With the determination of the nuclear densities of the isotopes, it is assumed that uranic oxide displaces the water. Calculated on the basis of this assumption was the course of the relationship between the weight concentration of the isotope U^{235} and the atomic $ratio$ ℓ_H / ℓ_5 for each density of uranic oxide. The corresponding curves are presented in Figures 3.39, 3.43, and 3.47. The oxide densities chosen for computation not only yield an idea of the influence of density on critical parameters with various dilutions of a system via the moderator, but also corresponds directly to a number of practically extant mixtures: aqueous solutions of uranium salts and suspensions of powders and compressed chemical compounds of uranium in the water. For example, a mixture of water and uranic oxide with an enrichment of five percent and with a density of 6 g/cm^3 corresponds directly to solutions of uranium salts UO_2F_2 and $UO_2(NO_3)_2$.

For the purposes of illustration, Figures 3.40, 3.41, and 3.42 present experimental data on solutions of UO_2F_2 $\underline{/\ 3\_/}$. The graphs make obvious that the difference between the computation of and the experiment with the critical mass does not exceed 10 percent, and with the critical volume, 15 percent. A comparison between the curves in Figures 3.36-3.47 and the corresponding curves in Figures 3.16-3.35 indicate that the relationship between the critical parameters (mass, volume of the sphere, and radius of the cylinder) and the moderation parameter for the mixture UO_2 + H_2O basically follows the regularities of the above-considered uranium-water mixtures, in which the density of uranium is 18.7 g/l cm^3: a reduction of the critical mass to the minimal value, with uranium concentrations corresponding to thermal neutron reactors; of the critical volume of a sphere and the radius of a cylinder, to the minimum lying within the range of the intermediate neutron spectrum; and then the monotonic increase of parameter values with further dilutions

of the moderator system. Drawing attention is the lack, in an $UO_2 + H_2O$ mixture, of a maximum in the fast range of all the densities of uranic oxide and enrichments considered here. Unlike mixtures of metallic uranium and water, mixtures of uranic oxide and water characteristically increase critical parameters in the range of a slight dilution of the moderator up to dry oxide.

From what has been stated, the following conclusions can be made:

1. The information presented in this article concerning mechanical mixtures of uranium-plutonium-water can be employed to determine the conditions of nuclear safety. The metallic-aqueous mixtures considered in this article surpass, in density, all the well-known real homogeneous hydrogen-containing systems, given corresponding moderation parameters; therefore, the data on the critical parameters of these mixtures can be applied in evaluating nuclear safety in all practical situations.

2. The results of the computations of the critical parameters of $UO_2 + H_2O$ mixtures with uranic-oxide densities of 3, 6, and 8.45 g/cm^3 can be expediently utilized in evaluating nuclear safety for mixtures of uranic-oxide powders with water, aqueous solutions of uranium salts, and suspensions of some compressed chemical compounds in water.

3. A general approach to the problem of applying the results contained herein for determining conditions of nuclear safety must be as follows: A weight concentration of fissionable isotopes is compared with a theoretical concentration in Figures 3.10, 3.15, etc. for uranium-plutonium-water mixtures, and in Figures 3.39, 3.43, and 3.47 for mixtures of uranic oxide and water. The application of the computed results is feasible, provided that the real concentrations do not exceed theoretical concentrations, given uniform moderation parameters. In the most general situation in which a comparison is difficult, it is necessary to utilize the results of the calculations for uranium-plutonium-water mixtures, from which the maximum initial densities of mixture components were taken (uranium - - 18.7 g/cm^3, plutonium - - 19.6 $g/cm3$, and water - - 1 g/cm^3).

4. The computation methods used here are not universal and provide satisfactory results only in a certain range of moderations: a) the effective one-group theory is for fast and thermal neutron spectrums, b) the P_1-multigroup approximation is for systems whose dimensions greatly exceed the length of the free range of neutrons in a given environment.

In the range of moderation characteristic of the intermediate neutron spectrum, the presented approximations are inadequate in a general situation; therefore, an evaluation of nuclear safety in accordance with the above-mentioned critical parameters in this range is

possible only from the qualitative point of view.

5. In the evaluations of nuclear safety in accordance with the critical relationships in Figures 3.36-3.47, it is necessary to pay attention to the 15-percent overestimation of the critical masses, compared with the experimental results.

Figure 3.6

The critical mass for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an aqeously reflected sphere. The uranium is natural.
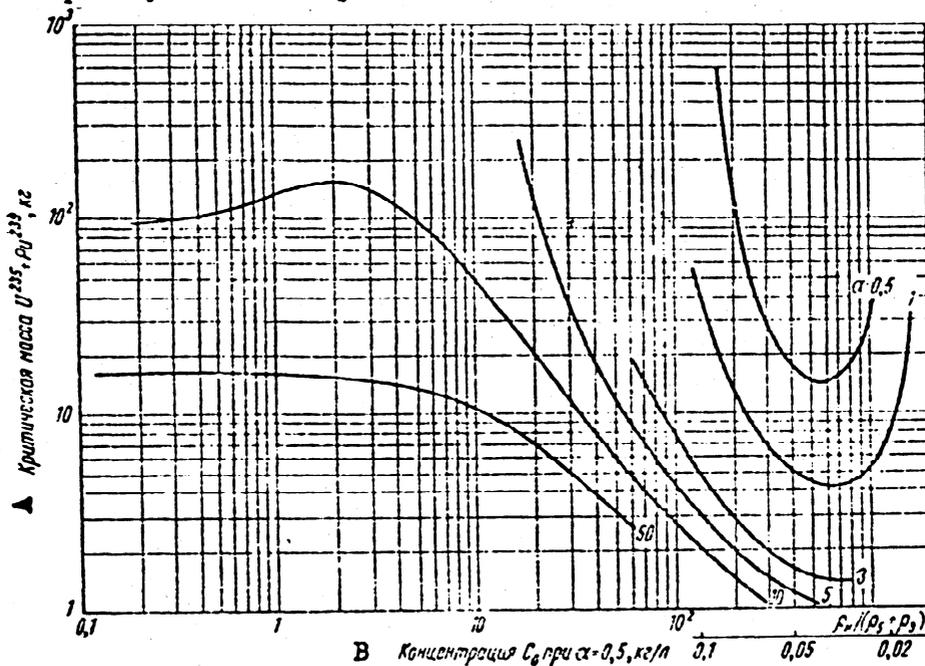
Legend: A = Critical mass of U^{235} + Pu^{239}, B = Concentration of C_0 with α = $O.5$, kg/l.

Figure 3.7

The critical volume for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an aqueously reflected sphere. The uranium is natural.
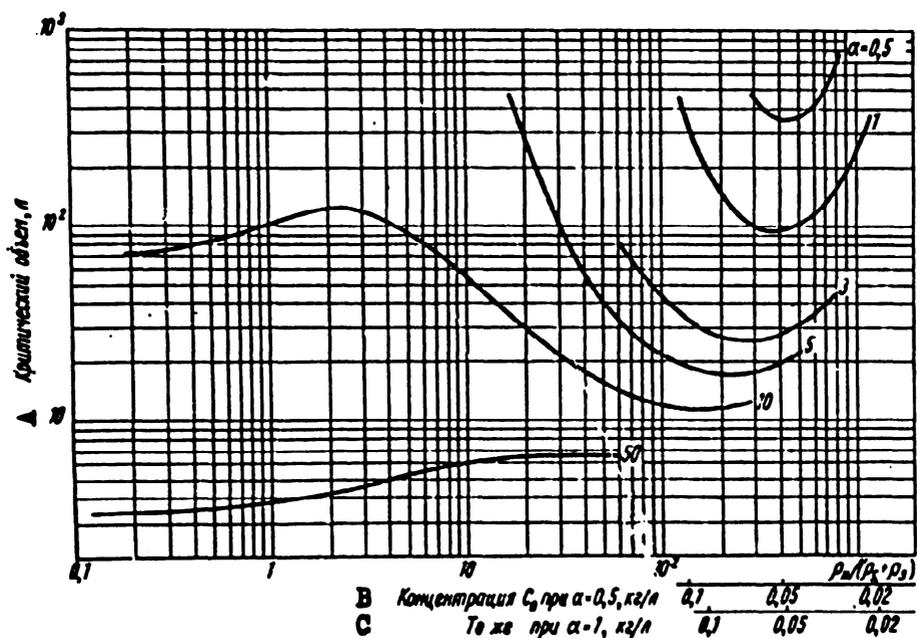
Legend: A = Critical volume, l; B = Concentration of C_0 with $\alpha = 0.5$, kg/l; C = Concentration of C_0 with $\alpha = 1$, kg/l

Figure 3.8

The critical radius of an aqueously reflected infinite cylinder
for a mechanical mixture of uranium, plutonium, and water as a function
of moderation. The uranium is natural.
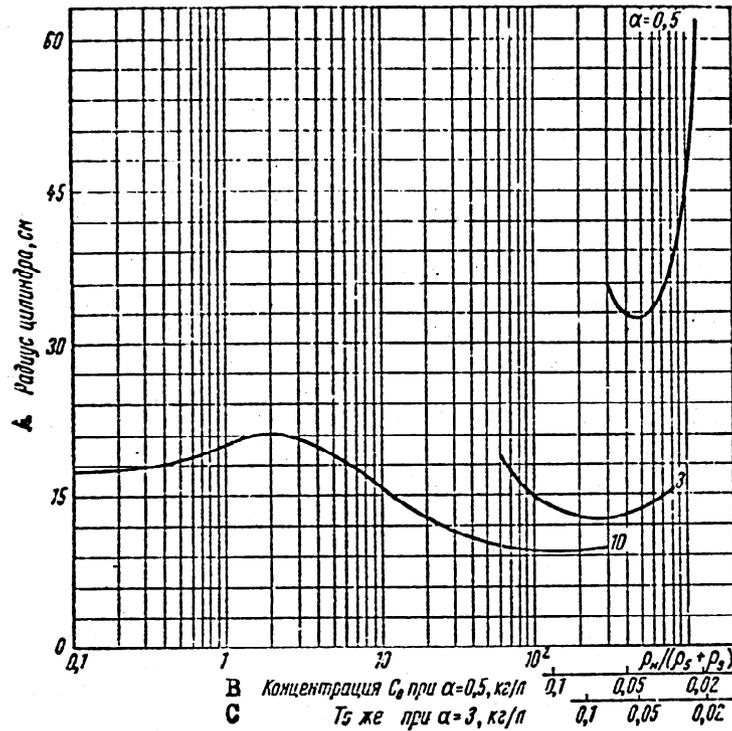
Legend: A = Radius of the cylinder, cm; B = Concentration of C_0 with $\alpha = 0.5$;
C = Concentration of C_0 with $\alpha = 3$, kg/l.

Figure 3.9

The critical half-thickness of an aqueously reflected infinite plate for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The uranium is natural.
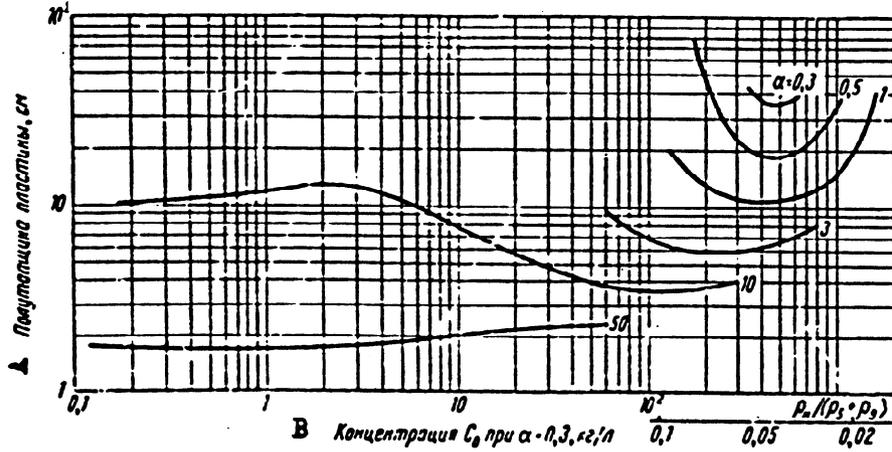
Legend: A = Half-thickness of the plate, cm; B = Concentration of C_0 with $\alpha = 0.3$, kg/l

Figure 3.10

The relationship between a concentration of U^{239} in a mechanical mixture of uranium, plutonium, and water and the ratio of atoms $\theta_H/(\theta_5 + \theta_9)$ with various α values. The uranium is natural.
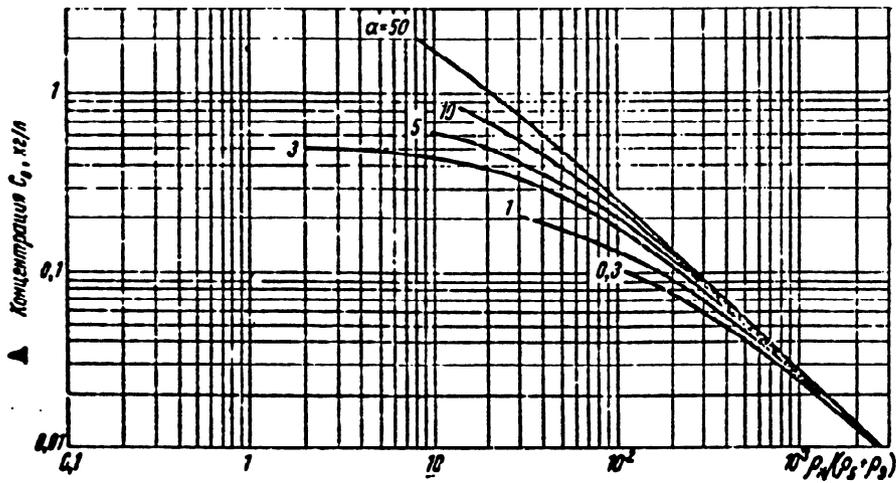
Legend: A = Concentration of C_0, kg/l.

Figure 3.11

The critical mass for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an acqueously reflected sphere. The enrichment of uranium is one percent.
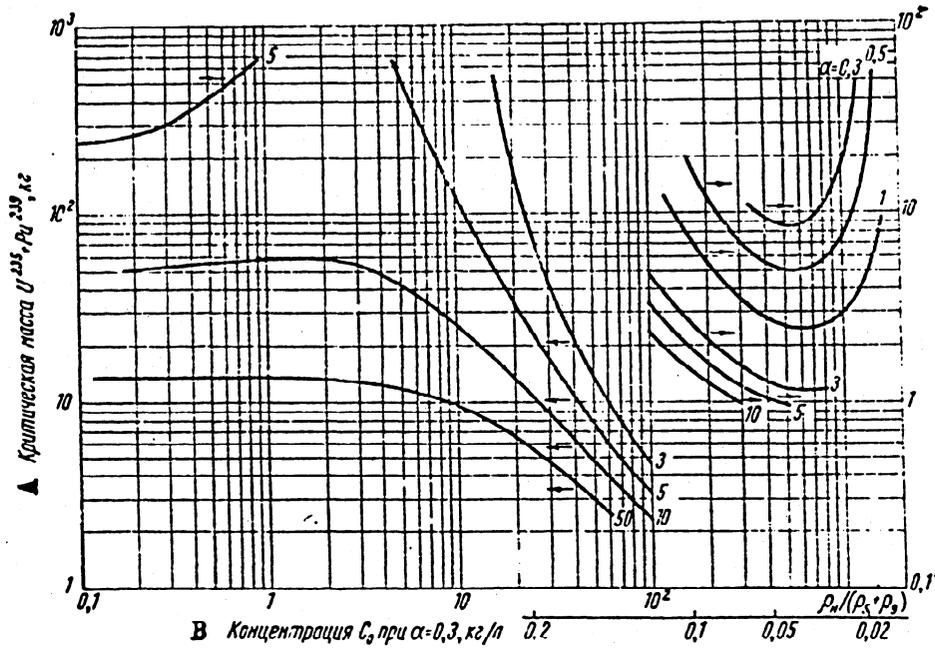
Legend: A = Critical mass of U^{235} + Pu^{239}, kg; B = Concentration of C_0 with $\alpha = 0.3$, kg/l.

Figure 3.12

The critical volume for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an acqueously reflected sphere. The enrichment of uranium is one percent.
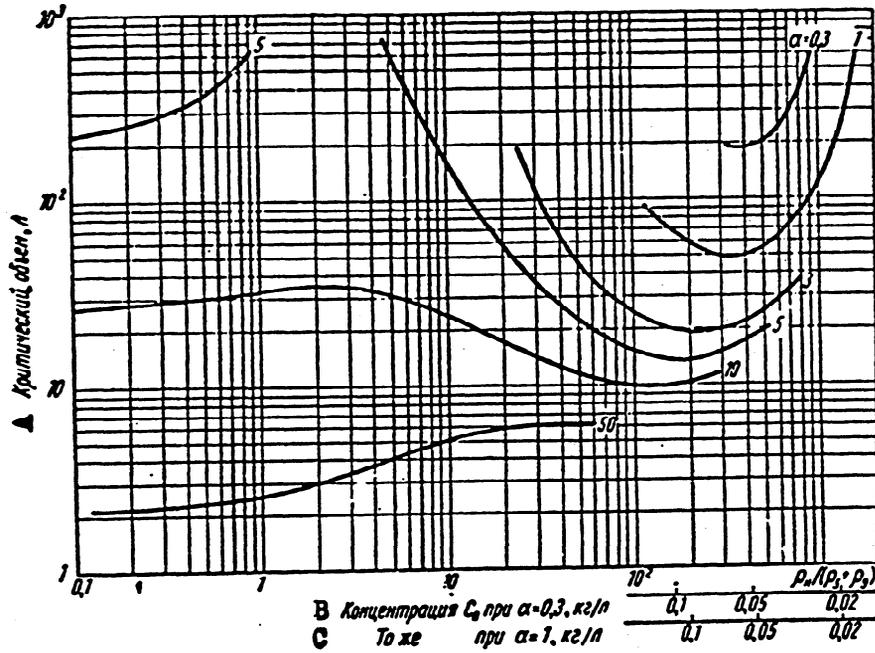
Legend: A = Critical volume, l; B = Concentration of C_0 with $\alpha = 0.3$, kg/l, C = Concentration of C_0 with $\alpha = 1$, kg/l.

Figure 3.13

The critical radius of an aqueously reflected infinite cylinder
for a mechanical mixture of uranium, plutonium, and water as a function
of moderation. The enrichment of uranium is one percent.
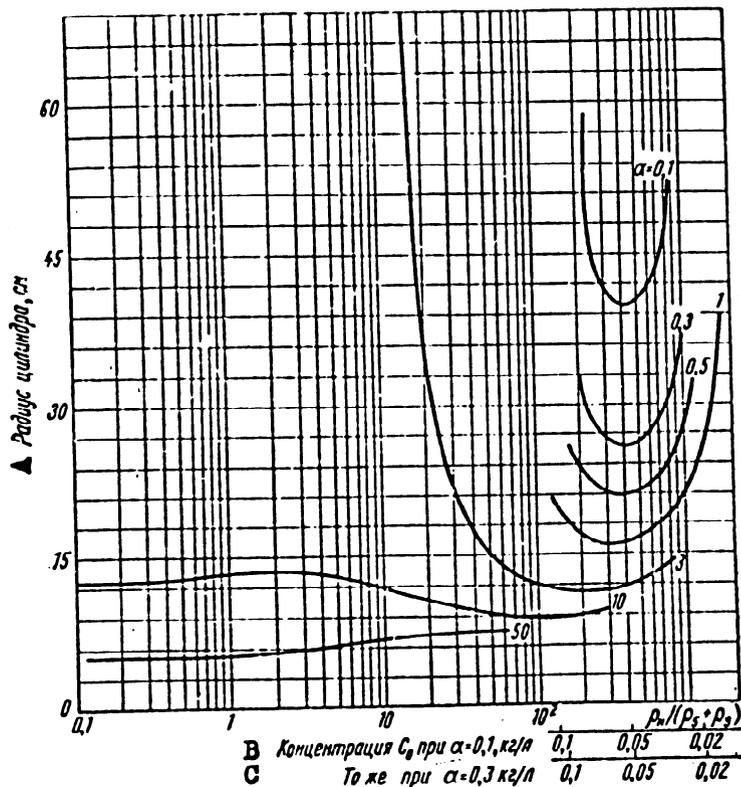
Legend: A = Radius of the cylinder, cm; B = Concentration of C_0 with α =
$0.1, kg/l$; C = Concentration of C_0 with $\alpha = 0.3, kg/l$.

Figure 3.14

The critical half-thickness of an aqueously reflected infinite plate for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The enrichment of uranium is one percent.
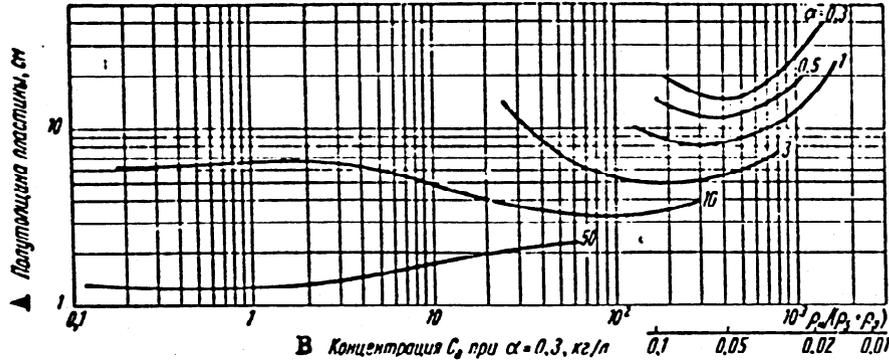
Legend: A = Half-thickness of the plate, cm; B = Concentration of C_0 with $\alpha = 0.3$, kg/l.

Figure 3.15

The relationship between a concentration of $U^{235} + Pu^{239}$ in a mechanical mixture of uranium, plutonium, and water and the ratio of atoms to $\ell_g/(\ell_5 + \ell_9)$ with various a values. The enrichment of uranium is one percent.
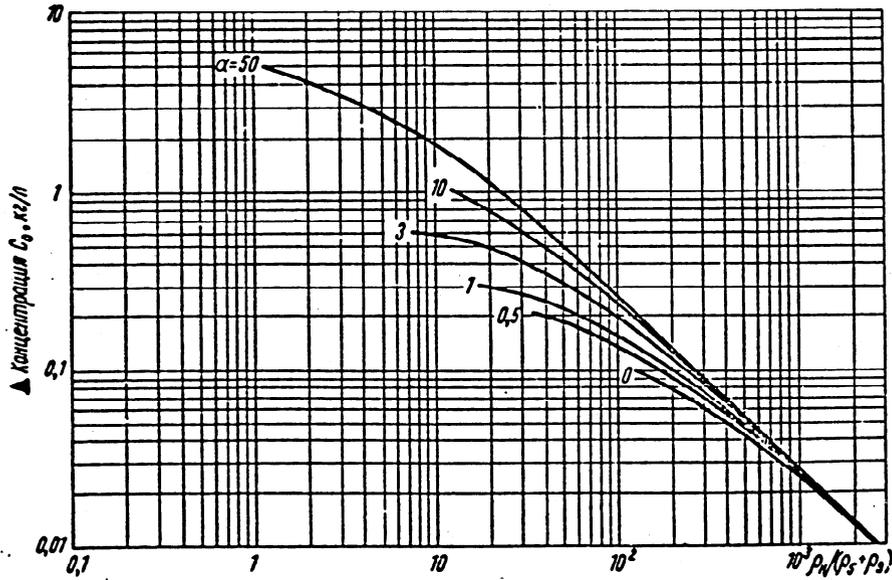
Legend: A = Concentration of C_0, kg/l.

Figure 3.16

The critical mass for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an aqueously reflected sphere. The enrichment of uranium is two percent.
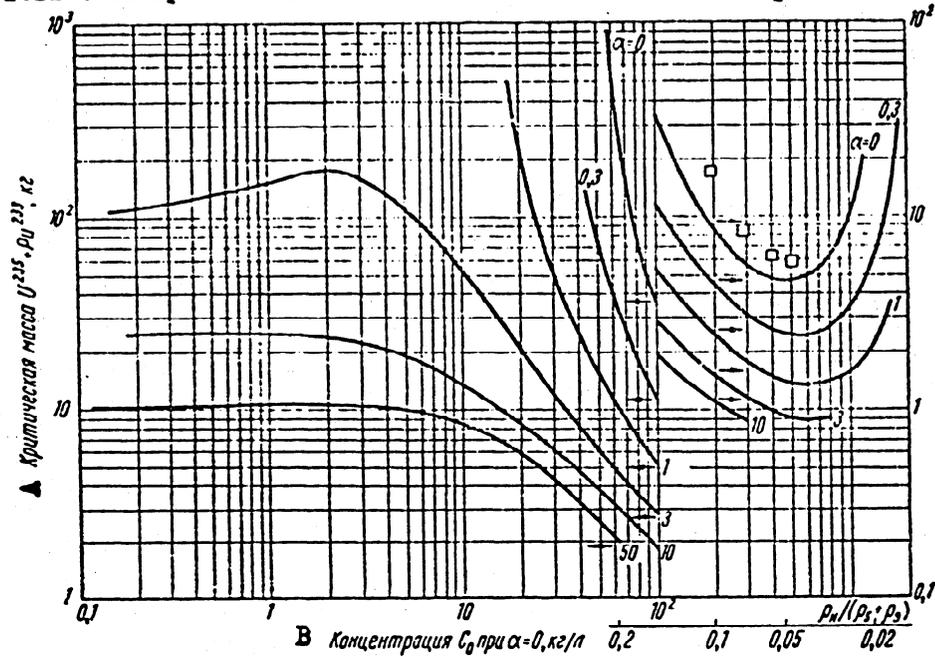
Legend: $\square$ is the experiment $\left[U \ (2\% \ U^{235}) \ F_4 + C_{25}H_{52} \right] \left[3 \right]$; A = Critical mass of $U^{235} + Pu^{239}$, kg; B = Concentration of C_0 with $\alpha = 0$, kg/l.

Figure 3.17

The critical volume for a mechanical m'xture of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an aqueously reflected sphere. The enrichment of uranium is two percent.

B   Концентрация C_0 при $\alpha=0$, кг/л   0,2   0,1   0,05   0,02
C   То же при $\alpha=0,3$, кг/л   0,2   0,1   0,05   0,02
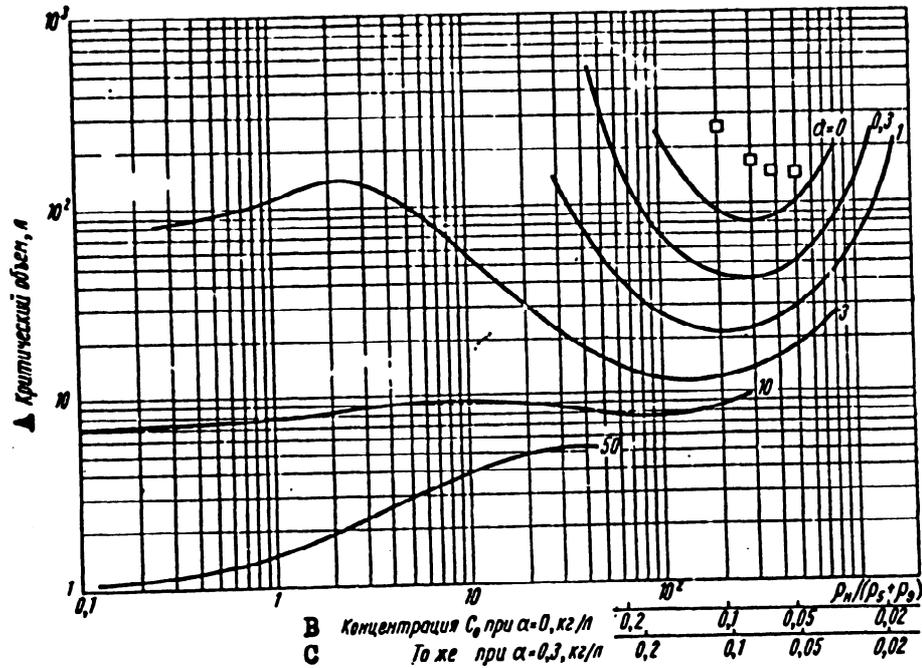
Legend: ▢ is the experiment $[U\ (2\%\ U^{235})\ F_4 + C_{25}H_{52}]\ [3]$; A = Critical volume, 1; B = Concentration of C_0 with $\alpha = 0$, kg/l.

Figure 3.18

The critical radius of an aqueously reflected infinite cylinder
for a mechanical mixture of uranium, plutonium, and water as a function
of moderation. The enrichment of uranium is two percent.
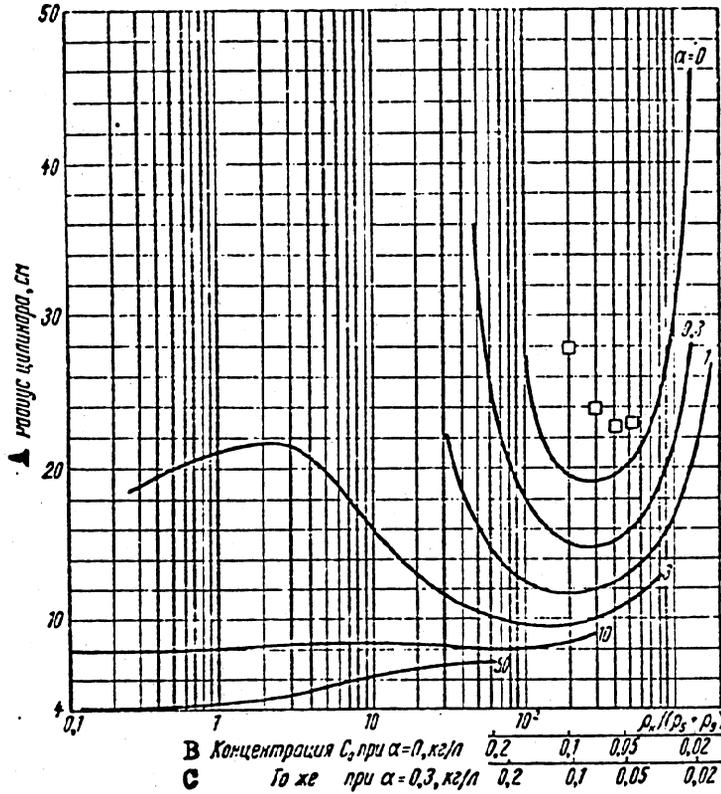
Legend: □ = the experiment $\int U$ (2% U^{235}) F_4 + $C_{25}H_{52}\int$ $\int 3\int$; A = Radius
of cylinder, cm; B = Concentration of C_0 with $\alpha = 0$, kg/l.

Figure 3.19

The critical half-thickness of an aqueously reflected infinite
plate for a mechanical mixture of uranium, plutonium, and water as a function
of moderation. The enrichment of uranium is two percent.
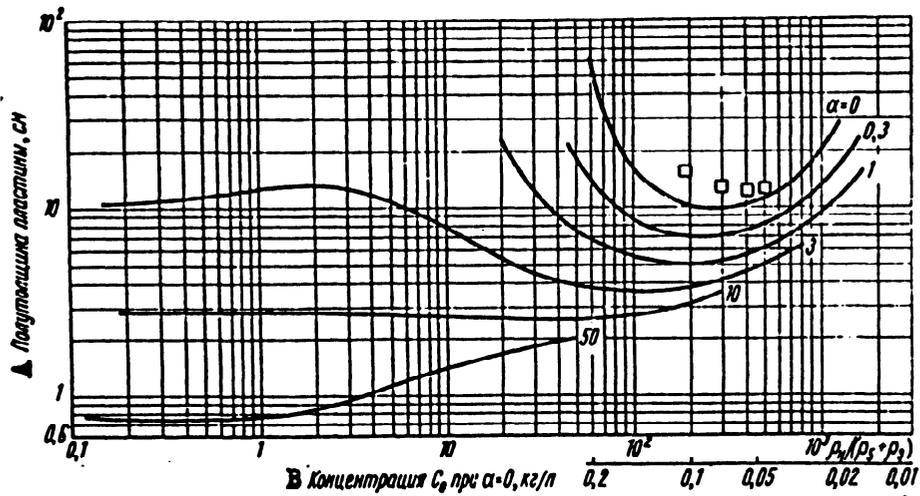
Legend: □ = the experiment $\int U \ (2\% \ U^{235}) \ F_4 + C_{25}H_{52} \ \int \ \int 3 \int$; A =
Half-thickness of the plate, cm; B = Concentration of C_0 with $\alpha = 0$,
kg/l.

Figure 3.20

The relationship between a concentration of $U^{235} + Pu^{239}$ in a mechanical mixture of uranium, plutonium, and water and the ratio of atoms to $\ell_g/(\ell_5 + \ell_9)$ with various α values. The enrichment of uranium is two percent.
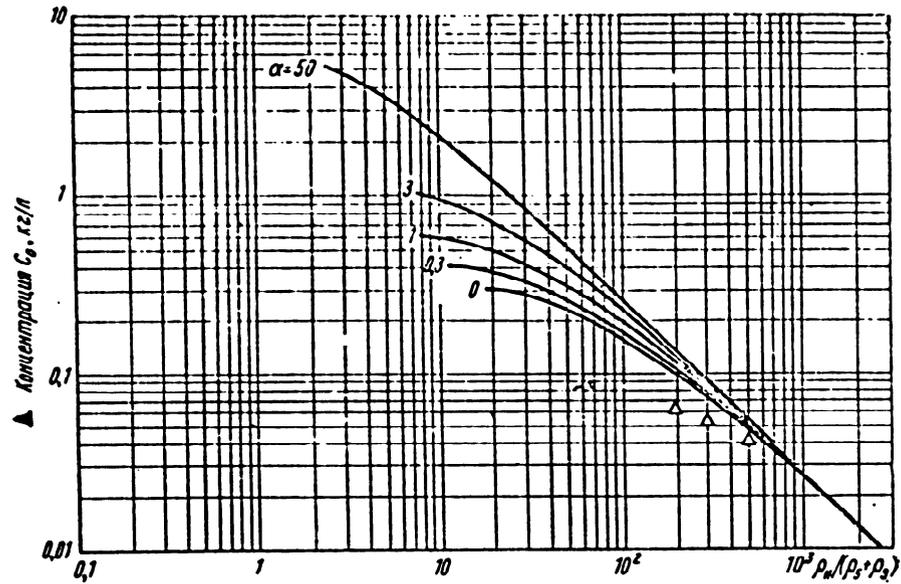
Legend: $\triangle$ is the experiment $\int U$ $(2\% \ U^{235})$ $P_4 + C_{25}H_{52}$ $\mathcal{J}\mathcal{L}3\mathcal{J}$;
A = Concentration of C_0, kg/l.

- 48 -

Figure 3.21

The critical mass for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an aqueously reflected sphere. The enrichment of uranium is five percent.
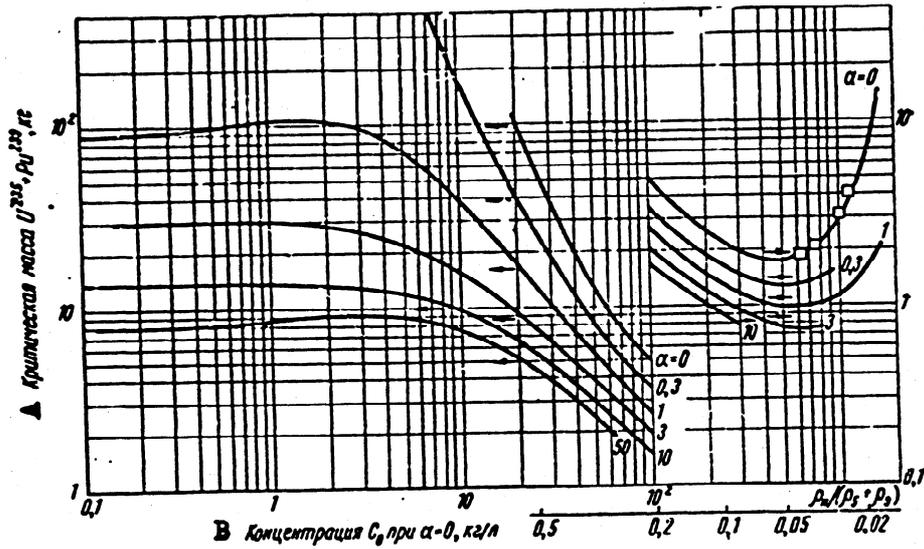
Legend: $\square$ is the experiment [solution of U (4.9% U^{235}) O_2F_2] [3]; A = Critical mass of U^{235} + Pu^{239}, kg; B = Concentration of C_0 with $\alpha = 0$, kg/l.

Figure 3.22

The critical volume for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an aqueously reflected sphere. The enrichment of uranium is five percent.
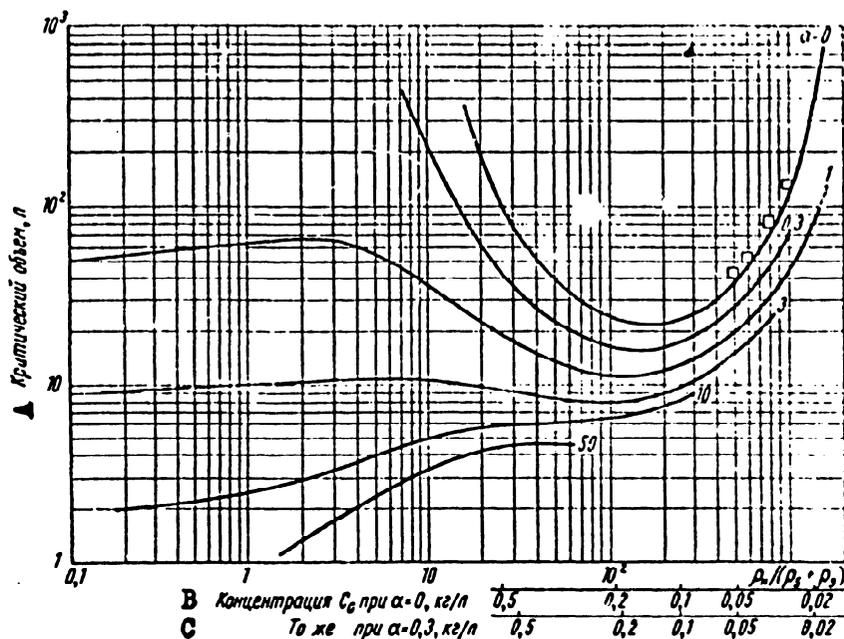
Legend: $\square$ is the experiment $\sqrt{\,}$solution of U (4.9% U^{235}) O_2F_2 $J[3J]$;
A = Critical volume, 1; B = Concentration of C_0 with $\alpha = 0$, kg/l;
C = Concentration of C_0 with $O.3$, kg/l.

Figure 3.23

The critical radius of an aqueously reflected infinite cylinder
for a mechanical mixture of uranium, plutonium, and water as a function of
moderation. The enrichment of uranium is five percent.

Legend: ▢ is the experiment $\sqrt{\text{solution of U}}$ (4.9% U^{235}) O_2F_2 $\cancel{}$ $[3]$;
A = Radius of the cylinder, cm; B = Concentration of C_0 with $\alpha = 0, kg/l$;
C = Concentration of C_0 with $\alpha = 0.3$, kg/l.

- 51 -
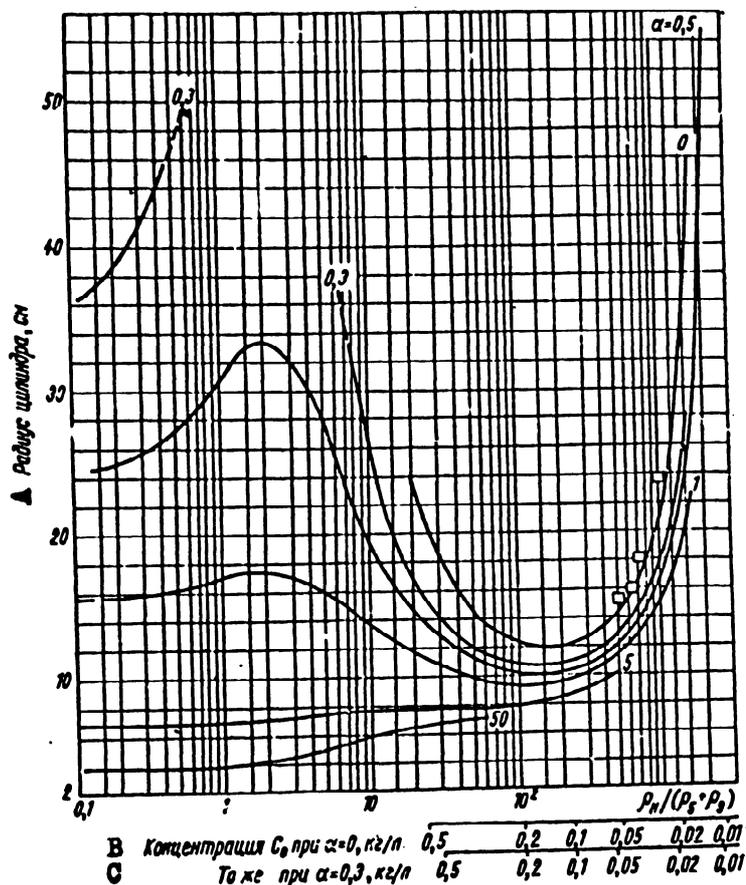
Figure 3.24

The critical half-thickness of an aqueously reflected infinite plate for a mixture of uranium, plutonium, and water as a function of moderation. The enrichment of uranium is five percent.
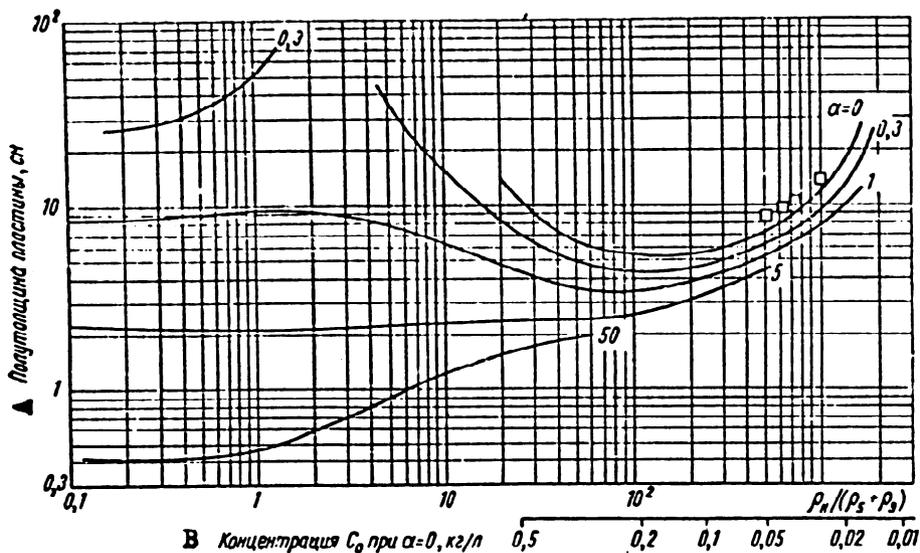
Legend: $\square$ is the experiment $\lceil$ solution of U (4.9% U^{235}) O_2F_2 $\rfloor$ $[3]$; A = Half-thickness of the plate, cm; B = Concentration of C_0 with $\alpha = 0$, kg/l.

Figure 3.25

The relationship between a concentration of $U^{235} + Pu^{239}$ in a mechanical mixture of uranium, plutonium, and water and the ratio of atoms to $\mathcal{P}_H/(\mathcal{P}_5 + \mathcal{P}_9)$ with various α values. The enrichment of uranium is five percent.
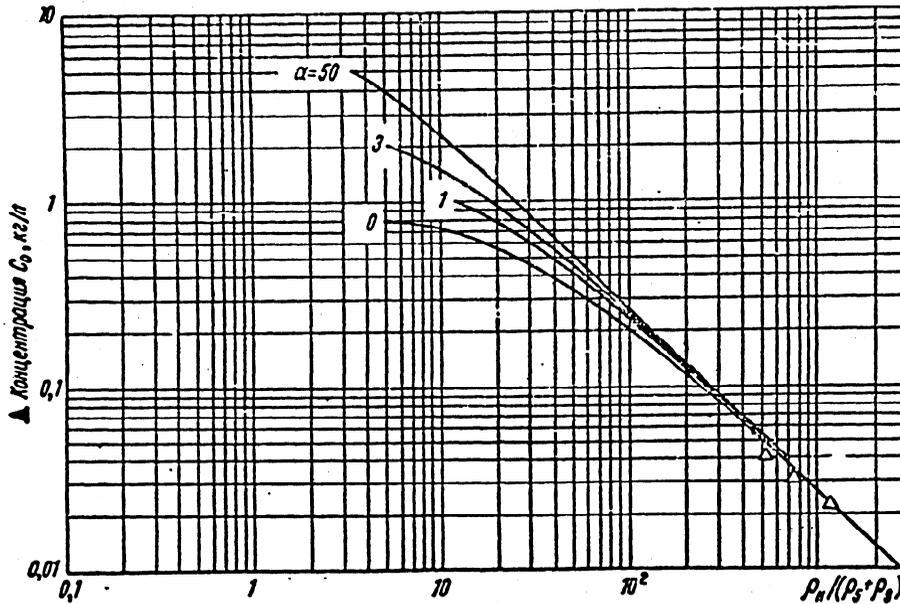
Legend: $\triangle$ is the experiment $\int$ solution of U $(4.9\% \ U^{235})$ O_2F_2 $\int$ $\int 3 \int$; $\blacktriangle$ = Concentration of C_0, kg/l.

Figure 3.26

The critical mass for a mechanical solution of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an aqueeously reflected sphere. The enrichment of uranium is ten percent.
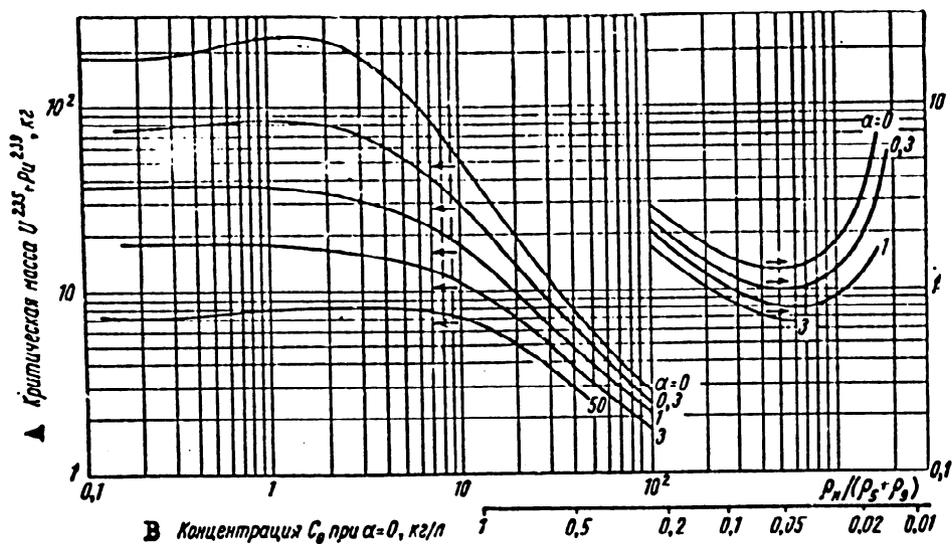
Legend: A = Critical mass of U^{235} + Pu^{239}, kg; B = Concentration of C_0 with $\alpha = 0, kg/l$.

Figure 3.27

The critical volume for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an aqueously reflected sphere. The enrichment of uranium is ten percent.
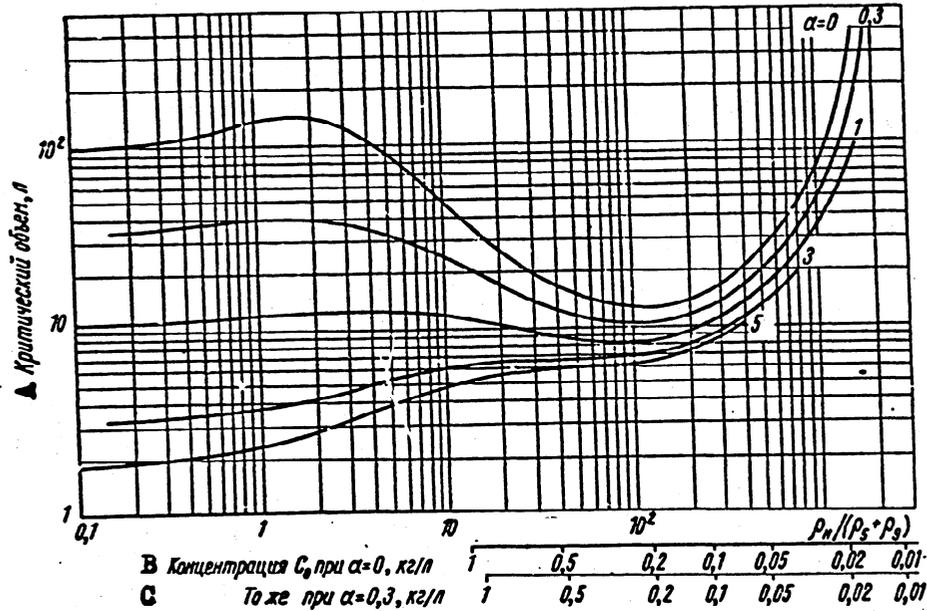
Legend: A = Critical volume, l; B = Concentration of C_0 with $\alpha = 0$, kg/l;
C = Concentration of C_0 with 0.3; $kg = l$.

Figure 3.28

The critical radius of an aqueously reflected infinite cylinder
for a mechanical mixture of uranium, plutonium, and water as a function of
moderation. The enrichment of uranium is ten percent.
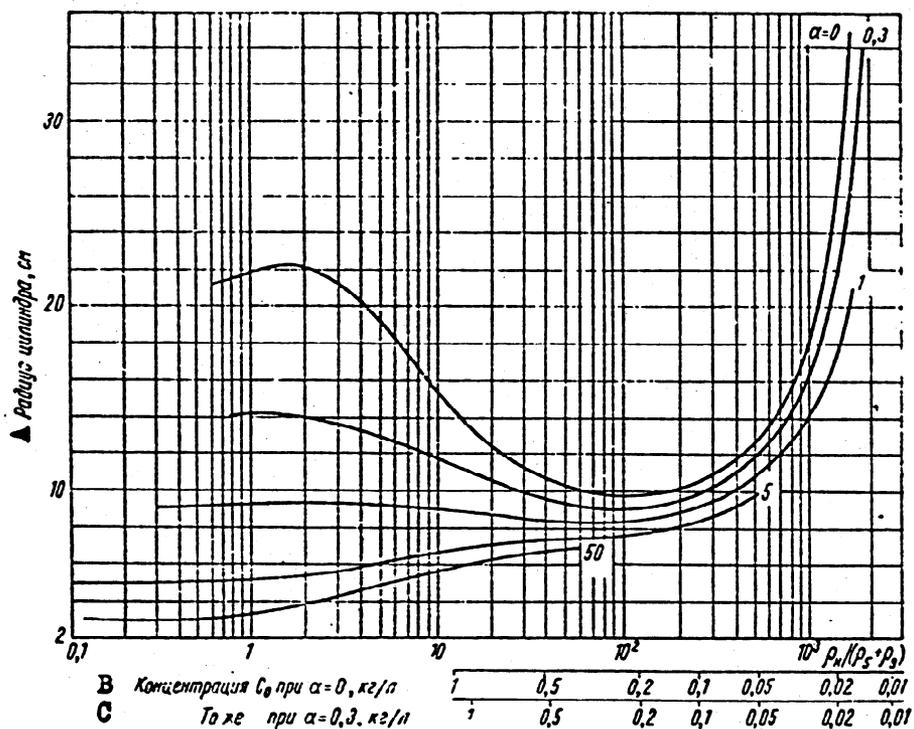
Legend: A = Radius of the cylinder, cm; B = Concentration of C_0 with $\alpha = 0$, kg/l;
C = Concentration of C_0 with $\alpha = 0.3$, kg/l

Figure 3.29

The critical half-thickness of an aqueously reflected infinite
plate for a mechanical mixture of uranium, plutonium, and water as a function
of moderation. The enrichment of uranium is ten percent.
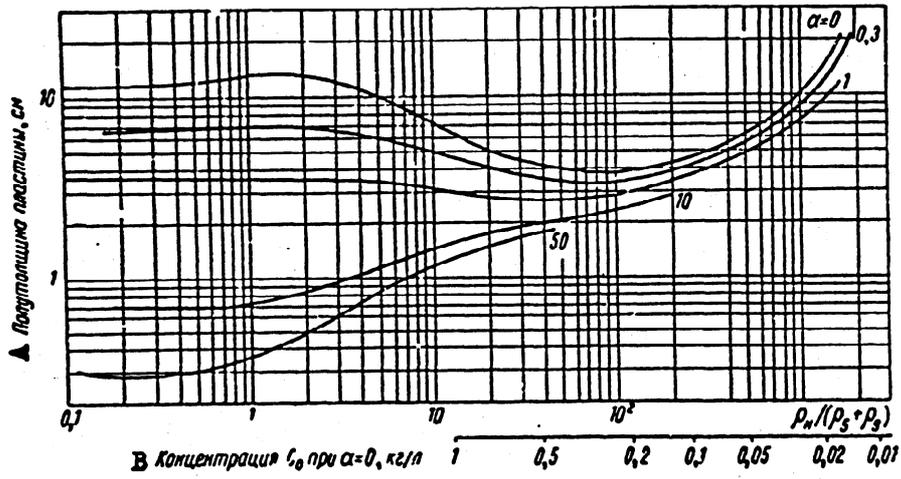
Legend: A = Half-thickness of the plate, cm; B = Concentration of C_0
with $\alpha = 0$, kg/l.

Figure 3.30

The relationship between a concentration of $U^{235} + Pu^{239}$ in
a mechanical mixture of uranium, plutonium, and water and the ratio of
atoms to $\rho_H/(\mathcal{E}_5 + \rho_9)$ with various α values. The enrichment of uranium
is ten percent.
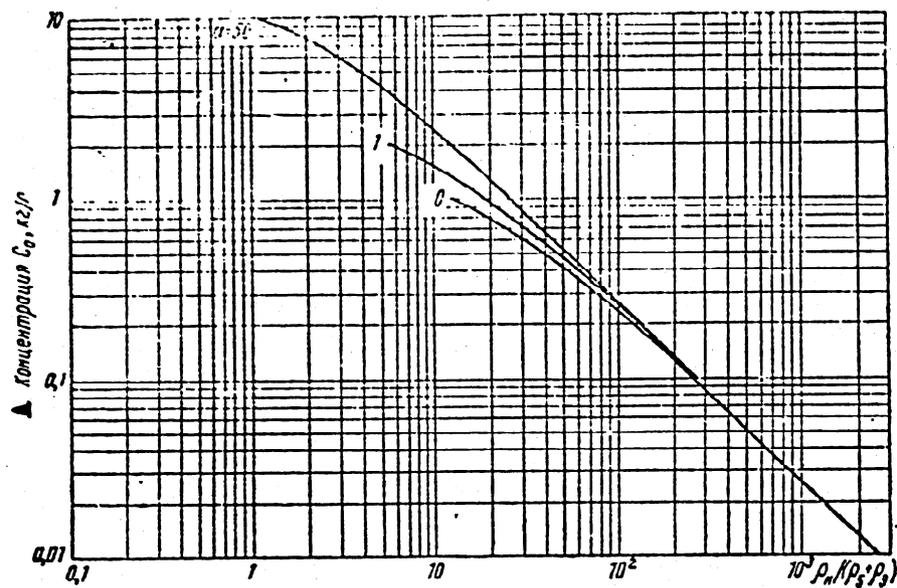
Legend: A = Concentration of C_0, Kg/l.

Figure 3.31

The critical mass for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an aqueously reflected sphere. The enrichment of uranium is 23 percent.
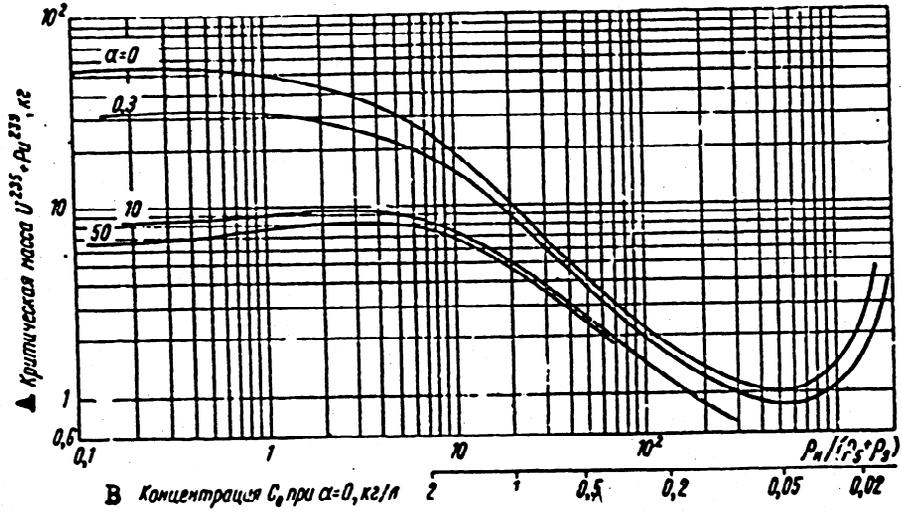
Legend: A = Critical mass of U^{235} + Pu^{239}, kg; B = Concentration of C_0 with $\alpha = 0$, $kg./l$.

Figure 3.32

The critical volume for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The shape of the fuel core is an aqueously reflected sphere. The enrichment of uranium is 23 percent.
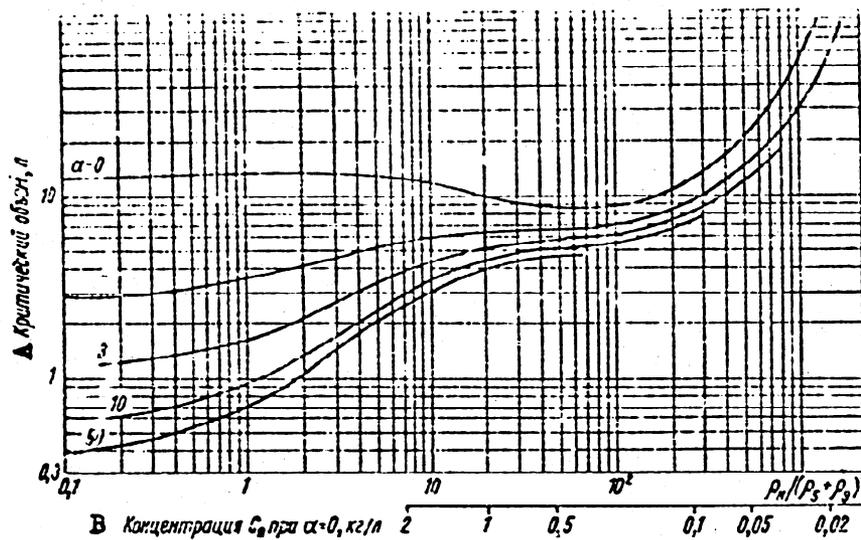
Legend: A = Critical volume, l; B = Concentration of C_0 with $\alpha = o, kg/\ell$.

Figure 3.33

The critical radius of an aqueously reflected infinite
cylinder for a mechanical mixture of uranium, plutonium, and water
as a function of moderation. The enrichment of uranium is 23 percent.
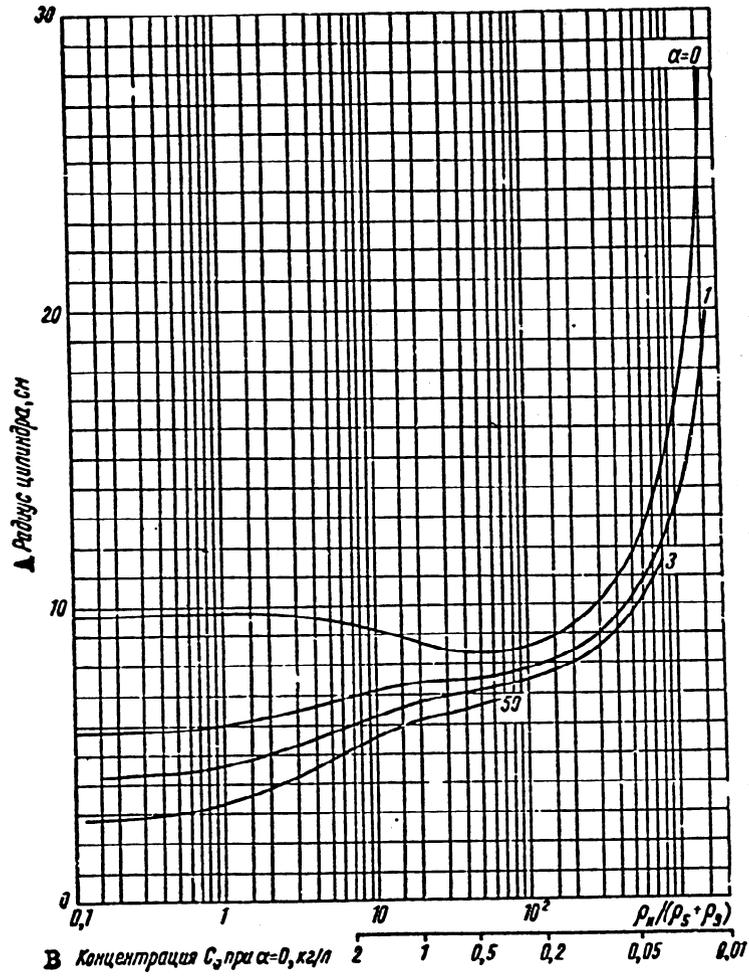
Legend: A = Radius of the cylinder; B = Concentration of C_0 with $\alpha = 0, kg/l$.

Figure 3.34

The critical half-thickness of an aqueously reflected infinite plate for a mechanical mixture of uranium, plutonium, and water as a function of moderation. The enrichment of uranium is 23 percent.
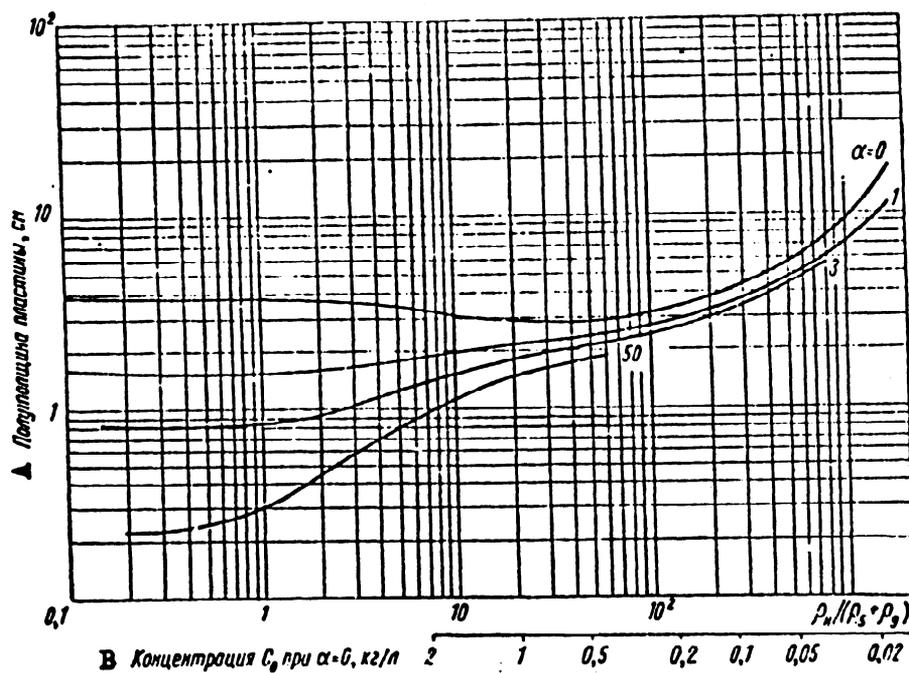
Legend: A = Half-thickness of the plate, cm; B = Concentration of C_0 with $\alpha = o$, kg/l.

Figure 3.35

The relationship between a concentration of $U^{235} + Pu^{239}$ in a mechanical mixture of uranium, plutonium, and water and the ratio of atoms to $\ell_H / (\ell_5 + \ell_9)$ with various α values. The enrichment of uranium is 23 percent.
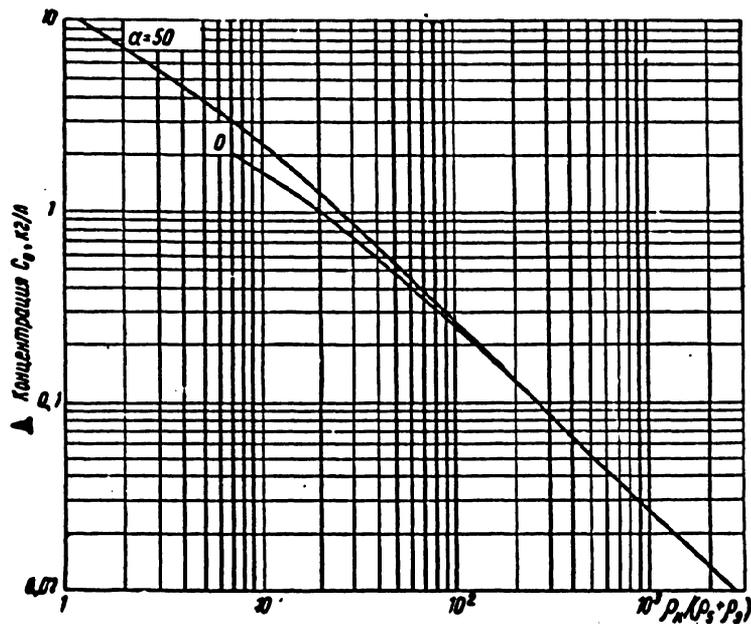
Legend: A = Concentration of C_0, kg/l.

Figure 3.36

The critical mass for a mechanical mixture of UO_2 + H_2O as a function of moderation, with various enrichments (U^{235}, %). The shape of the fuel core is an aqueously reflected sphere. The density of the UO_2 is 3 g/cm^3.
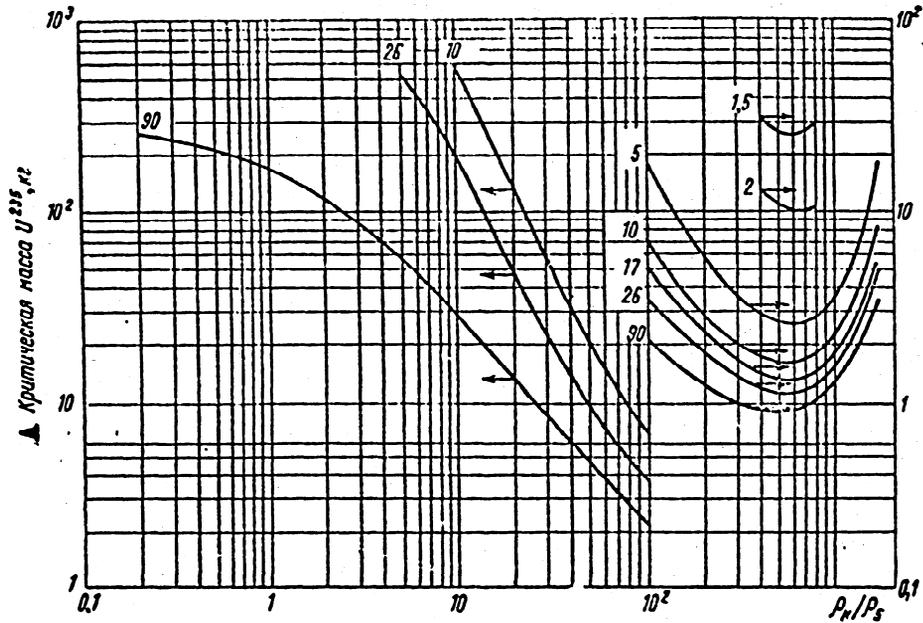
Legend: A = Critical mass of U^{235}, kg.

Figure 3.37

The critical volume for a mechanical mixture of UO_2 + H_2O as a function of moderation, with various enrichments (U^{235}, %). The shape of the fuel core is an aqueously reflected sphere. The density of the UO_2 is 3 g/cm^3.
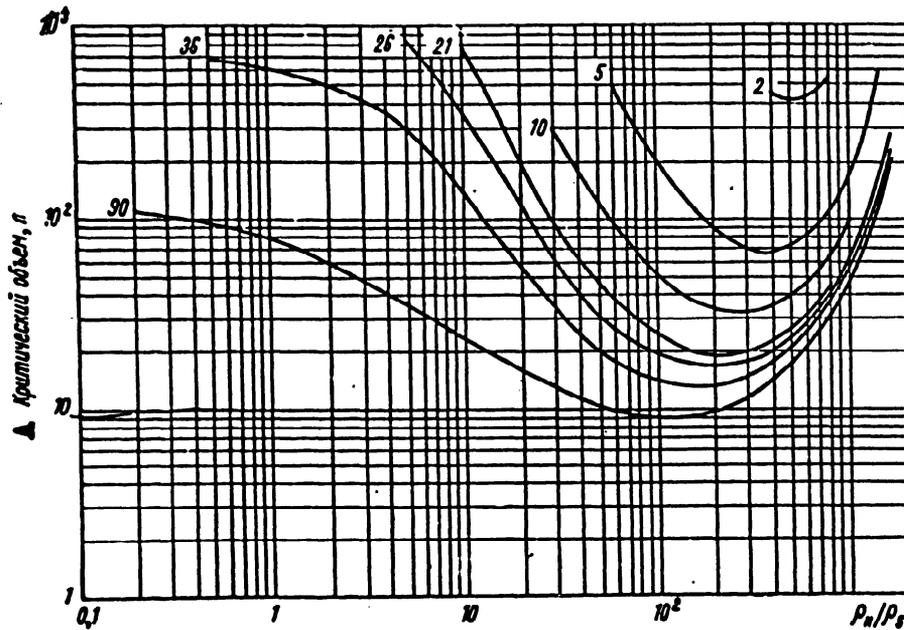
Legend: A = Critical volume, l.

Figure 3.38

The critical radius of an aqueously reflected infinite cylinder for a mechanical mixture of UO_2 + H_2O as a function of moderation, with various enrichments (U^{235}, %). The density of the UO_2 is 3 g/cm^3.
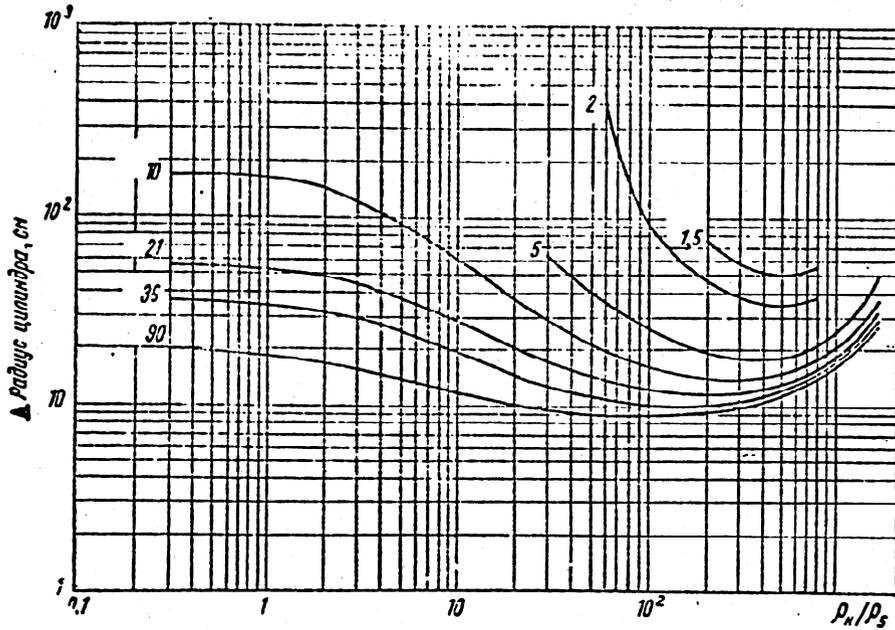
Legend: A = Radius of the cylinder, cm.

Figure 3.39

The relationship between a concentration of U^{235} in a mechanical mixture of UO_2 + H_2O and the ratio of the atoms to $\dfrac{\rho_H}{\rho_5}$, with various enrichments (U^{235}, %). The density of the UO_2 is 3 g/cm^3.
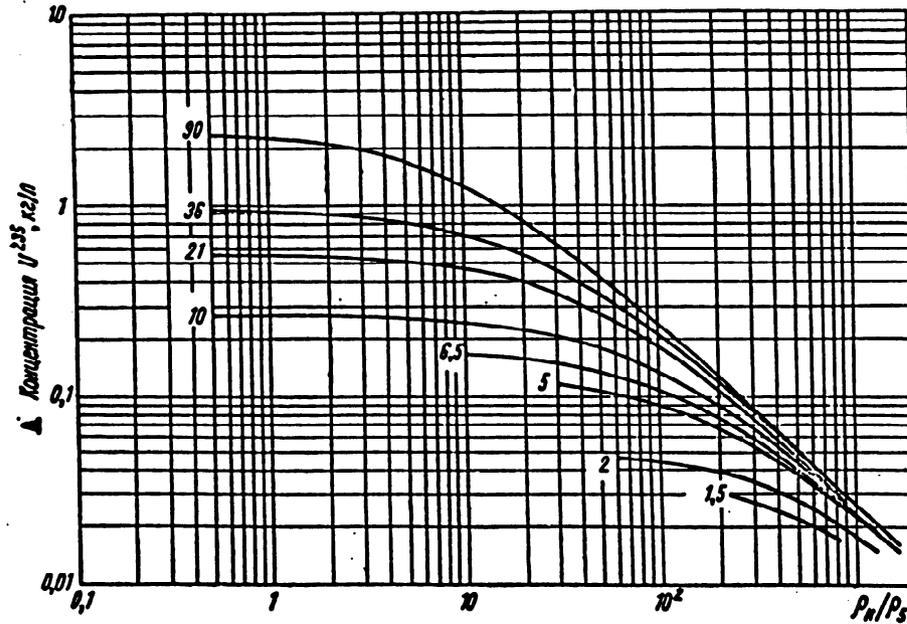

Legend: A = Concentration of U^{235}, kg/l.

Figure 3.40

The critical mass for a mechanical mixture of $UO_2 + H_2O$ as a
function of moderation, with various enrichments (U^{235}, %).[2] The shape
of the fuel core is an aqueously reflected sphere. The density of the
UO_2 is 6 g/cm³ .
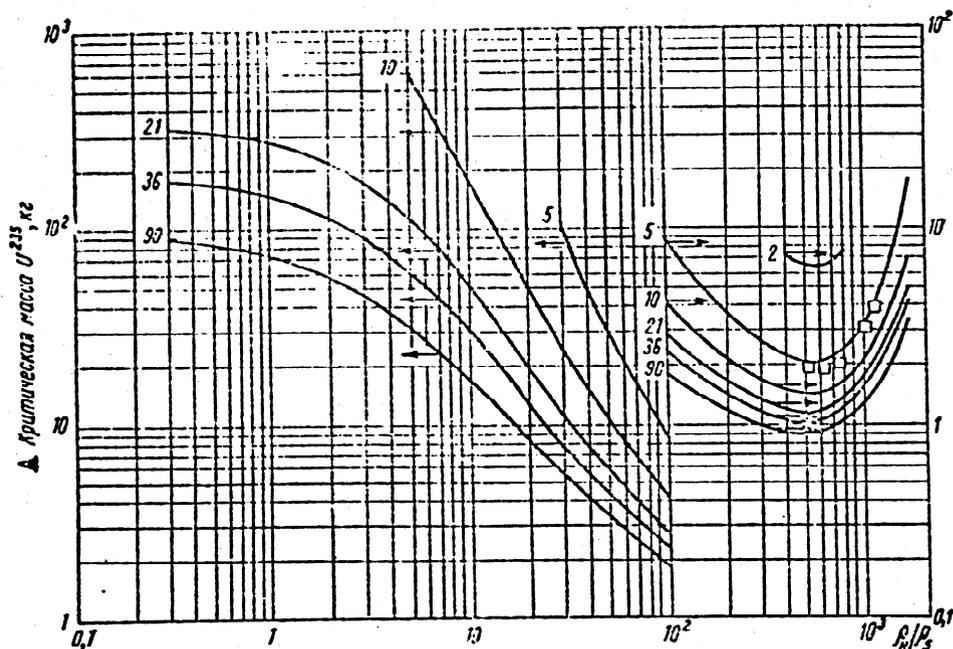
Legend: = ◻ is the experiment [a solution of U (4.9% U^{235}) O_2F_2] [3];
A = Critical mass of U^{235}, kg.

Figure 3.41

The critical volume for a mechanical mixture of UO_2 + H_2O as a function of moderation, with various enrichments (U^{235}, %). The shape of the fuel core is an aqueously reflected sphere. The density of the UO_2 is 6 g/cm³.
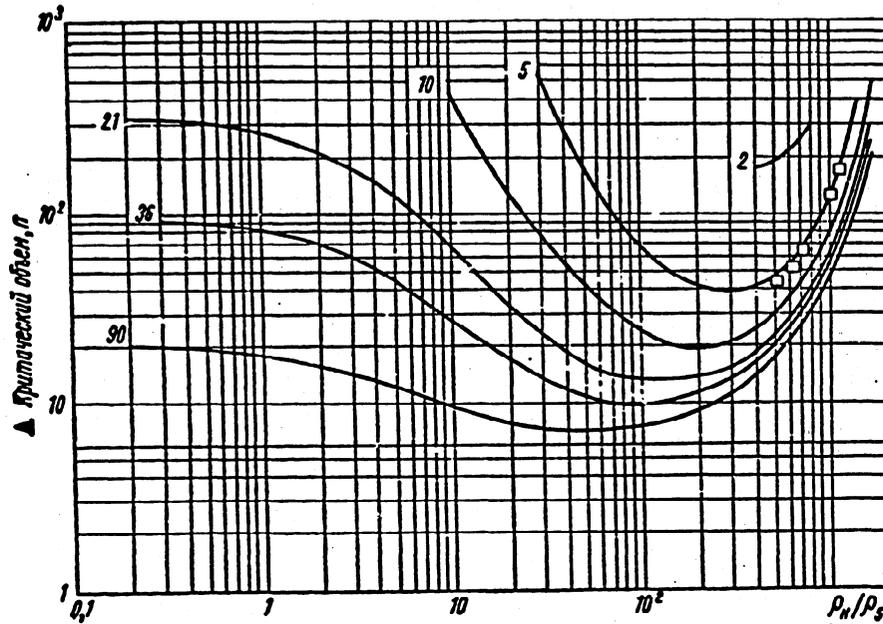
Legend: □ is the experiment ⌐ a solution of U (4.9% U^{235}) O_2F_2 ⌐ [3];
A = Critical volume, l.

Figure 3.42

The critical radius of an aqueously reflected infinite cylinder for a mechanical mixture of UO_2 + H_2O as a function of moderation, with various enrichments (U^{235}, %). The density of UO_2 is 6 g/cm^3.
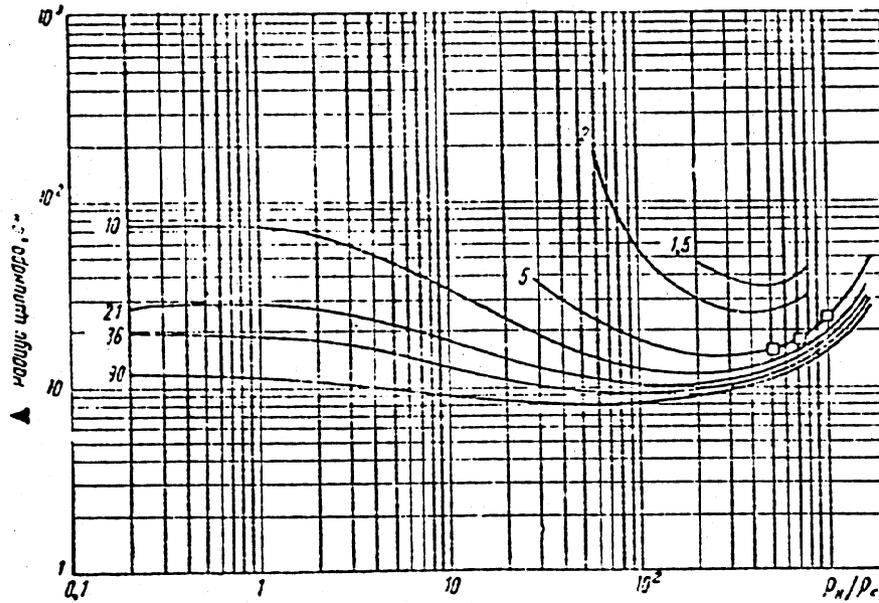
Legend: □ is the experiment [a solution of U (4.9% U^{235}) O_2F_2] [3];
A = Radius of the cylinder, cm.

Figure 3.43

The relationship between the concentrations of U^{235} in a mechanical mixture of UO_2 + H_2O and the ratio of atoms to $\frac{\rho H}{\rho_5}$ with various enrichments (U^{235}, %). The density of the UO_2 is 6 g/cm³.
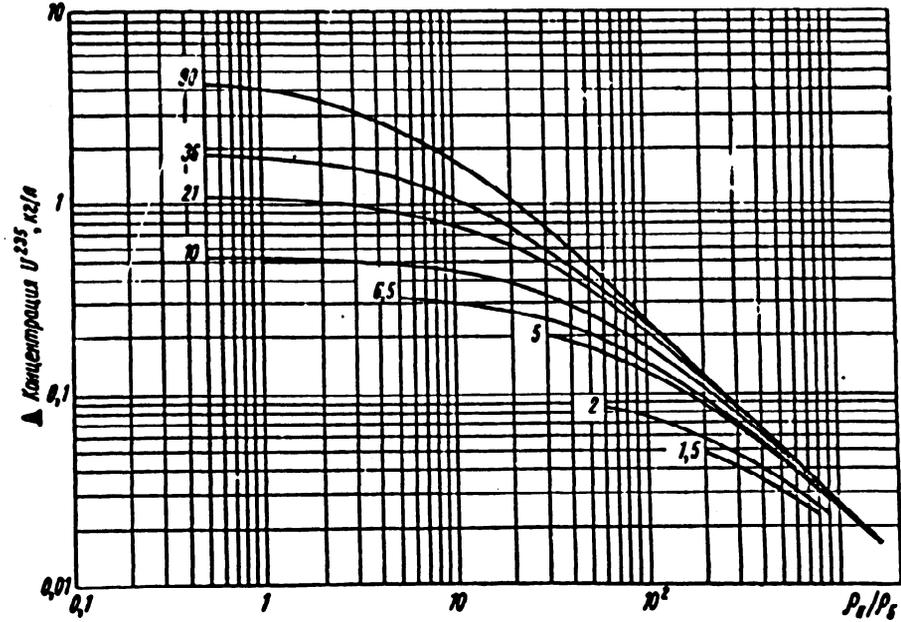


Legend: A = Concentration of U^{235}, kg/l.

Figure 3.44

The critical mass for a mechanical mixture of UO_2 + H_2O as a
function of moderation, with various enrichments (U^{235}, %). The shape
of the fuel core is an aqueously reflected sphere. The density of the
UO_2 is 8.45 g/cm^3 .
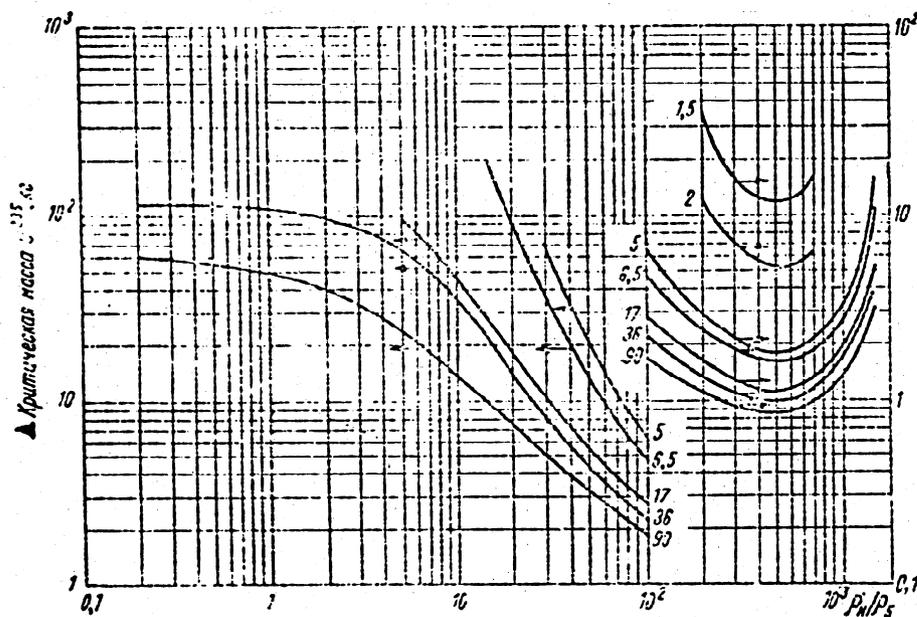
Legend: A = Critical mass of U^{235}, kg.

Figure 3.45

The critical volume for a mechanical mixture of UO_2 + H_2O as a function of moderation, with various enrichments (U^{235}, %). The shape of the fuel core is an aqueously reflected sphere. The density of the UO_2 is 8.45 g/cm^3.
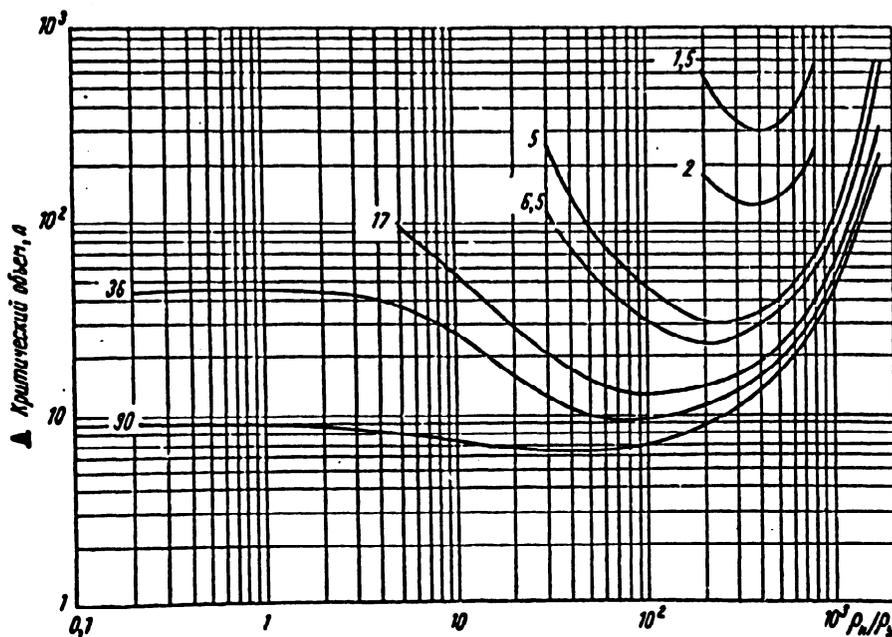

Legend: A = Critical volume, l.

Figure 3.46

The critical radius of an aqueously reflected cylinder for a mechanical mixture of UO_2 + H_2O as a function of moderation, with various enrichments (U^{235}, %). The density of the UO_2 is 8.45 g/cm^3.
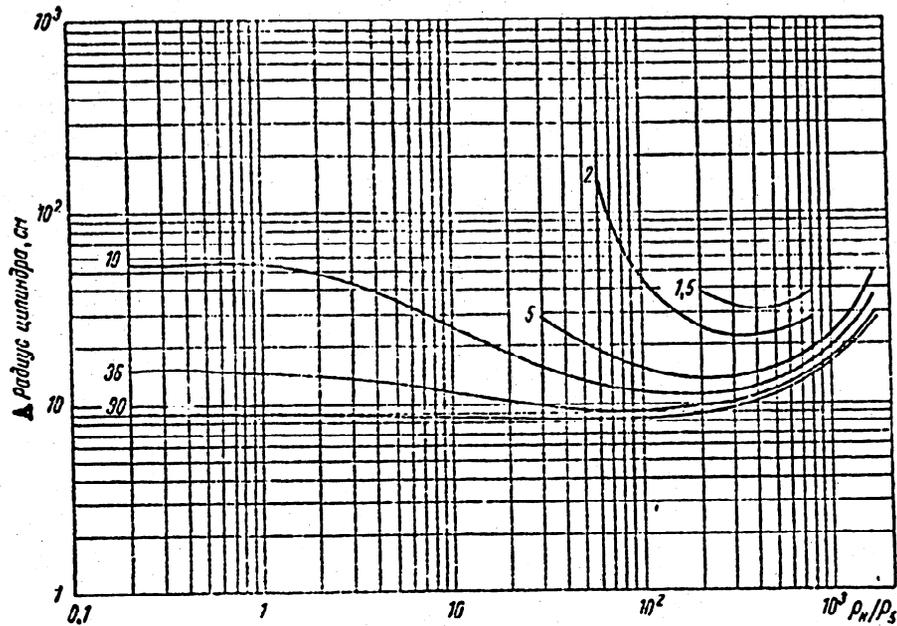
Legend: A = Radius of the cylinder, cm.

Figure 3.47

The relationship between a concentration of U^{235} in a mechanical mixture of $UO_2 + H_2O$ and the ratio of atoms to $\frac{\rho_{H}}{\rho_{S}}$ with various enrichments $(U^{235}, \%)$. The density of the UO_2 is 8.45 g/cm^3.
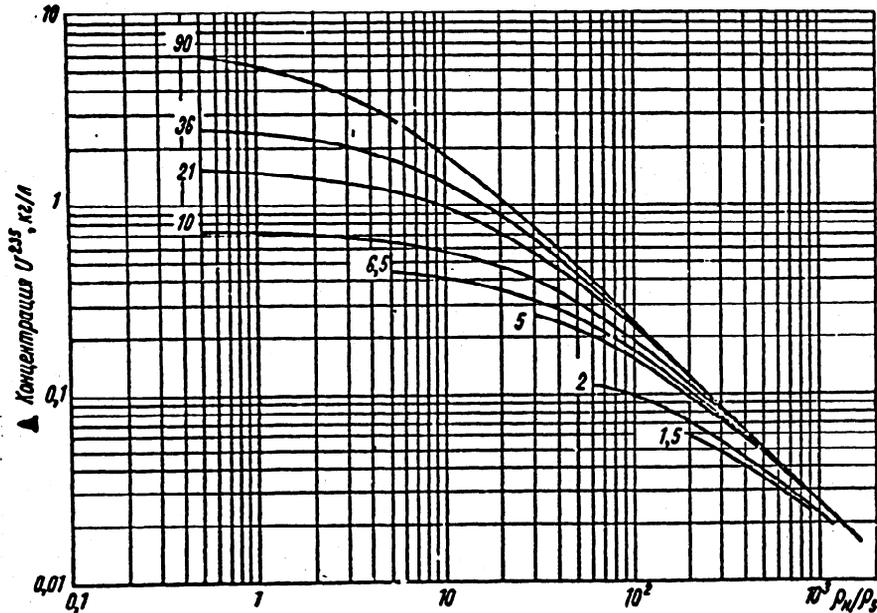
Legend: A = Concentration of U^{235}, kg/l.

c) Critical Parameters of Mechanical Mixtures of Uranium with Water and Solid Moderators

Presented below are the critical masses of mechanical mixtures of uranium, water, and carbon; uranium, water, and beryllium; and uranium, water and beryllium oxide. These masses were computed in a P_1-approximation (Figures 3.48-3.63).*

* The computations were made in the Mathematics Department of the FEI on a machine of the BESM-2 type.

- 75 -

Figure 3.48

The critical mass of U^{235} for a mechanical mixture of U + C + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is two percent.
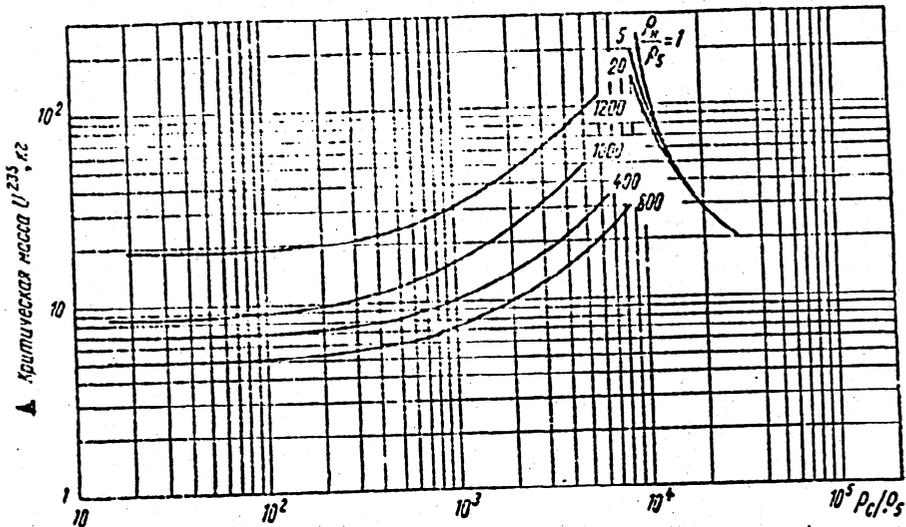
Legend: A = Critical mass of U^{235}, kg.

Figure 3.49

The critical mass of U^{235} for a mechanical mixture of U + C + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is 6.5 percent.
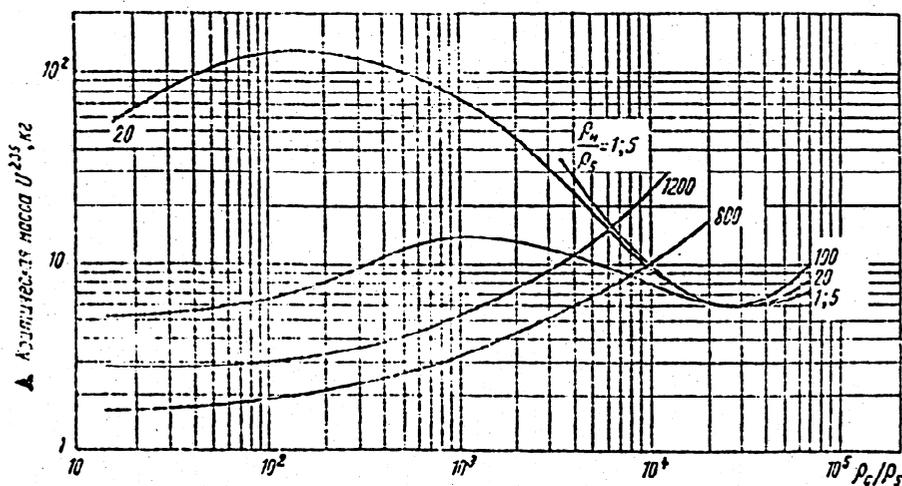
Legend: A = Critical mass of U^{235}, kg.

Figure 3.50

The critical mass of U^{235} for a mechanical mixture of $U + C + H_2O$ as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is ten percent.
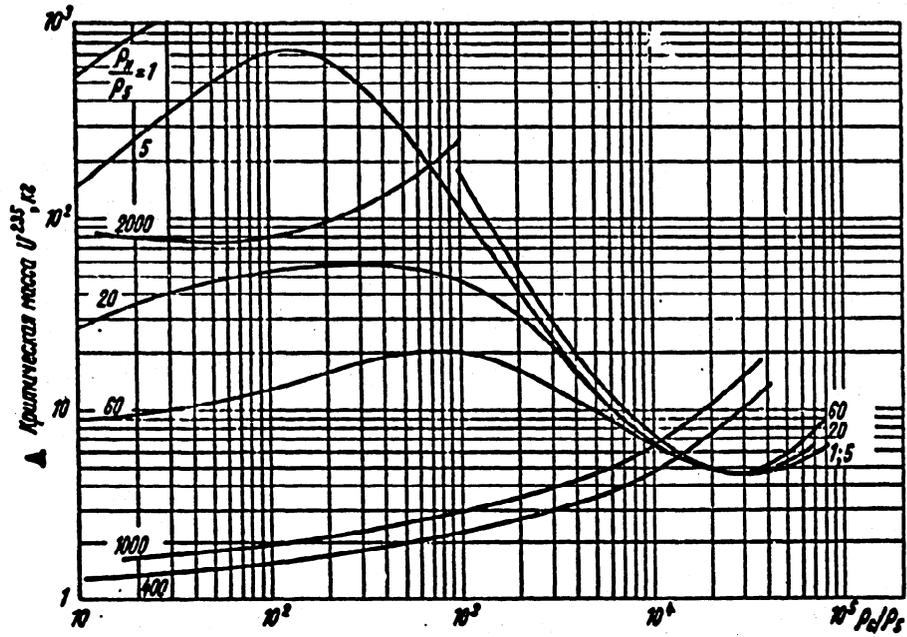
Legend: A = Critical mass of U^{235}, kg.

Figure 3.51

The critical mass of U^{235} for a mechanical mixture of U + C + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is 20 percent.
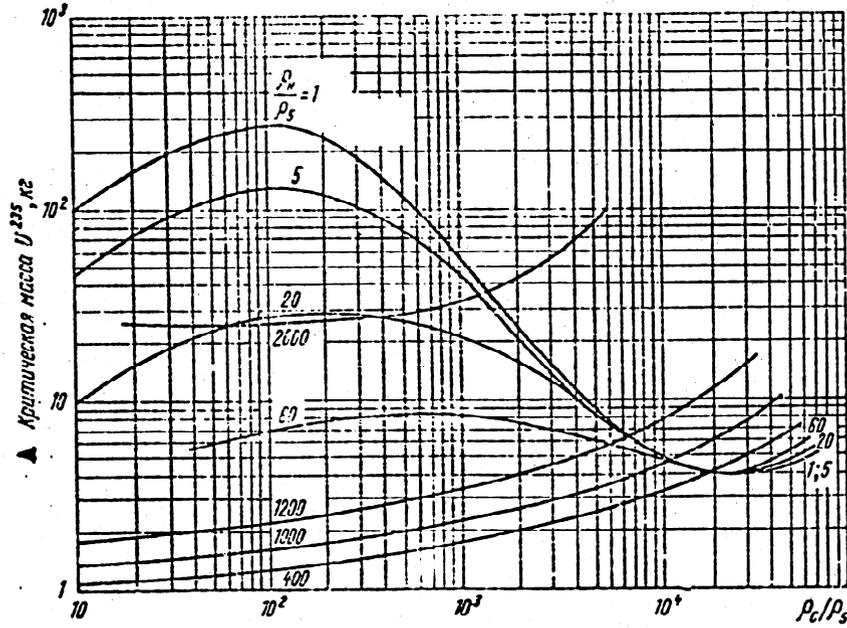
Legend: A = Critical mass of U^{235}, kg.

Figure 3.52

The critical mass of U^{235} for a mechanical mixture of U + C + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is 36 percent.
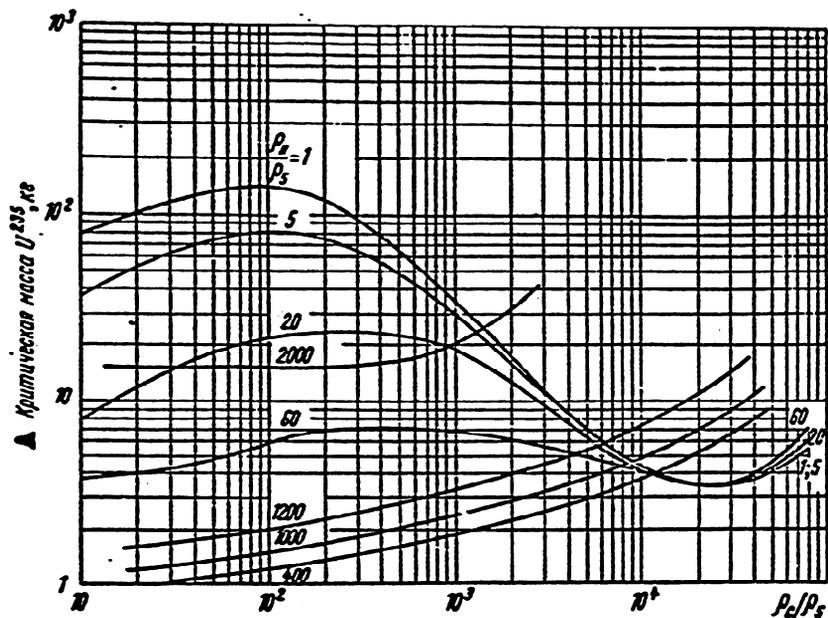
Legend: A = Critical mass of U^{235}, kg.

Figure 3.53

The critical mass of U^{235} for a mechanical mixture of U + C + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is 95 percent.

Legend: A = Critical mass of U^{235}, kg.

Figure 3.54

The critical mass of U^{235} for a mechanical mixture of U + Be + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is two percent.

Legend: A = Critical mass of U^{235}, kg.

Figure 3.55

The critical mass of U^{235} for a mechanical mixture of U + Be + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is 6.5 percent.

Legend: A = Critical mass of U^{235}, kg.

Figure 3.56

The critical mass of U^{235} for a mechanical mixture of U + Be + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is
20 percent.
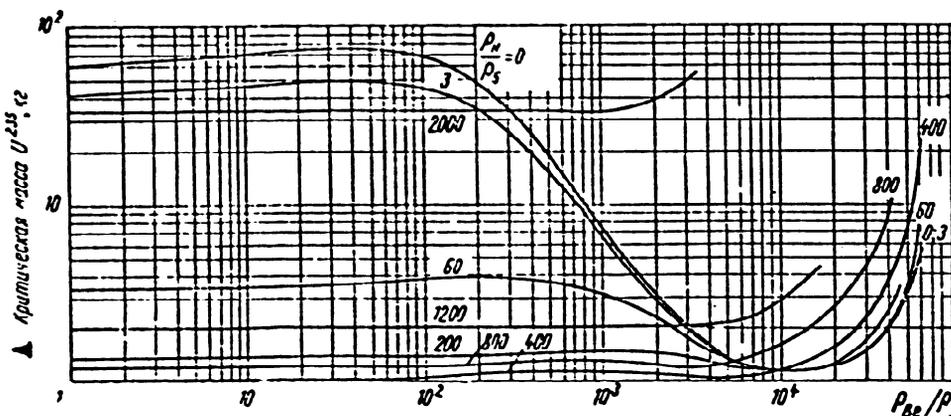
Legend: A = Critical mass of U^{235}, kg.

Figure 3.57

The critical mass of U^{235} for a mechanical mixture of U + Be + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is
36 percent.
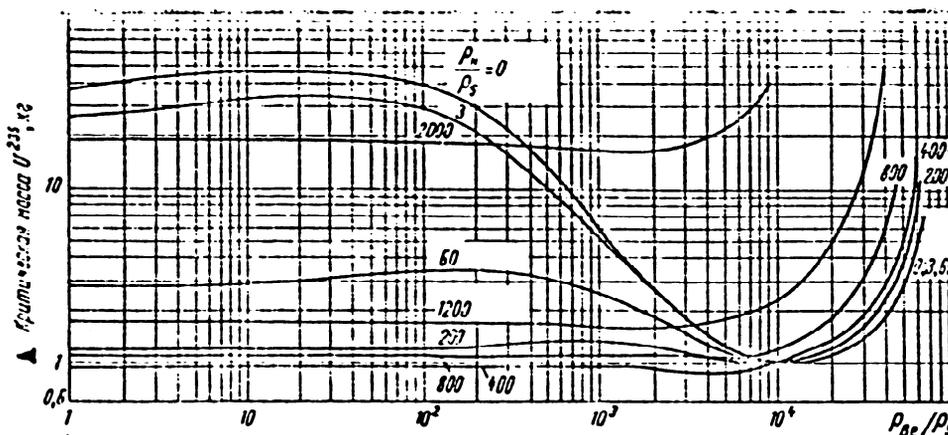
Legend: A = Critical mass of U^{235}, kg.

Figure 3.58

The critical mass of U^{235} for a mechanical mixture of U + Be + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is 90 percent.
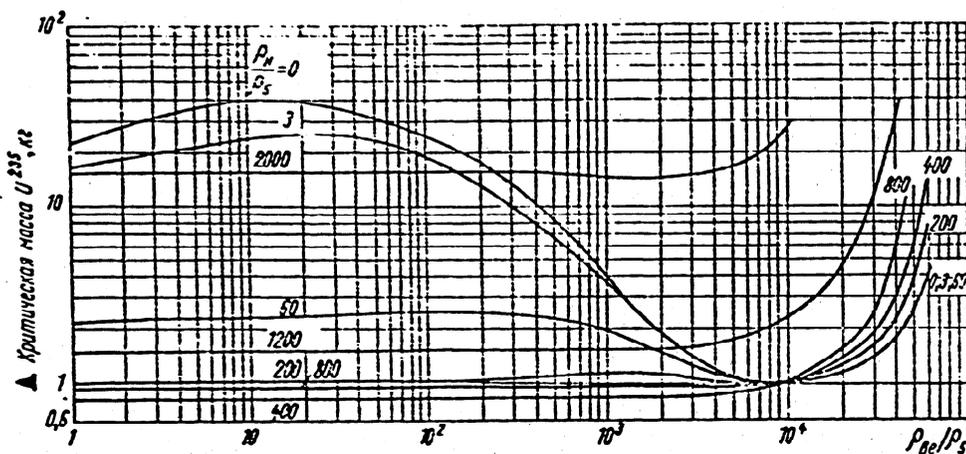
Legend: A = Critical mass of U^{235}, kg.

Figure 3.59

The critical mass of U^{235} for a mechanical mixture of U + Be + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is two percent.
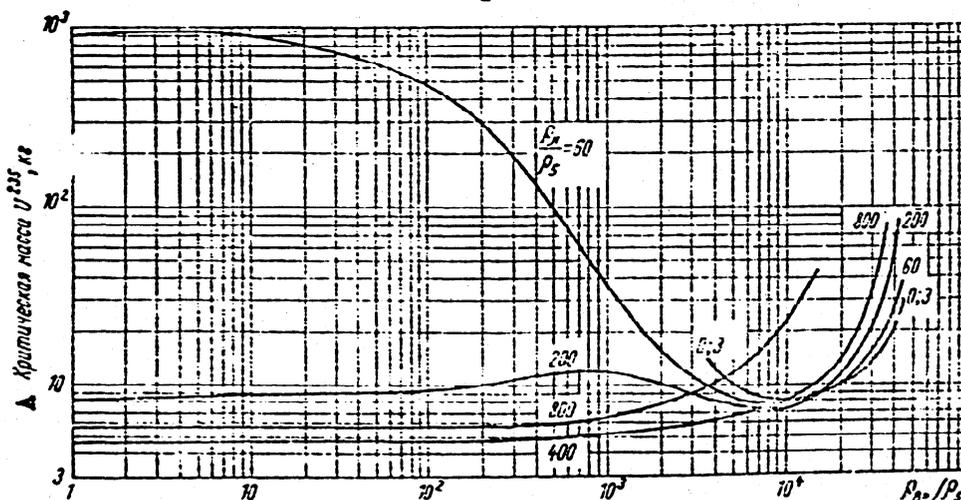

Legend: A = Critical mass of U^{235}, kg.

The critical mass of U^{235} for a mechanical mixture of U + BeO + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is 6.5 percent.
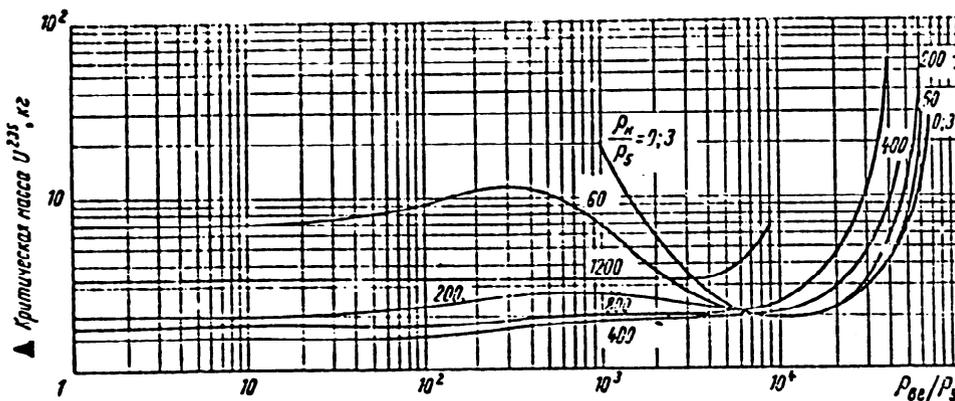
Legend: A = Critical mass of U^{235}, kg.

Figure 3.61

The critical mass of U^{235} for a mechanical mixture of U + BeO + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is 20 percent.
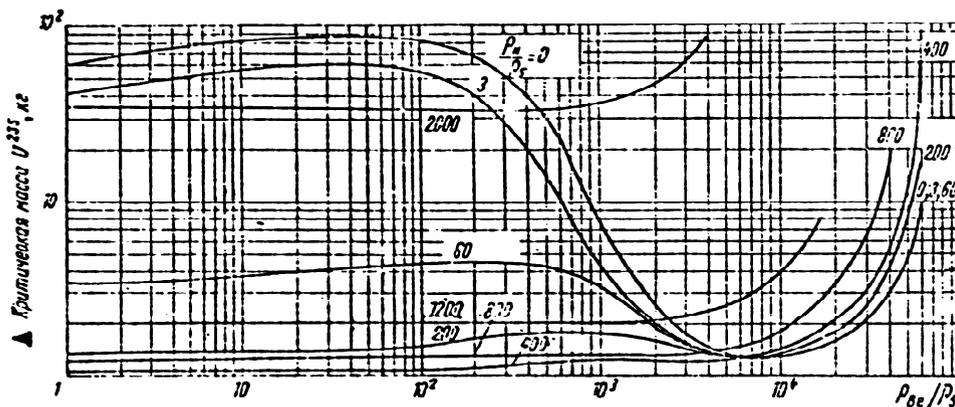
Legend: A = Critical mass of U^{235}, kg.

Figure 3.62

The critical mass of U^{235} for a mechanical mixture of U + BeO + H2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is 36 percent.
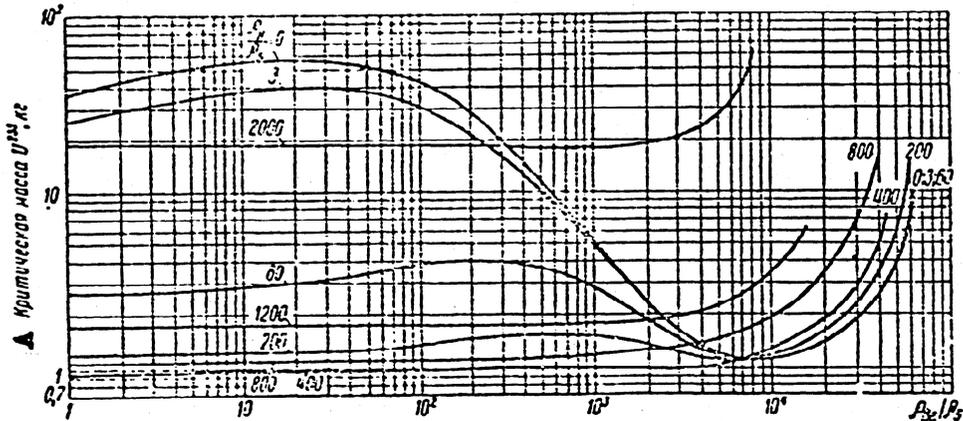
Legend: A = Critical mass of U^{235}, kg.

The following values of the densities of the components of a mixture were applied for the computations:

	γ_1 g/cm^3
Uranium	18.7
Carbon	1.65
Beryllium	1.85
Beryllium oxide	2.85
Water	1.00

The results of the computations are presented in the form of graphic relationships between the critical masses of variously enriched uranium (2, 6.5, 20, 36, and 90 percent) for the mixtures of uranium, water, and beryllium and uranium, water, and beryllium oxide, and 2, 6.5, 10, 20, 36, and 90 percent for mixtures of uranium, water, and carbon on the one hand and the corresponding parameters of moderation: ℓ_H/ℓ_s, ℓ_c/ℓ_s, ℓ_{c_3}/ℓ_s on the other.

Figure 3.63

The critical mass of U^{235} for a mechanical mixture of U + BeO + H_2O as a function of moderation. The shape of the fuel core is an infinite aqueously reflected sphere. The enrichment of uranium is 90 percent.
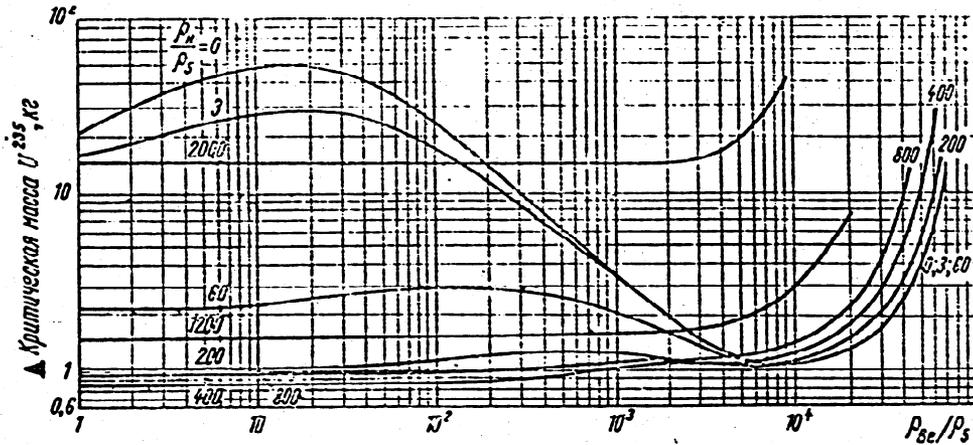
Legend: A = Critical mass of U^{235}, kg.

For each value of a critical mass, it is possible to compute the following critical volume and density of a mechanical mixture by the following equations:

$$V = m \left(\frac{5.35}{P} + 0.0383\beta_{\text{H}} + 0.0309\beta_{\text{c}} \right) ;$$
$$\gamma = \frac{100 + 0.0383\beta_{\text{H}}P + 0.0510\beta_{\text{c}}P}{5.35 + 0.0383\beta_{\text{H}}P + 0.0309\beta_{\text{c}}P}$$

(3.32)

for a mixture of uranium, water, and carbon;

$$V = \left(\frac{5.35}{P} + 0.0383\beta_{\text{H}} + 0.0208\beta_{\text{Be}} \right) m ;$$
$$\gamma = \frac{100 + 0.0383\beta_{\text{H}}P + 0.0385\beta_{\text{Be}}P}{5.35 + 0.0383\beta_{\text{H}}P + 0.0208\beta_{\text{Be}}P}$$

(3.33)

for a mixture of uranium, water, and beryllium;

$$V = m \left(\frac{5.35}{P} + 0.0383\beta_{11} + 0.0373\beta_{BeO} \right) ;$$

$$Y = \frac{100 + 0.0383\beta_{11}P + 0.1033\beta_{11e0}P}{5.35 + 0.038\beta_{11}P + 0.0373\beta_{11e0}P}$$

(3.34)

for a mixture of uranium, water, and beryllium oxide, since V is the critical volume, 1; m is the critical mass of U^{235}, kg; γ is the density of the mechanical mixture, g/cm^3; P is the enrichment of uranium, %; β_H is the ratio ρ_H / ℓ_5; β_{Be} is the ratio ℓ_{Be} / ℓ_5; β_{BeO} is the ratio ℓ_{BeO} / ℓ_5.

For mixtures of uranium, water, and carbon; uranium, water, and beryllium; and uranium, water, and beryllium oxide, there are no experimental data with an aqueous reflector; therefore, it is impossible to verify the results of the computations. The applied method of computation apparently results in a maximum deviation of ± 30 percent from experimental data. Especially great is this deviation for systems having small dimensions and utilizing uranium of low enrichment (2 and 6.5 percent).

On the basis of the presented graphical relationships between the critical masses and ℓ_H/ℓ_5, ℓ_{c}/ℓ_5, ℓ_{c}/ℓ_5, it is possible to obtain the general regularity of the behavoir of curves. When there is very little or no water in a system ($\ell_H/\ell_5 = 0-5$), the critical masses increase commensurately with the addition of a solid moderator (carbon or beryllium); they attain their maximum value, following which they are reduced until they reach the minimum and then again increase. The growth of critical masses with small values of the relationship of ℓ_c/ℓ_5 or ℓ_c/ℓ_5 is explained by the fact that the small quantities of solid moderator, while essentially not changing the neutron spectrum, substantially reduces the density of uranium.

A subsequent addition of solid moderator reduces the critical mass, for the neutron spectrum is substantially softened. After the neutron spectrum has become thermal ($\ell_c/\ell_5 = 6,000 - 8,000$; $\ell_c/\ell_5 = 30,000$ 40,000), an addition of a solid moderator increases the critical mass, since the neutron spectrum is essentially unchanged. However, the density of the uranium is reduced and neutron capture in the moderator is increased.

With the presence of an insignificant amount of water in the system ($\ell_H/\ell_5 = 60$ 300), the critical mass is reduced commensurately with the addition of a solid moderator, and after having attained the minimum, it increases. This occurs because the addition of the solid moderator nevertheless results in this case in softening the neutron spectrum.

After the system has become thermal, the subsequent addition of solid moderator increases the critical mass because of the reduction of the density of uranium and the increase of moderator absorption. With the presence of the optimal amount of water in the system ($\rho_H/\rho_s =$ 400 plus), the addition of a solid moderator leads only to a reduction in the density of the uranium and the additional capture of neutrons, and the critical mass increases.

4. The Computation of Size-Shape Factors of Complicated Geometric Configurations

In solving problems of the nuclear safety of the containers of complicated geometric configurations, the calculation of size-shape factors is of substantial significance. The size-shape factor $\Big/$ geometric parameter $\Big/$ of a container in a critical state is equal to the material parameter of the solution filling the container. When the geometric parameter of a container is less than the material parameter of a solution, the filling of the container with this solution may result in a chain reaction. Conversely, when the geometric parameter is greater than the material parameter, filling is safe. Since the geometric parameter is equal to the material one in a critical state, the most convenient and accurate method of calculating the material parameter is through the employment of experimental data on critical masses. When the radius of the critical sphere is designated by the letter R, the material parameter $\varkappa^2$ (equal to the geometric parameter) is calculated with the equation

$$\varkappa^2 = \frac{\pi^2}{(R + 0.71\lambda_t)^2},$$

(3.35)

since $0.71\,\lambda_t$ is the length of extrapolation determined via the transport length λ_t.

When it is usual to assume the same value $0.71\,\lambda_t$ in computing the geometric and material parameters of various figures via the equation (3.35), even a 30-percent error in the extrapolation length does not yield an error of over 2 percent in calculating the dimension of a container.

A geometric parameter is computed exactly only for a small number of accurate figures. The best known geometric parameters are the sphere, the cylinder, and the parallelepiped. Also known are the geometric parameters of some other less employable figures, for which the problem of the calculation of geometric parameters can be solved accurately in the spherical, cylindrical, and Cartesian coordinates $\Big/38\Big/$. Presented below are the figures, whose exact values of the geometric shape-size factor $\varkappa^2$, as well as the distribution of neutron flux ϕ, are known.

The sphere

$$\varkappa^2 = \frac{\pi^2}{R^2}; \quad \Phi = \frac{\sin \varkappa r}{r}$$

(r is the point coordinate).

The cylinder

$$\varkappa^2 = \varkappa_1^2 + \varkappa_2^2;$$

$$\varkappa_1 = \frac{2,405}{R}; \quad \varkappa_2 = \frac{\pi}{H};$$

$$\Phi = J_0(\varkappa_1 r) \sin \varkappa_2 h$$

(r and h are the point coordinates).

The infinite flat layer of thickness H

$$\varkappa^2 = \frac{\pi^2}{H^2}, \quad \Phi = \sin \varkappa h$$

(h — координата точки).

(h is the point coordinate).

The parallelepiped

$$\varkappa^2 = \varkappa_1^2 + \varkappa_2^2 + \varkappa_3^2;$$

$$\varkappa_1 = \frac{\pi}{a}; \quad \varkappa_2 = \frac{\pi}{b}; \quad \varkappa_3 = \frac{\pi}{c};$$

$$\Phi = \sin \varkappa_1 a' \sin \varkappa_2 b' \sin \varkappa_3 c'$$
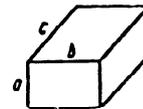
(a', b', c' are the point coordinates).

The infinite cylinder with the central cylindrical cavity
The magnitude $\varkappa^2$ is determined by the solution of the equation

$$J_1(\varkappa r) - \frac{J_0(\varkappa R)}{Y_0(\varkappa R)} \cdot Y_1(\varkappa r) = 0;$$

$$\Phi = J_0(\varkappa \varrho) - \frac{J_0(\varkappa R)}{Y_0(\varkappa R)} \cdot Y_0(\varkappa \varrho)$$
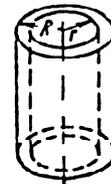
(is the point coordinate).

The sphere with a central spherical cavity
The magnitude $\varkappa^{k}$ is determined by the solution of the equation

$$\frac{\varkappa \cos \varkappa (R-r)}{r} + \frac{\sin \varkappa (R-r)}{r^2} = 0;$$

$$\Phi = \sin \varkappa (R-\varrho)/\varrho$$

(Q is the point coordinate).
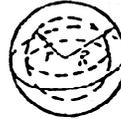
Individual parts of a cylinder
The magnitude n takes the values 0, 1, 2, 3, ...

$$\varkappa^2 = \varkappa_1^2 + \varkappa_2^2; \quad \varkappa_1 = \frac{\pi}{H} ,$$
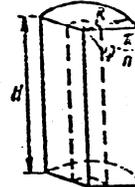
$\varkappa^{k}$ is found from solving the equation
with n = 0, 1, 2, 3; $\varkappa R$ is equal to 2.405, 3.83, 5.13, 617;

$$\Phi = \sin \varkappa_1 h \cdot J_n (\varkappa \varrho) \cdot \cos n\varphi$$

$(h, \varrho, \varphi$ are the point coordinates$)$

Sectors of a sphere
The magnitude θ assumes the values determined by the equation

$$P_n (\cos \theta) = 0 \quad \text{при} \quad n = 1, 2, 3, 4, ... ,$$

whereupon the maximum value will be taken in accordance with the magnitude
of the root. The first four values of $\cos \theta$ are

$$0; \quad 1/\sqrt{3}; \quad \sqrt{3/5}; \quad \sqrt{\frac{3}{7} + \sqrt{\frac{24}{245}}} .$$

is found through the equation $J_{n+\frac{1}{2}} (\varkappa R) = 0$; for the first four
values, $\varkappa R$ equals 4.493, 5.762, 6.990, 8.184. $\Phi = \frac{1}{\sqrt{\varrho}} J_{n+\frac{1}{2}} (\varkappa \varrho) P_n$

$(\cos \theta)$ $(\varrho, \theta$ are the point coordinates$)$.

It is useful to employ the exact values of geometric parameters
in evaluating the criticality of figures somewhat differing in shape from
those whose exact geometric-parameter value is known. For example, the
geometric parameter of a cone can be sucessfully calculated by using the

precise value of the geometric parameter of a spherical
sector. Insofar as this method of comparison is definitely
arbitrary, good results are obtained only for figures having
geometric configurations approximating those of the original.

There is a general procedure for computing a geometric
parameter of an arbitrary body -- -- a procedure based on
variation methods $\underline{/}$ 38, 39$\underline{/}$. However, to obtain a fair
degree of accuracy, many computations are required.
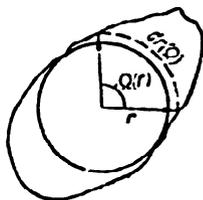
Figure 3.64.
Toward a con-
clusion of an
equation for
a geometric
parameter.

Presented below is a relatively simple method of
calculating the geometric parameters of complicated
figures $\underline{/}$ 40$\underline{/}$.

We will consider a convex body of arbitrary shape
(Figure 3.64). In this body we will inscribe the sphere
of maximum radius. It is obvious that the geometric
parameter x^2 of a body of arbitrary shape can be presented in the form of
the sum of

$$x = x_0 + \Delta x, \tag{3.36}$$

where x_0 is the geometric parameter of the sphere; Δx is the contribution
of the remaining part of the body to the geometric parameter.

We will consider that the elementary contribution to the geometric
parameter dx from the layer of the thickness dr (Ω) is expressed in the
form of

$$dx = x'(r) dr \frac{\Omega(r)}{4\pi}. \tag{3.37}$$

This is tantamount to saying that the volumetric elements within the sphere
make an identical contribution to the geometric parameter regardless of its
angular coordinates θ, φ. Here r is the radius drawn from the center
of the sphere, and

$$x(r) = \frac{\pi}{r}, \quad x'(r) = -\frac{\pi}{r^2}. \tag{3.38}$$

The equation (3.37) somewhat approximates actuality and is fairly accurate.
The accuracy of this approximation will be indicated by a concrete computation

and a comparison with the precise result.

Using the equation (3.37), the equation (3.36) can be presented in the form of

$$x = x_0 + \int_{r_0}^{r_{max}} x'(r)\,dr\,\frac{\Omega(r)}{4\pi},$$ (3.39)

where r_{max} is the maximum value of r.

We will integrate the equation (3.39) by parts:

$$x = x_0 + x(r_{max})\frac{\Omega(r\quad)}{4\pi} - x_0 - \int_{r_0}^{r_{max}} x(r)\,\frac{d\Omega(r)}{4\pi}.$$ (3.40)

(The expression $x(r)\,\Omega(r)/4\pi = 0$ with $r = r_{max}$, since with r_{max} the solid angle becomes zero; $x(r)\cdot\Omega(r)/4\pi = x_0$ with $r = r_0$, since with $r = r_0$ the solid angle becomes 4π, while $x(r_0) = \pi/r_0$). Replacing the expression $x(r)$ with its value π/r, we get $x = \int_{r_0}^{r_{max}} \frac{\pi}{r}\,\frac{d\ \Omega(r)}{4} = \frac{1}{4}\int_{r_0}^{r_{max}}\frac{d\ \Omega^*(r)}{r}.$

Here $d\Omega^*(r)$ is understood to mean the quantity obtained by the subtraction of the solid angles $\Omega(r) - \Omega(r + \Delta r)$: that is, the solid angle $d\Omega^*$ is formed by two conical surfaces.

The integration of r can be substituted simply by the integration of the solid angle. The value does not change in this situation, since the integrand will run through the same values provided that $d\Omega'$ is understood by its usual definition of $d\Omega' = \sin\theta\,d\theta\,d\varphi$. In this situation, the single integral becomes a double one; the second integration appears in solving the problem to find $d\Omega(r)$. Thus, the final equation for can be written in the form

$$x = \frac{1}{4}\int_{\Omega}\frac{d\Omega'}{r(\Omega')}.$$ (3.41)

Similar equations for x^2, x^4, etc. can be evolved in an analogous manner; therefore, another criterion is necessary in selecting the indicator of degree.

From the formulation of the equation for $\varkappa$ one ought to expect that the result, closer to reality, should be obtained for figures whose shapes approximate the spherical. This phenomenon, as a computation indicates, is actually observed. Therefore, it is natural to demand that, in choosing the indicator of the n degree, the equation yield an accurate result for a figure whose shape substantially differs from the spherical. An infinite flat layer was chosen as such a figure. In this case, the equation with the indicator of the degree 1 yields a result accurately coinciding with the actual result.

Thus, for computations it is best to use the equation (3.41), where $d\Omega'$ is the element of the solid angle emerging from the point of maximum flux (we will subsequently call this point the integration center); $r(\Omega')$ is the ray emitted from this point lying in the solid angle $d\Omega'$ and extending to the extrapolated border of the body. Subsequently, we will mean everywhere the dimensions of bodies, including the extrapolation length. We will consider an example of computing the geometric parameter of an infinite flat layer with thickness H. The center of integration must be placed in the middle of the layer. We will direct the axis of the spherical coordinates perpendicularly to the plane. It is obvious that

$$ r(\Omega) = \frac{H}{2\cos\theta}; \quad d\Omega = \sin\theta\, d\theta\, d\varphi. $$

Hence

$$ \varkappa = 2 \cdot \frac{1}{4} \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\dfrac{H}{2\cos\theta}} = \frac{\pi}{H}, \tag{3.42} $$

which coincides with the exact value of $\varkappa$.

The numerous computations made with the equation (3.41) for the most diverse figures with the known, exact values of a geometric parameter, as well as a comparison of the computations and the experimental data of many complicated figures, have indicated that the deviation stemming from computations of the geometric parameter $\varkappa$ via the equation (3.41) is no worse that four percent. It should be noted that the error is, in a majority of cases, propitious from the point of view of nuclear safety. The equation (3.41) is nearly insensitive to the selection of the center of integration. It is easy to be convinced of this, for example, by making computations for a flat layer with a displaced integration center. For a flat layer, the error stemming from the selection of an inaccurate integration center is the greatest, but even in this case a ten-percent deviation from the exact point yields an error of one percent in the final result. In those cases in which the finding of maximum flux is difficult, it is possible, taking advantage of this conservative characteristic, to choose it at random within the regions of the maximum flux point. The

given characteristic is also usefully employed to simplify calculations during computations.

We will present a method of finding the integral (3.41) in a situation in which the equations of the surfaces bordering the body in spherical coordinates are known. Let, for example, the body be limited to two surfaces $r_1 = f_1(\theta, \varphi)$ and $r_2 = f_2(\theta, \varphi)$ (Figure 3.65). Via the equation $f_1(\theta, \varphi) = f_2(\theta, \varphi)$, it is possible to find the range of integration by θ in its relationship to φ. Let this be the function $\theta = \xi(\varphi)$. It is now possible to write the value of the integral (3.41) in an equation easy to solve:

$$\varkappa = \frac{1}{4}\left[\int_0^{2\pi} d\varphi \int_0^{\theta=\xi(\varphi)} \frac{\sin\theta\,d\theta}{f_1(\theta,\varphi)} + \int_0^{2\pi} d\varphi \int_{\theta=\xi(\varphi)}^{\pi} \frac{\sin\theta\,d\theta}{f_2(\theta,\varphi)}\right] \qquad (3.43)$$

It is possible to proceed in analogous fashion in other situations.

Figure 3.65

For the computation of the geometric parameter of a common situation

All the figures encountered in practice are usually formed by the intersection of the flat, cylindrical, spherical, and conical surfaces. The computation of the integral (2.41) separates into a calculation of integrals in the form of (3.43) according to the individual elements of these surfaces.

Presented below is the computation of the integral of a flat triangle, whose two sides are rectilinear and one side is curvilinear (Figure 3.66):

$$\theta = f(\varphi);$$

$$J = \frac{1}{4}\int_0^{\varphi_0} d\varphi \int_0^{\theta=f(\varphi)} \frac{\cos\theta\cdot\sin\theta\,d\theta}{a} = \frac{1}{4a}\int_0^{\varphi_0} \left\{\frac{1}{2} - \frac{\cos^2[f(\varphi)]}{2}\right\} d\varphi = \frac{1}{4}\int_0^{\varphi_0}\left[\frac{1}{2} - \frac{1}{2}\cdot\frac{a^2}{f(\varphi)^2 + a^2}\right] d\varphi, \qquad (3.44)$$

where $\rho(\varphi)$ is the equation of the curve AB in the polar coordinates.

When all the sides of a triangle are rectilinear, it is possible to effect integration according to φ and obtain an expression for the integral in a distinct form (Figure 3.67):

$$\theta = f(\varphi) = \operatorname{arctg} \frac{b \sin \alpha}{a \sin(\alpha + \varphi)};$$

$$\varrho(\varphi) = \frac{b \sin \alpha}{\sin(\alpha + \varphi)};$$

$$I = \frac{A}{8a\sqrt{A^2+1}}\left[\operatorname{arctg} \frac{\sqrt{A^2+1}}{A}\operatorname{tg}(\alpha+\varphi_0) - \operatorname{arctg}\frac{\sqrt{A^2+1}}{A}\operatorname{tg}\varphi_0\right], \quad (3.45)$$

where $A = \frac{b}{a}\sin\alpha$.

When the curve AB (see Figure 3.66) is polynomially expressed, it is possible to effect integration according to φ.
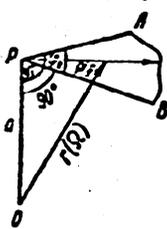
Figure 3.66. For the computation of an integral of a flat triangle.
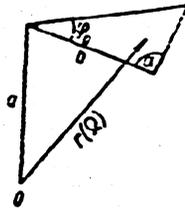
Figure 3.67 For the computation of a flat triangle whose sides are rectilinear.
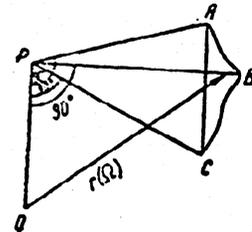
Figure 3.68. For the computation of an integral of an arbitrary flat figure.

For example, with $\varrho^2(\varphi) = b_0 + b_1\varphi + b_2\varphi^2$

$$I = \frac{1}{4a}\left[\frac{\varphi_0}{2} - \frac{a^2}{\sqrt{\Delta}}\left(\operatorname{arctg}\frac{2b_2\varphi_0 + b_1}{\sqrt{\Delta}} - \operatorname{arctg}\frac{b_1}{\sqrt{\Delta}}\right)\right], \quad (3.46)$$

где $\Delta = 4b_2(a^2 + b_0) - b_1^2$.
При $\varrho^2(\varphi) = b_0 + b_1\varphi$
with

$$I = \frac{1}{4a}\left[\frac{\varphi_0}{2} - \frac{a^2}{2b_1}\ln\frac{b_1\varphi + b_0 + a^2}{b_0 + a^2}\right]. \quad (3.47)$$

При $\rho^2(\varphi) = b_0$
with

$$I = \frac{1}{4\pi} \Psi_0 \left[\frac{1}{2} - \frac{a^2}{2(b_0 + a^2)} \right].$$ (3.48)

При $\rho^2(\varphi) = \rho^2 = const$ и $\varphi_0 = 2\pi$
with

$$I = \frac{\pi}{4\pi} - \frac{\rho^2}{\rho^2 + a^2}.$$ (3.49)

The integral of any figure lying on a plane can be expressed with the equation (3.44) in the following manner (Figure 3.68):

$$\int_{ABC} \frac{d\Omega}{r(\Omega)} = \int_{PAB} \frac{d\Omega}{r(\Omega)} + \int_{PBC} \frac{d\Omega}{r(\Omega)} - \int_{PAC} \frac{d\Omega}{r(\Omega)} .$$

Analogously, with the aid of similar additions and subtractions via integrals of triangles, it is possible to compute the integrals of the cylindrical, spherical, and conical surfaces of any figure.

The integrals for triangles lying on such surfaces are presented below.

The cylindrical surface (Figure 3.69)

$$\left. \begin{aligned} r(\Omega) &= \frac{\rho(\varphi)}{\sin\theta}, \\ I &= \frac{1}{4} \int_{0}^{\varphi_0} d\varphi \int_{\pi - \sigma}^{\pi/2} \frac{\sin^2\theta \, d\theta}{\rho(\varphi)} - \frac{1}{3} \int_{0}^{\varphi_0} \frac{-f(\varphi) \left[\pi/2 + \frac{1}{2}\sin 2f(\varphi) \right]}{\rho(\varphi)} d\varphi. \end{aligned} \right\}$$ (3.50)

For a round cylinder (Figure 3.70), we obtain the following expression for $\rho(\varphi)$:

$$\rho(\varphi) = \pm \sqrt{r^2 - a^2 + a^2\cos^2\varphi} + a\cos\varphi.$$ (3.51)

The sign before the root is assumed according to the significance relating to the position of point O.
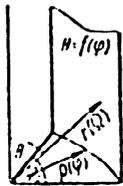
Figure 3.69. For the computation of an integral of a triangle lying on a cylindrical surface.
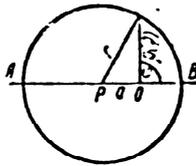
Figure 3.70. For the computation of an integral of a triangle lying on the surface of a round cylinder.
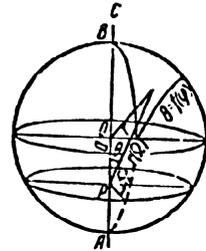
Figure 3.71. For the computation of the integral of a triangle lying on a spherical surface.

When in the case of a round cylinder $\rho(\varphi) = r$ and $f(\varphi) = \Theta$ then

$$I = \frac{\varphi_0}{8r}\left(\frac{\pi}{2} - \Theta + \frac{1}{2}\sin 2\Theta\right).$$

(3.52)

The spherical surface (Figure 3.71)

$$PO = a; \quad r^2 = a^2 + r^2(\Omega) - 2ar(\Omega)\cos\Theta;$$
$$r(\Omega) = \pm\sqrt{r^2 - a^2 + a^2\cos^2\Theta} + a\cos\Theta. \quad \Big\}$$

(3.53)

The sign before the root is assumed according to the significance relating to the position of point P

$$I = \int_0^{\varphi_0} d\varphi \int_0^{\Theta = f(\varphi)} \frac{d\cos\Theta}{\sqrt{r^2 - a^2\cos^2\Theta} + a\cos\Theta} = \frac{1}{2}\cdot\frac{b}{a}\cdot\int_0^{\varphi_0}\left\{ t(\varphi)\sqrt{t^2(\varphi) + b} + \right.$$

$$\left. + b\ln\left[t(\varphi) + \sqrt{t^2(\varphi) + b}\right] - t^2(\varphi) - \frac{2}{a} - b\ln\left(1 + \frac{2}{a}\right) + 1\right\} d\varphi, \quad (3.54)$$

где $b = \frac{r^2 - a^2}{a^2}$, $t(\varphi) = \cos f(\varphi)$.
where
with При $\Theta = \mathrm{const}$

$$I = \frac{1}{2}\cdot\frac{b\varphi_0}{a}\cdot\left[\cos\Theta\sqrt{\cos^2\Theta + b} + b\ln\frac{\cos\Theta + \sqrt{\cos^2\Theta + b}}{1 + r/a} - \frac{r}{a} + \sin^2\Theta\right]. \quad (3.55)$$

For a hemisphere,

$$\cos\Theta = \frac{a}{\sqrt{a^2 + r^2}}; \quad \varphi_0 = 2\pi;$$

$$I = \frac{\pi a}{r^2 - a^2}\left[2 - \frac{r^2}{a^2 + r^2} + \frac{r^2 - a^2}{u^2}\ln\frac{\sqrt{a^2 + r^2}}{a + r} - \frac{r}{a}\right]. \quad (3.56)$$

The conical surface (Figure 3.72)

$$\Theta = f(\varphi); \quad OA = a; \quad AB = r; \quad AC = h; \quad PO = r(\Omega)\cos\Theta;$$

$$DM = [h + r(\Omega)\cos\Theta]\,\mathrm{tg}\,\alpha; \quad P\varepsilon = \sqrt{DM^2 - a^2 + a^2\cos^2\varphi} + a\cos\varphi = r(\Omega)\sin\Theta,$$

hence $\quad \frac{1}{r(\Omega)} = \frac{b \pm \sqrt{b^2 - dc}}{c}\quad$ **(the sign before the root is assumed according**

to the significance relating to the position of point O), since

$$b = a\sin\Theta\cos\varphi + r\,\mathrm{tg}\,\alpha\cos\Theta; \quad c = a^2 - r^2; \quad d = \sin^2\Theta - \cos^2\Theta\,\mathrm{tg}^2\alpha;$$

$$I = \frac{1}{4}\int_0^{\varphi_0} d\varphi \int_{\Theta = f(\varphi)}^{\pi/2} \frac{b(\Theta,\varphi) - \sqrt{b^2(\Theta,\varphi) - d(\Theta)c}}{c}\sin\Theta\, d\Theta = I_1 - I_2;$$

$$I_1 = \frac{1}{4}\int_0^{\varphi_0} d\varphi \int_{\Theta = f(\varphi)}^{\pi/2} \frac{b(\Theta,\varphi)}{c}\sin\Theta\, d\Theta = \frac{1}{4c}\int_0^{\varphi_0}\left[\frac{a\cos\varphi}{2}\left(\frac{\pi}{2} - \right.\right.$$

$$\left.\left. - f(\varphi) + \frac{1}{2}\sin 2f(\varphi)\right) + \frac{2\,\mathrm{tg}\,\alpha\cos^2 f(\varphi)}{2}\right]d\varphi;$$

$$I_2 = \frac{1}{4c}\int_0^{\varphi_0} d\varphi \int_{\Theta = f(\varphi)}^{\pi/2}\sqrt{b^2(\Theta,\varphi) - d(\Theta)c}\sin\Theta\, d\Theta.$$

I_2 can lead to the computation of elliptical integrals, but the expression of computation is obtained in a very cumbersome manner; it is therefore more convenient, in concrete cases, to separate the radicand into a series and compute the integral numerically.

When **a = 0, then**

$$I_1 = \frac{\lg \alpha}{8r} \int_0^{\varphi_0} -[\cos^2 f(\varphi)]\, d\varphi;$$

$$I_2 = \frac{1}{8r} \int_0^{\varphi_0} \left[\frac{\pi}{2} - f(\varphi) + \frac{1}{2}\cdot 2f(\varphi)\right] d\varphi. \quad (3.57)$$

With
Прн $\Theta = \mathrm{const}$

$$I_1 = \frac{\lg \alpha \cos^2\Theta\,\varphi_0}{8r}; \qquad I_2 = \frac{\left(\frac{\pi}{2} - \Theta + \frac{1}{2}\sin 2\Theta\right)\varphi_0}{8r}. \quad (3.58)$$

Figure 3.72. Toward the computation of the integral for a triangle lying on a conical surface.

The integral of the cone (the range of integration according to Θ from Θ to π and according to $\widetilde{\pi}$ from 0 to 2π is equal to

$$I = \frac{\pi}{4r}\left(\pi - \Theta + \frac{1}{2}\sin 2\Theta + \lg \alpha \sin^2\Theta\right). \quad (3.59)$$

The above mentioned integrals make possible the computation of geometrically complicated figures. It is also useful to combine the numerical computation of integrals with the tabular. This substantially reduces the number of computational operations. For containers with reflectors, computations are made in an analogous manner, provided the effective addition of the reflector is taken into account in the dimension. The equation (3.41) is used to compute multiply connected figures [40] (the computation of interaction, etc.). Results concur favorably with those of an experiment. A careful analysis indicates that in this case the equation yields results with a small reserve in favor of nuclear safety. Presented below is an example of a computation made with the use of tabular integrals of a one-connected figure (Figure 3.73) and a two-connected figure (interaction of two spheres, Figure 3.74). Also presented are examples of calculating the size-shape factor.

The one-connected figure

$$x = I_1 + I_2 + I_3 + I_4$$

I_1 is calculated via the equation (3.49):

$$I_1 = \frac{\pi}{4h} \cdot \frac{R^2}{R^2 + h^2} .$$

I_2 and I_3 are calculated according to the equation (3.52):

$$I_2 = \frac{\pi}{4R} \left(\frac{\pi}{2} - \Theta_1 + \frac{1}{2} \sin 2\Theta_1 \right) \quad \text{при} \quad \Theta_1 = \operatorname{arctg} \frac{R}{h} ;$$

$$I_3 = \frac{\pi}{4R} \left(\frac{\pi}{2} - \Theta_2 + \frac{1}{2} \sin 2\Theta_2 \right) \quad \text{при} \quad \Theta_2 = \operatorname{arctg} \frac{R}{H-h} .$$

I_4 is calculated according to the equation (3.59):

$$I_4 = \frac{\pi}{4R} \left(\pi - \Theta_3 + \frac{1}{2} \sin 2\Theta_3 + \operatorname{tg} \sin^2 \Theta_3 \right) \quad \text{при} \quad \Theta_3 = \frac{\pi}{2} + \Theta_2.$$

The two-connected figure

$$r(\Omega) = r + PQ; \qquad AO_1 = d \sin \Theta;$$

$$PA = \sqrt{r^2 - d^2 \sin \Theta}; \qquad PQ = 2\sqrt{r^2 - d^2 \sin^2 \Theta};$$

$$\varkappa = \frac{1}{4} \int_0^{2\pi} d\varphi \int_{\Theta = \arcsin \frac{r}{d}}^{\pi} \frac{\sin \Theta \, d\Theta}{r} + \frac{1}{4} \int_0^{2\pi} d\varphi \int_0^{\Theta = \arcsin \frac{r}{d}} \frac{\sin \Theta \, d\Theta}{r + 2\sqrt{r^2 - d^2 \sin^2 \Theta}} .$$

When there is a replacement of $\sin \Theta$ by Θ and the arcsin $\dfrac{r}{d}$ by $\dfrac{r}{d}$, the newly obtained integral differs by no more than 0.1 percent from the former. Hence, we obtain

$$\varkappa = \frac{\pi}{r} \sqrt{1 - \frac{r^2}{d^2}} +$$

$$+ \frac{\pi r}{2d^2} \left(1 - \frac{1}{2} \ln 3 \right) .$$

Figure 3.73. For a computation of the geometric parameter of a one-connected figure.
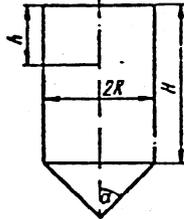
Figure 3.74. For the computation of the geometric parameter of interacting spheres.
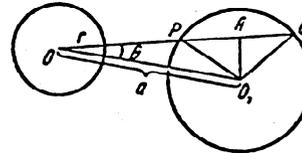
We will also consider the calculation of the size-shape factor of several important practical cases, e.g., T- and cross-shaped pipelines. In the situation under consideration, the pipes of the radius r and R ($R \geqslant r$) are intersected at a right angle. Evaluations indicate that with a T-shaped intersection the error in computing $\varkappa$ will be less than 1.5 percent in the worst of cases ($R = r$). Using the equations of the surfaces of cylinders in spherical coordinates, the integration range can be found according to

$$
\left.
\begin{aligned}
\Theta_1 &= f_1(\varphi) = \arcsin \sqrt{\frac{r^2}{R^2 + r^2 \cos^2 \varphi}} \\[2mm]
\Theta_2 &= f_2(\varphi) = \arcsin \sqrt{\frac{R^2}{r^2 + R^2 \cos^2 \varphi}}\,.
\end{aligned}
\right\}
\tag{3.60}
$$

Using the equations (3.50) and (3.52) for a T-shaped intersection, we obtain the value

$$
\varkappa = \frac{1}{2r} I_1 + \frac{1}{2R} I_2 + I_3,
\tag{3.61}
$$

and for a cross-shaped intersection

$$
\varkappa = \frac{1}{r} I_1 + \frac{1}{R} I_2 + I_4,
\tag{3.62}
$$

since

$$
I_1 = \int_0^{\frac{\pi}{2}} \left[f_1(\varphi) - \frac{1}{2} \sin 2 f_1(\varphi) \right] d\varphi;
$$

$$
I_2 = \int_0^{\arccos \sqrt{1 - \frac{r^2}{R^2}}} \left[f_2(\varphi) - \frac{1}{2} \sin 2 f_2(\varphi) \right] d\varphi;
$$

$$
I_3 = \frac{\left(\pi - \arccos \sqrt{1 - \frac{r^2}{R^2}} \right) \pi}{4R};
$$

$$
I_4 = \frac{\left(\pi - 2 \arccos \sqrt{1 - \frac{r^2}{R^2}} \right) \pi}{4R}\,.
$$

The values of the integrals I_1 and I_2 are presented in Table 3.2.

Table 3.2

Values of the Integrals I_1 and I_2

For I_1

$\frac{r}{R}$	0	0,15	0,20	0,25	0,30	0,35	0,40	0,45	0,50	0,55
I_1	0	0,0035	0,0114	0,0193	0,0272	0,0503	0,0734	0,0965	0,1196	0,1603
$\frac{r}{R}$	0,60	0,65	0,70	0,75	0,80	0,85	0,90	0,95	1,00	
I_1	0,201	0,257	0,312	0,386	0,460	0,561	0,661	0,83	0,999	

For
Для I_2

$\frac{r}{R}$	0	0,15	0,20	0,25	0,30	0,35	0,40	0,45	0,50	0,55
I_2	0	0,205	0,254	0,302	0,351	0,379	0,407	0,435	0,463	0,482
$\frac{r}{R}$	0,60	0,65	0,70	0,75	0,80	0,85	0,90	0,95	1,00	
I_2	0,502	0,523	0,544	0,563	0,581	0,614	0,647	0,873	0,999	

CRITICALITY OF SYSTEMS OF INTERACTING SUBCRITICAL ASSEMBLIES OF FISSIONABLE MATERIALS

Pages 169-201

Kriticheskiye Parametry Sistem
s Delyashchimisya Veshchestvami
i Yadernaya Bezopasnost'

Introduction

When a certain number of subcritical assemblies of fissionable material make up a certain system, the exchange of neutrons among the assemblies -- -- interaction -- -- may make the system supercritical and cause an uncontrolled release of substantial energy.

The ensurement of nuclear safety in interacting systems of fissionable materials is a very important problem, which is often encountered in practice by specialists working with fissionable materials. The problem of the interaction of a large number of subcritical assemblies in a common set-up is extremely complex.

Developed at this time are exact methods for the calculation of interacting systems: Ackroid's unified theory of interaction $/47\,/$, the Monte-Carlo method, V. G. Zagrafov's method of evaluating the critical parameters of bodies of arbitrary shape $/48\,/$, etc. However, these methods, although yielding good results, are quite complicated and several of them require the application of high-speed computers.

Frequent attempts are made to find not exact solutions but reliable, appraisable, semi-empirical methods which provide an assured reserve when evaluating the conditions of the nuclear safety of a system of interacting subcritical assemblies. Presented below are the following methods for computing interacting systems:

a) the equivalent dimensions method,
b) the method of the safe solid angle,
c) the interaction parameter method,
d) the homogenization method.

Presented also is a summary of published experimental data on interaction. The results of some experiments are presented in the sections describing the computation method.

A. The Equivalent Dimensions Method

1. The determination of equivalent dimensions

We will assume that to every combination of subcritical assemblies with definite nuclear properties and size-shape factors corresponds a Laplacian-equivalent single assembly with changed geometric and former nuclear characteristics. We are attempting to write an equation for finding equivalent geometric dimensions.

The effective dimensions of an equivalent assembly depend on the probability P_{ij} to the effect that neutrons escaping from an i assembly falls into j. The value of P_{ij} is determined by the solid angle which substends the surface j of the assembly at a point on the i assembly. This solid angle must be computed with a consideration of the angular distribution of the neutrons escaping from the assembly. We will discuss the link between the equivalent dimension and the solid angle.

Figure 5.1

Toward a computation of the interaction of assemblies in the shape of parallelepipeds
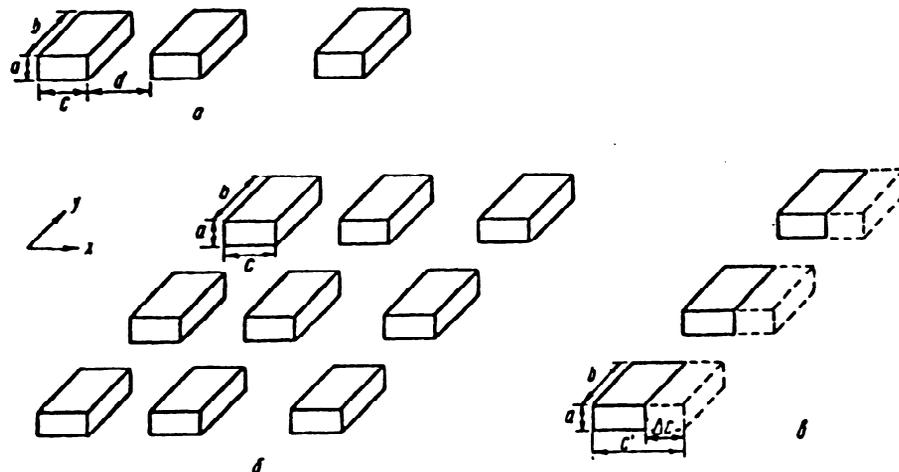
We will consider a case in which n of the uniform, parallelepiped-shaped assemblies are spaced in a line at the uniform distance d from each other and have neither a common nor a separate reflector. (Figure 5.1, a). It is advisable that in the given situation the equivalent reactor have the dimension a x b x $c_{экв}$, i.e., only the dimension in the direction of interaction is changed. The desired relationship must meet the following, obvious conditions:

$$\text{при } \begin{array}{ll} d \to 0 & c_{экв} = nc; \\ d \to \infty & c_{экв} = c; \\ n \to \infty & c_{экв} \to \infty \end{array} \right\} \quad \partial K\beta = e q r \qquad (5.1)$$

for any fixed d.

If the solid angle is standardized so the $\Omega = I$ at d = 0, the following equation for finding the equivalent dimension meets the above-mentioned conditions

$$c_{экв} = c \left[1 + (n-1) \Omega \right]. \qquad (5.2)$$

The significance of this expression is completely clear: the first term to the right is the real dimension and the second is the addition to the dimension via interaction.

We will consider a flat lattice of uniform parallelepipeds (see Figure 5.1, b). It is essential to reduce the given set to a single equivalent parallelepiped with sides a x $b_{экв}$ x $c_{экв}$. Two sequential approximations are possible to find the values $b_{экв}$ and $c_{экв}$. In the first approximation, each of the series along the direction x and y is considered independently. Then

$$c_{экв,n} = c [1 + (n_x - 1) \Omega_x]; \qquad (5.3a)$$
$$b_{экв,n} = b [1 + (n_y - 1) \Omega_y]. \qquad (5.36)$$

since n_x and n_y constitute the number of assemblies in the series in the direction of x and y; Ω_x, Ω_y are the solid angles between the closest assemblies in the direction of x and y, respectively.

Diagonal interaction is taken into account in the second approximation. The equivalent dimensions are found via the following equation:

$$c' = c [1 + (n_x - 1) \Omega_x]. \qquad (5.4a)$$

This corresponds to the "bunching" of all the assemblies in the series x and to the transformation of the initial system into the system shown in Figure 5.1, b. Now, we find that

$$b_{\mathfrak{z}_{K\mathfrak{a}}} = b\left[1 + (n_y - 1)\Omega_y'\right].\qquad (5.46)$$

$where\ \Omega_y'$ is the solid angle on the surface of a x c'.

To determine $c_{\mathfrak{z}_{q\nu}}$, we "will bunch" all the assemblies in the series y:

$$b' = b\left[1 + (n_y - 1)\Omega_y\right].\qquad (5.5a)$$

Теперь
Now

$$c_{\mathfrak{z}_{K\mathfrak{a}}} = c\left[1 + (n_x - 1)\Omega_x'\right].\qquad (5.56)$$

$where\ \Omega_x'$ is the solid angle on the surface of a x b'.

We will consider the spatial lattice of a parallelepiped. In the first approximation, the equivalent dimensions are found via the equation

$$a_{\mathfrak{z}_{K\mathfrak{a}}} = a\left[1 + (n_z - 1)\Omega_z\right];\qquad (5.6a)$$

$$b_{\mathfrak{z}_{K\mathfrak{a}}} = b\left[1 + (n_y - 1)\Omega_y\right];\qquad (5.66)$$

$$c_{\mathfrak{z}_{K\mathfrak{a}}} = c\left[1 + (n_x - 1)\Omega_x\right].\qquad (5.6\text{в})$$

The equivalent dimensions can also be found by a system of sequential approximations. Usually sufficient is the first approximation, irrespective of the fact that in this situation we disregard diagonal interaction.

The above-mentioned system for computing equivalent dimensions is quite common, for each irregular deployment of uniform assemblies can be rearranged into a regular one, creating a more reactive configuration. The spherical and cylindrical assemblies can be replaced by the described parallelepipeds. In this case, the technique of calculating the solid angle from the point at the plane a x b is simple:

$$\Omega = \frac{2}{\pi}\ \text{arctg}\ \frac{ab}{2d\sqrt{a^2 + b^2 + 4d^2}}\ .\qquad (5.7)$$

Presented below are several equations for calculating solid angles. The expression for Ω is obtained in the assumption of the isotropism and homogeneity of the escape of neutrons from the surface of an assembly.

1) Lines

$$\Omega = \frac{d}{h}\sin\theta.$$

2) Disks

$$\Omega = 2\pi(1-\cos\theta).$$

3) Planes
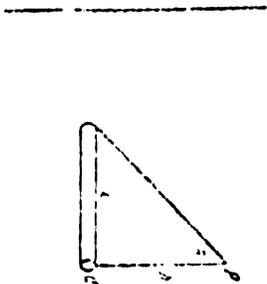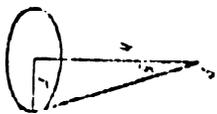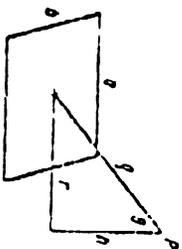
$$\Omega = \frac{ab\cos\theta}{q^2};$$
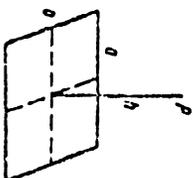
$$\Omega = \frac{2}{\pi}\arctan\frac{ab}{2h\sqrt{a^2+b^2+4h^2}}.$$

4) Cylinders
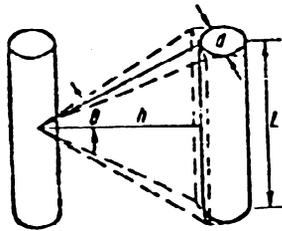
$$\Omega = \frac{2d}{h}\sin\theta.$$

5) Spheres
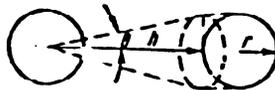
$$\Omega = 2\pi(1 - \cos\theta).$$

Table for the Transfer of the Proportion of the Solid Angle to the Sterad (Ster)

Ω_f	$\frac{Ster}{cm \cdot p}$	Ω_f	$cm \cdot p$	Ω_f	$\frac{Ster}{cm \cdot p}$
1,000	12,56 (4π)	0,350	4,40	0,100	1,26
0,750	9,42 (3π)	0,250	3,14 (π)	0,050	0,63
0,500	6,28 (2π)	0,150	1,88	0,000	0,00

For frequently encountered cylindrical reactors, it is necessary to use more accurate equations for the solid angle. For the interaction on the end planes of cylinders, the solid angle from the point to the circle of the radius R of the previous assumptions is calculated according to the equation

$$\Omega = 1 - \frac{d}{\sqrt{d^2 + R^2}} \cdot \qquad\qquad (5.8)$$

The solid angle from the point to the lateral surface of the cylinder is computed by the equation

$$\Omega \simeq \frac{1}{2\pi}\left[\frac{H}{\sqrt{\left(\frac{H}{2}\right)^2 + d^2}} - \frac{H}{\sqrt{\left(\frac{H}{2}\right)^2 + d^2 + 2dR}}\right]\arcsin\frac{R}{d+R}. \qquad (5.9)$$

Here, d is the shortest distance from the point to the surface, H is the height of the cylinder, and R is the radius of the cylinder.

Following the determination of the equivalent dimensions, the system obtained should be transformed into a sphere by equating the size-shape factors:

$$x^2 = \frac{\pi^2}{(a_{эк в} + 2\lambda)^2} + \frac{\pi^2}{(b_{эк в} + 2\lambda)^2} + \frac{\pi^2}{(c_{эк в} + 2\lambda)^2} = \frac{\pi^2}{(R_{эк в} + \lambda)^2}, \qquad (5.10)$$

where λ is the extrapolation length.

To solve the problem of the criticality of a system of assemblies, it is necessary to compare the sphere's critical mass of the given material m_{cr} taken from an experiment and the mass in the volume of the obtained sphere $m_{эк в}$. The condition of subcriticality will be

$$m_{cr} > m_{эк в} = \frac{4}{3}\pi R_{эк в}^3 \rho, \qquad (5.11)$$

where ρ is the known density of the fuel core in the given system.

The method under consideration can be applied to the computation of the interaction of subcritical assemblies entirely enclosed in a moderator. In this situation, the relationship (5.2) is

$$c_{эк в} \simeq c\left[1 + (n-1)\Omega e^{-\frac{d}{M}}\right], \qquad (5.12)$$

where M^2 is the migration area and d is the distance between assemblies. Under the condition of subcriticality (5.11), the critical mass of the sphere with a reflector of the given moderator must function in the vicinity of ...

2. Comparison of the Results of Experiments and Computations

Conducted for the purpose of explaining some regularities of interaction were experiments with subcritical assemblies in the shape of cylinders and parallelepipeds. An aqueous solution of salts of $UO_2(NO_3)_2$ with uranium of 90-percent enrichment, served as the fuel core of the assemblies. The cylinders and parallelepipeds were made of stainless steel of 1.5- to 2-mm thickness; the diameter of the cylinders was 30 cm, and the basis of the parallelepiped was a square with a side of 30 cm.

Figure 5.2

The interaction of assemblies shaped as parallelepipeds
with the square of the basis amounting to 30 x 30 cm in the
air. The number of the assemblies:
2 (x), 3 ($\square$). Three in line (o), 4 ($\triangle$), 5 ($\bullet$).
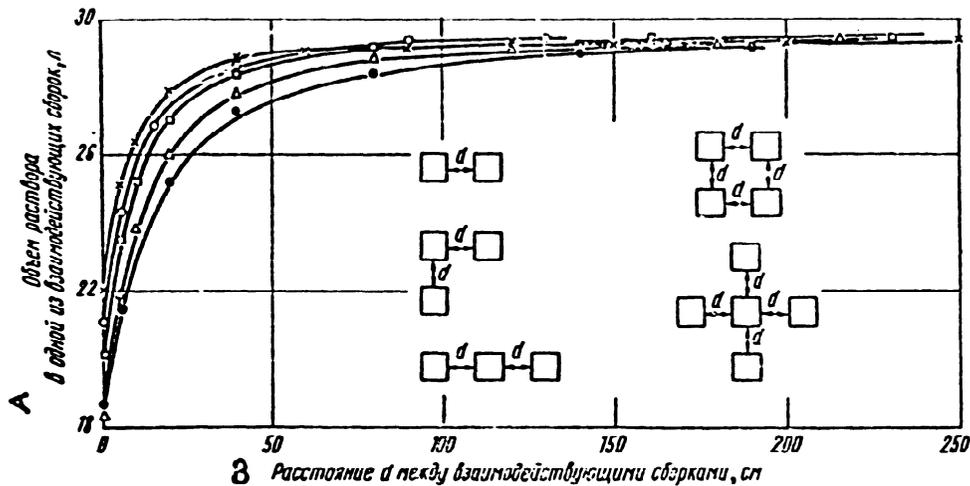
Legend: A = Volume of the solution in one of the interacting assemblies, l;
B = Distance d between interacting assemblies, cm.

Presented in Figures 5.2-5.6 are the results of experiments aimed at determining the effectiveness of two, three, four, and five assemblies in the air. The figures point out the relationship between the volume of the solution in one assembly and the distance between them. The entire system of assemblies was critical, and the quantity of the solution in all the assemblies was uniform. The concentration of uranium in the solution was 71 g/l, save for the test on interaction in water (Figure 5.7), in which the concentration of uranium in the solution constituted 113 g/l. Determined via the experiments was the critical state of the system composed of several homogeneous assemblies spaced at various distances from one another. Obviously, with the presence of interaction, each assembly

individually was subcritical while the entire system as a whole was critical. The analogous relationship of two cylindrical assemblies of 25-cm diameter — — assemblies interacting in water — — is shown in Figure 5.7. For the purposes of comparison Figure 5.3 also contains curves computed according to the equivalent dimensions method.

Figure 5.3

Comparison of the interaction of four assemblies: the interaction was computed by the equivalent dimensions method, with an experiment: ● is the experiment, Δ is the computation, first approximation; ▢ is the computation with a consideration of oblique interaction in the directions A and B.
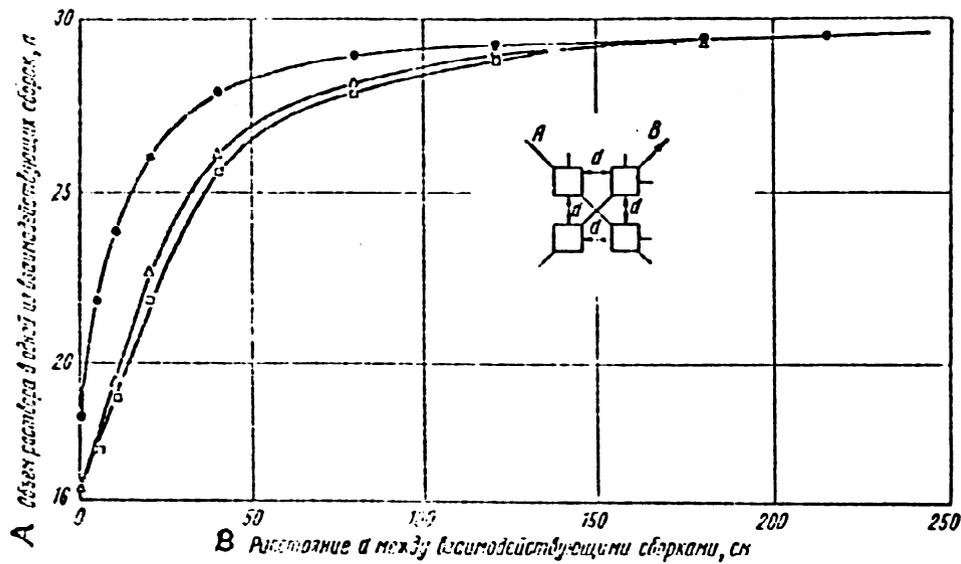
Legend: A = Volume of the solution in one of the interacting assemblies, l; B = Distance d between interacting assemblies, cm.

Figure 5.4

The interaction of cylindrical assemblies of 30-cm diameter in
the air. The number of assemblies:
2 (x), 3 (0), three in line ($\triangle$), 4 ($\square$).
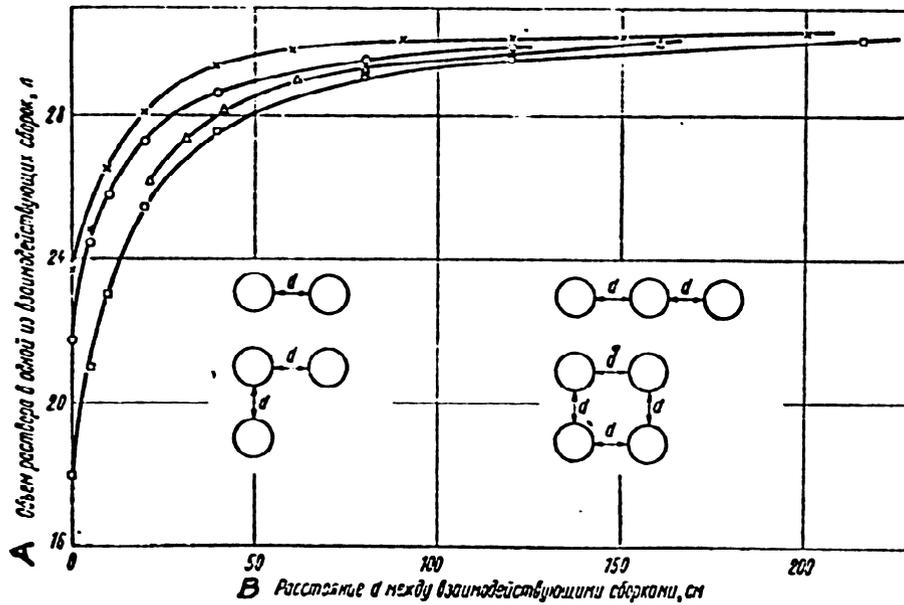
Legend: A = Volume of the solution in one of the interacting assemblies, l;
B = Distance d between interacting assemblies, cm.

Figure 5.5

The interaction of two assemblies
in the shape of parallelepipeds,
with a basis of 15 x 60 cm in
the air (X) and in water (O).
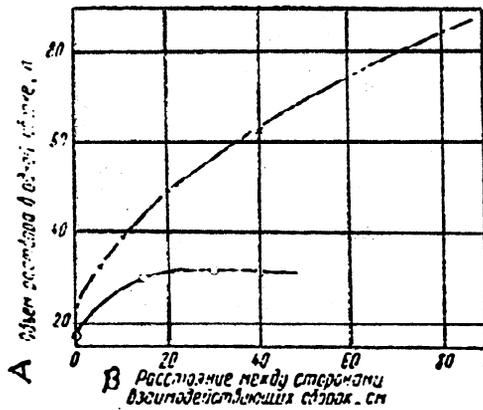
Figure 5.6

The interaction of two
cylindrical assemblies
of 30-cm diameter with-
out a reflector (O)
and with an aqueous
reflector of 5-cm
thickness (◉).
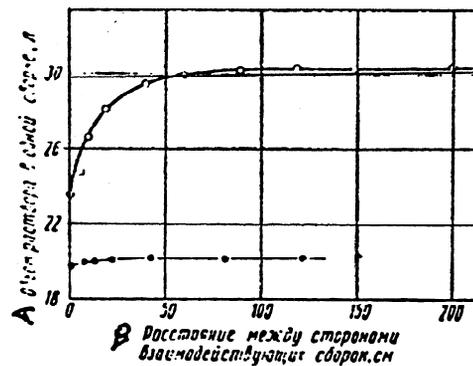
Legend: A = Volume of solution
in one assembly, l; B = Distance
between the sides of interacting
assemblies, cm.

Legend: A = Volume of
solution in one assembly,
l; B = Distance between
the sides of interacting
assemblies, cm.

Employed for the study of the interaction of a large number of sub-
critical assemblies in a spatial lattice were six-liter, cylindrical glass
containers filled with an aqueous solution of salts of $UO_2(NO_3)_2$. The diameter
of the container was about 18 cm, the height of the solution 24 cm, and the
thickness of the walls 0.5 cm. The device employed for the study of the
interacting assemblies in a spatial lattice is shown in Figure 5.8. The
support and the frame of the device were made of steel, and the charging
hole-equipped loading platforms on which the containers were deployed were
made of duralumin. The results of these experiments are shown in Table 5.1.

The values d_x and d_y presented in the table are the distances in the plane and d_z is the distance between planes. All the distances are indicated with account taken of the walls, i.e., between solutions.

Figure 5.7

The interaction of two cylindrical assemblies of 25-cm diameter
in water
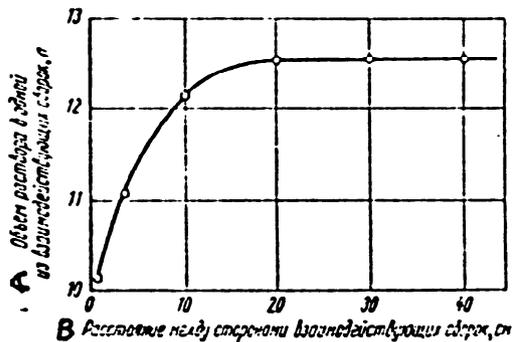
Legend: A = Volume of the solution in one of the interacting assemblies, 1;
B = Distance between the sides of interacting assemblies, cm.

Neutron capture by the construction materials of the device were disregared in the computations. The calculation of neutron capture approximates the computed values of those of the experiment. A comparison of the results of computations and experiments indicate that the equivalent dimensions method yields in all cases a reserve in favor of safety and can be employed in evaluations of the nuclear safety of interacting systems.

With the equivalent dimensions method, the interaction of several subcritical assemblies is computed with an accuracy of about 20 percent; the interaction of many assemblies is evaluated with an accuracy of 100 percent, and in all cases computation overstate the effectiveness of interaction. The equivalent dimensions method, being very simple, does not cede, in terms of accuracy, to the methods of the parameter of interaction and homogenization.

Table 5.1

The Interaction of Six-Liter Containers in a Spatial Lattice
without Reflector

(the concentration of uranium in the solution is 96 g/l; enrichment is 90 percent).

1 Расположение емкостей	2 Количество установленных емкостей	3 Критическое значение (эксп.)	4 Расчетное значение
5 В двух плоскостях: $d_x=1$ см, $d_y=4,5$ см, $d_z=12$ см	**10** 23 (в 1-й плоскости 4×3=12, во 2-й (1×3)—1=11) **11**	23	12
6 В трех плоскостях: $d_x=d_y=6,5$ см, $d_z=12$ см	**12** 52 (в 1-й плоскости 4×4=16, во 2-й—4×5=20, в 3-й— 4×4=16)	54±0,5 21 (экстраполяция)	18
7 В трех плоскостях: $d_x=d_y=4,5$ см, $d_z=12$ см	**14** 39 **13** (в 1-й плоскости 3×4=12, во 2-й—3×4+3=15, в 3-й—3×4=12) **15**	39,5±0,3 (экстраполяция)	12
8 В четырех плоскостях: $d_x=d_y=9$ см, $d_z=11$ см	**17** 67 **16** (в 1-й плоскости 4×4=16, во 2-й—4×4=16, в 3-й—4×5— —1=19, в 4-й—4×4=16)	80±5 (экстраполяция)	36
9 В четырех плоскостях: $d_x=d_y=7,5$ см, $d_z=12$ см	**18** 64 **19** (в каждой плоскости 4×4=16) **20**	72±5 (экстраполяция)	27

Legend: 1 = Container location, 2 = amount of prescribed capacity,
3 = critical value (exponential), 4 = computed value, 5 = In two planes,
6 = In three planes, 7 = In three planes, 8 = In four planes, 9 = In
four planes, 10 = ∫in the first plane..., 11 = in the second plane...∫,
12 = (in the first plane..., 13 = in the second plane...), 14 (in the
first plane..., 15 = in the second plane..., 16 = in the third plane...),
17 = (in the first plane..., 18 = in the second plane...and in the third
plane..., 19 = ...in the fourth plane...),20 = in each plane, 21 =
(extrapolation).

Figure 5.8

Device for studying the interaction of assemblies in a spatial lattice:
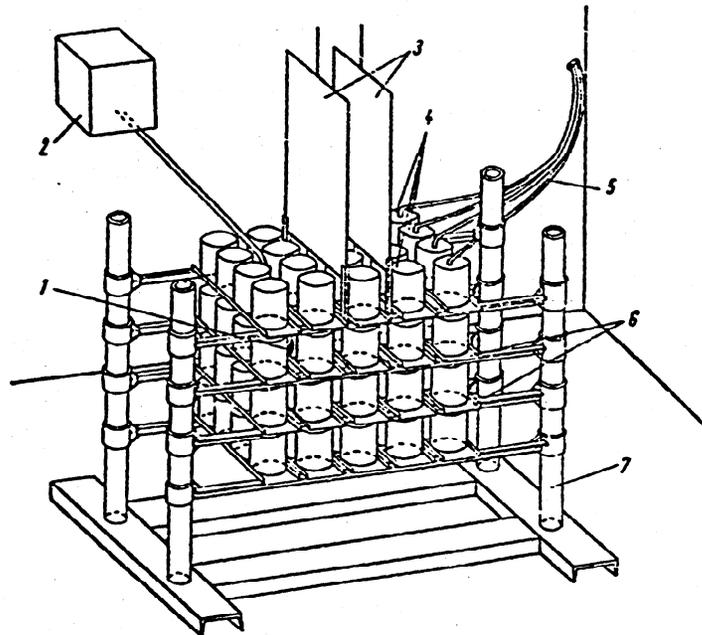
Legend: 1 = neutron source, 2 = container for neutron source, 3 = cadmium screen, 4 = containers with solution, 5 = hoses for pouring the solution, 6 = charging hole-equipped loading platforms for the containers, 7 = support.

B. __The Method of the Safe Solid Angle__

1. __The Essence of the Method__

Three basic conditions underlie this method of evaluating safety during interaction.

1. Each assembly must be safe when it is surrounded completely by an aqueous reflector.

Figure 5.9

The interaction of assemblies containing solutions of uranium of 90-percent enrichment in accordance with the K_{eff} of each assembly:
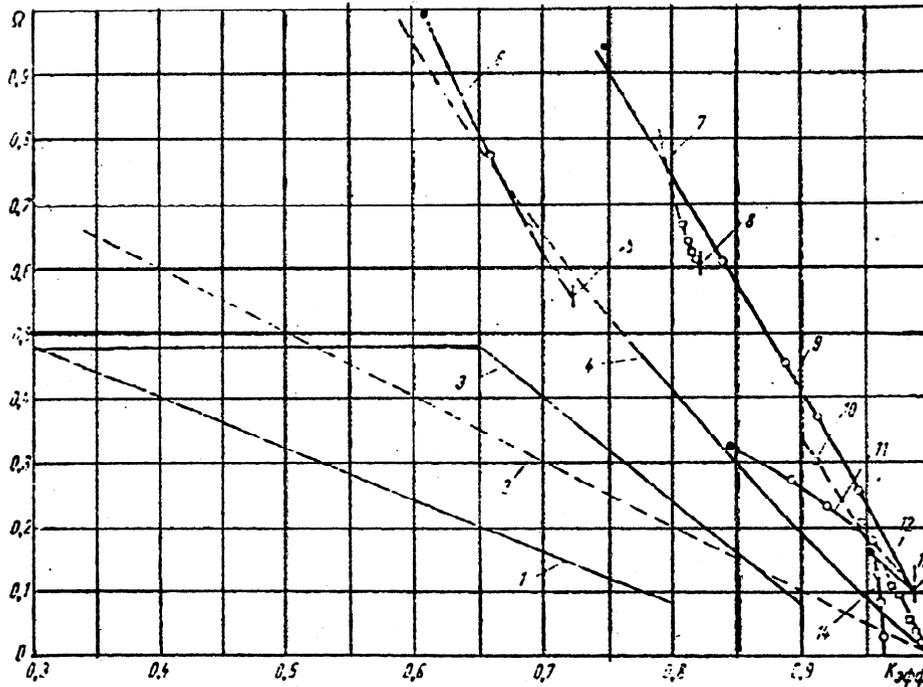
Legend: 1 = the curve safe for any container; 2 = the curve safe for a solution containing $\frac{P_H}{P_5}$ = 100; 3 = the curve safe for unreflected cylinders, 4 = the computed minimal critical curve for two infinite cylinders, 5 = an infinite cylinder of 15.24-cm; 6 = 7 cylinders of 15.24-cm diameter; 7 = 3 plates with a bisis of 7.62 x 120.65 cm, 8 = a plate with a basis of 7.62 x 120.65 cm of infinite height, 9 = 7 cylinder of 20.32-cm diameter, 10 = 2 plates with a basis of 15.24 x 120.65 cm, 11 = 3 cylinders of 20.32-cm diameter, 12 = 2 cylinders of 20.32-cm diameter, 13 = an infinite cylinder of 20.32 cm diameter, 14 = 2 cylinders of 50.8-cm diameter;

$\frac{P_H}{P_5}$ = 44.3 for cylinders with diameters of 15.24 and 20.32 cm;

$\frac{P_H}{P_5}$ = 169 for cylinders of 50.8-cm diameter; $\frac{P_H}{P_5}$ = 330 plates;

t_p = closely placed cylinders.

2. The distance between assemblies must be no less than 30 cm.

3. The distance between assemblies must be such that the maximum, general solid angle subtended to the central assembly from all the other assemblies does not exceed a certain safe magnitude. The first two conditions provide safety during the flooding of the system of assemblies with water, since a water layer of 30-cm thickness ensures the complete absence of interaction (see, e.g., Figure 5.7). The third condition requires the determination of the certain maximum solid angle at which interaction is safe. The magnitude of the maximum safe solid angle is determined by the effective multiplication factor K_{eff} of the individual assemblies of a system. The K_{eff} can be found via the two-group diffusion theory.

Experimentation is basic for the purpose of determining the relationship between the solid angle and the multiplication factor. Two or more uniform assemblies are set up at definite distance and reach a critical state, after which the critical solid angle Ω_{cr} and the K_{eff} are also determined for each assembly. The relationships between Ω_{cr} and the K_{eff} found in this manner are shown in Figure 5.9.

2. Safety Criteria for a System of Unreflected Assemblies

With the graphs in Figure 5.9, it is possible to conclude that, for un-reflected assemblies with a uniform K_{eff} but of various shapes interaction is increased with an increase of the solid angle. The assumption that the general solid angle subtended to the central assembly can be applied to multi-assembly systems is conservative.* Figure 5.9 also presents the computed minimal critical curve. It pertains to the pair of infinite cylinders containing the most reactive solution (90 percent enrichment, $\mathcal{C}_{H}/\mathcal{C}_{u} = 44.3$).

Although the maximum solid angle for the two infinite cylinders is 20 percent of 4 , a part of the curve for solid angles, greater than this magnitude, yields a conservative theoretical limit for a multicomponent system (e.g., for the seven cylinders).

Curve 3 for unreflected systems is a safe range for the solid angle. It is quite conservated under any condition, provided that neutron reflection in a system is completely excluded. To determine the spacing of containers (as-semblies), the K_{eff} of each assembly is computed; the angle is determined from curve 3, and distance between containers, by the magnitude of the solid angle. Table 5.2 presents the K_{eff} values of some assemblies.

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* Here and subsequently, the word "conservative" will mean "with a great reserve in favor of nuclear safety."

Table 5.2

K_{eff} Values of Unreflected Cylinders Containing an Aqueous Solution of UO_2F_2

(uranium is of 90 percent enrichment)

Диаметр, см (1)	Высота, см (2)	ρн/ρ5	Концентрация U235, кг/л (3)	Кэфф (4)
12,7	60	50	0,48	0,55
12,7	∞	50	0,48	0,58
12,7	60	150	0,17	0,50
12,7	∞	150	0,17	0,52
12,7	60	350	0,075	0,42
12,7	∞	350	0,075	0,43
20,3	15,4	50	0,48	0,68
20,3	15,4	150	0,17	0,63
20,3	30,8	150	0,17	0.81
30,8	15,4	150	0,17	0,83
30,8	15,4	350	0,075	0,72

Legend: 1 = Diameter in cm, 2 = Height in cm, 3 = Concentrations of U^{235} in kg/l, 4 = K_{eff}.

Maximum K_{eff} Values for Aqueous Solutions of UO_2F_2

Система (1)	Кэфф (2)
3 Цилиндр, D = 12,7 см	0,58
4 Сфера, D = 20,3 см	0,67
5 Пластина, H = 8,175 см	0,91
6 350 г U235 в сферическом объеме 12,5 л	0,65

Legend: 1 = System, 2 = K_{eff}, 3 = Cylinder, 4 = Sphere, 5 = Plate, 6 = 350 g of U^{235} in a spherical volume of 12.5 l.

3. Effects of Complete and Partial Reflectors

Safety during the flooding of a system with water is taken into account and provided for by the first and second conditions (see 1). However, there are situations in which two assemblies have a partial reflector. The interaction in such a case will be substantially greater than in a situation in which assemblies have no reflector or are completely flooded with water and the distance between assemblies is 30 cm.

Most dangerous is a set-up whereby the partial reflector is located on the external side of two interacting assemblies. In this situation, the magnitude K_{eff} of each assembly with a "half" reflector $(K_{\frac{1}{2}})$ lies between the values $K_{c.r}$ (for a completely reflected assembly) and the value $K_{c.r}$ (for an unreflected assembly):

$$K_{1/2} \approx {}^1/_2 (K_{c.r} + K_{c.r}).$$

The most dangerous situation arises when one completely reflected assembly is critical. Consequently, the safety magnitude K with partial neutron reflection is equal to

$$K_{1/2} \approx {}^1/_2 (K_{c.r} + 1).$$

Experiments have indicated that this equation is quite conservative. In Figure 5.9, curve 1 takes into account safety when a complete reflector is available.

4. Interaction among Variously Shaped Assemblies

Experiments indicate that if assembly A is safe at distance from another such assembly A and assembly B is safe at distance from the analogous assembly B, then the assemblies A and B are safe with a distance between them of

$$d_{AB} = \frac{d_A + d_B}{2}. \tag{5.13}$$

Experiments aimed at substantiating the solid angle method have been conducted with fissionable materials having high concentrations of U^{235}. With the given magnitude of $K_{c.r}$ an assembly with a low uranium enrichment is less sensitive to changes in the intensity of an external neutron source, and curves 1 and 3 in Figure 5.9 are more conservative for assemblies with low enrichments. For a situation in which a change in the concentration of the fissionable material in a container was possible for some reason or another, a computation of the magnitude of $K_{c.r}$ was made during a change in the volume of an unreflected sphere containing 350 g of U^{235} (a safe amount).

It appeared that the maximum magnitude of K_{eff} was 0.65 with 12.5 l. Consequently, when a change in concentration is possible, a K_{eff} of no less than 0.65 should be assumed for evaluating the solid angle in spacing containers containing a safe quantity of uranium (350 g).

5. <u>Basic Conclusions and Equations for K_{eff}</u>

A system of uniform assemblies (containers) containing solutions of fissionable materials is safe, provided the following basic conditions are observed:

a) Each assembly is safe upon its complete submersion in water,
b) the distance between assemblies is no less than 30 cm,
c) the general solid angle between the central assembly and the remaining assemblies is less than the following values:

$$0,48 \cdot 4\pi \qquad K_{эфф} < 0,3,$$
$$(0,72 - 0,8 K_{эфф}) \cdot 4\pi \quad » \quad 0,3 < K_{эфф} < 0,8 \qquad (5.14)$$

(assemblies shielded from the central assembly by others are disregarded in calculating the solid angle).

For $K_{eff} > 0.8$, the distance between assemblies is determined by direct experimentation (see, e.g., Figures 5.2-5.6). With solid angles of 0.04 percent from 4π, the interaction of subcritical assemblies can be disregarded.

The magnitude of K_{eff} employed under these conditions can be determined with either known equations $\underline{/1/}$ or through experimentation.

For thermal neutron assemblies with high uranium enrichments and without a reflector, the magnitude of   can be determined by

$$K_{эфф} = \nu \theta U_j U_t, \qquad (5.15)$$

where ν is the average number of fast neutrons per one thermal neutron captured in U^{235}, θ is the probability of thermal neutron capture in U^{235}, U_j is the probability of capture up to thermalization, U_t is thermal neutron capture.

With low enrichments, it is essential to take into account the probability of resonance escape. For high enrichments, $\nu = 2.09$ and $\theta \approx$
when a solution does not have other powerful absorbers, save for uranium and hydrogen (e.g., a solution of UO_2F_2).

$$= \frac{1}{1 + 1.8 \cdot 10^{-4} \cdot \frac{\rho}{n} / \xi}$$

The magnitude U_f can be obtained from the following empirical rela-
tion, which is applicable at $\rho_H/\rho_S > 40$:

$$U_f = \frac{1}{(1+\varkappa_1^2)(1+4,2\varkappa_1^2)(1+20,16\varkappa_1^2)}, \qquad (5.16)$$

where ϑ is the Laplacian of a fast neutron system. The equation is also
applicable to low enrichments, with a corresponding selection of resonance
absorption:

$$U_t = \frac{1}{1+\varkappa_1^2 L^2},$$

where $\varkappa^2$ is the square of the diffusion length:

$$L^2 = 8,29(1-\theta)\,cm^2. \qquad (5.17)$$

The magnitude of U_f can be computed by more exact methods. It is
important that the methods yield a reserve in favor of nuclear safety.
In practice, it is reasonable to take advantage of simple approximation
methods.

C. The Interaction Parameter Method [51]

1. Criterion of Criticality

We will consider the system n of subcritical assemblies of a fission-
able material. The interaction parameter q_{ij} between the assembly i and the
assembly j we will determine as the total number of neutrons escaping from
the assembly i per one neutron leaving the assembly j. In other words, if
F_j is the total number of neutrons leaving the assembly j, then $q_{ij} F_j$ is
the number of neutrons forming in the assembly i upon interaction with the
assembly j. The interaction parameter q_{ij} can be presented as the product
of the P_{ij} probability that neutrons leaving the assembly j land in the
assembly i, as well as the probability of the surface multiplication factor
M_{si} of neutrons from the assembly j to the assembly i.

It is obvious that the value of P_{ij} depends on the shape of the i and j assemblies, the distance between them, the characteristics of the environment separating the assemblies, and the energy and angle of neutron distribution. The M_{si} factor is determined by the multiplying characteristics of the assembly i in respect to the neutrons leaving the assembly j. Hence,

$$q_{ij} = P_{ij} M_{si}. \tag{5.18}$$

A Case of Two Assemblies.

We will consider two subcritical assemblies. The number of neutrons leaving an assembly we will call the assembly's neutron yield. Let, when the assemblies are stationary and isolated from each other, the neutron yield be F_1 and F_2. When the assemblies are spaced at a certain finite distance from each other, they will interact and the neutron yields will increase to the values F'_1 and F'_2.

From a determination of the interaction parameter, it follows that the number of neutrons forming in the first assembly as a result of the presence of the second assembly is equal to $q_{12} F'_2$ i.e.,

analogously
аналогично

$$F'_1 = F_1 + q_{12} F'_2 \left. \right\}$$
$$F'_2 = F_2 + q_{21} F'_1. \left. \right\} \tag{5.19}$$

Consequently
Следовательно,

$$F'_1 = \frac{F_1 + q_{12} F_2}{1 - q_{12} q_{21}}, \quad F'_2 = \frac{F_2 + q_{21} F_1}{1 - q_{12} q_{21}}. \tag{5.20}$$

The subcritical state corresponds to a situation in which

$$q_{12} q_{21} < 1. \tag{5.21}$$

In a critical state $q_{12} \cdot q_{21} = 1$ and the finite values of F'_1 and F'_2 can be maintained without an extraneous neutron source. For the two uniform assemblies

$$F'_1 = F'_2 = F'; \quad F_1 = F_2 = F \quad \text{и} \quad q_{12} = q_{21} = q,$$

and the expression (5.20) can be formulated as follows:

$$F' = \frac{F}{(1 - q)}. \tag{5.22}$$

A Case of Numerous Assemblies.

Let F_i and F'_i be the neutron yields of the assembly i in an isolated state and in a system of assemblies, respectively. Then

$$F'_1 = F_1 + q_{12}F'_2 + q_{13}F'_3 + \ldots + q_{1n}F'_n,$$
$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$
$$F'_3 = q_{31}F'_1 + c_{32}F'_2 + F_3 + \ldots + q_{3n}F'_n$$

etc., and the condition of criticality can be written in the form of a determinant:

$$D \equiv \begin{vmatrix} -1 & q_{12} & q_{13} & \cdots & q_{1n} \\ q_{21} & -1 & q_{23} & \cdots & q_{2n} \\ q_{31} & q_{32} & -1 & \cdots & q_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ q_{n1} & q_{n2} & q_{n3} & \cdots & -1 \end{vmatrix} = 0, \qquad (5.23)$$

или
by
$$D \equiv \det [Q - I] = 0,$$

where I is the unit matrix of the n order and Q is the matrix of the interaction parameters in the system.

When the values q_{ij} are known, it is possible to evaluate the criticality of a system as a whole. However, the solution of such an equation is very difficult, especially as regards large, multicomponent systems.

The Approximate Solution of the Equation of Criticality.

It is possible to replace all the values q_{ij} by the maximum value encountered in a system. Under such a condition, we obtain an approximate though "safe" solution of the equation (5.23):

$$D = (-1)^{n-1} (1 + q_{\text{макс}})^{n-1} [(n-1) q_{\text{макс}} - 1],$$

and the state of criticality will be

$$(n-1) q_{\text{макс}} = 1. \qquad (5.24)$$

The equation (5.24) yields much reduced magnitudes of n — the number of assemblies, during which the system becomes critical.

2. An Experimental Evaluation of the Interaction Parameter q

Since the individual assemblies in a system are usually fundamentally subcritical, they have a very small effective multiplication factor. The magnitudes of q for such assemblies are usually low $(10^{-4}-10^{-2})$, thereby making it difficult to measure them with accuracy. The method of measurements is about as follows. A subcritical assembly, neutron source, and neutron detector are spaced as shown in Figure 5.10. To determine the neutron yield in the assembly located at point A -- a yield determined by the source at point B, counts are made with a neutron detector in the following positions:

a = source at point A without an assembly -- C_1;

b = assembly at point A without a source -- C_2;

c = source at point B without an assembly -- C_3;

d = assembly at point A, source at point B-C_4.

Figure 5.10

Test arrangement for determining the interaction parameter of
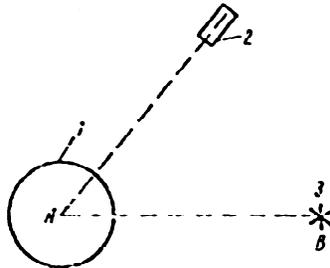
Legend: 1 = assembly, 2 = detector, 3 = source.

If F is the neutron yield of the assembly in an isolated state, Q is the intensity of the source, and E_1 and E_2 are the efficiencies of the counter with source position at points A and B, respectively, then

$$C_1 = E_1 Q;$$
$$C_2 = E_1 F;$$
$$C_3 = E_2 Q;$$
$$C_4 = E_1 (F + qQ) + E_2 Q,$$

откуда
whence

$$q = \frac{C_4 - (C_2 + C_3)}{C_1}.$$

- 124 -

Table 5.3

Interaction Parameters for Metallic Spheres at a Distance of 81 cm between the Source and the Center of the Sphere

a Сфера	b Среда	c Параметр взаимодействия	
		$q \cdot 10^4$	$q \frac{4\pi}{QM_c}$
20 кг U²³⁵ (93%)	Воздух d	58,4±3,9	1,05±0,07
20 кг U²³⁵ (93%)	Воздух. Контейнер на полу f	71,3±2,3	1,28±0,04
20 кг U²³⁵·(93%)	Песок. Сфера: g		
	покрыта кадмием h	4,5±0,4	0,08±0,01
	без кадмия i	29,0±0,4	0,52±0,07
20 кг U²³⁵ (93%)	Вода. Сфера: j		
	покрыта кадмием k	0,09±0,01	1,6·10⁻³
	без кадмия l	0,37±0,02	7,0·10⁻³
9 кг Pu (15,5 г/см³)	Воздух. Контейнер на полу m	83±6	1,44±0,10
9,0 кг Pu	Вода. Сфера: n		
	покрыта кадмием o	1,2±1,2	0,02±0,02
7,6 кг Pu	Воздух. Контейнер на полу p	60±4,0	1,48±0,1
4,6 кг Pu	То же q	29±3	1,58±0,16

Legend: a = Sphere, b = Environment, c = Interaction Parameters, d = Air, f = Air. Container on floor, g = Sand. Sphere:, h = coated with cadmium, i = without cadmium, j = Water. Sphere:, k = coated with cadmium, l = without cadmium, m = Air. Container on floor, n = Water. Sphere:, o = coated with cadmium, p = Air. Container on floor, q = Air. Container on floor.

Comment: 1. The parameters q are obtained at an angle of (see Figure 5.10). 2. In the second to the seventh tests, the neutron source and sphere were located in the center of a steel container (D = 30 cm, H = 60 cm, wall thickness = 2.5 cm). 3. In the third test, the containers were placed on the floor and surrounded by sand (the thickness of the sand layer was 30 cm in all directions). 4. In the fourth and sixth tests, the containers were placed in a box with water and completely submerged in the water. The magnitude $q \cdot \frac{4\pi}{QM_c}$ characterizes the magnitude of neutron diffusion and capture, since for systems in the air this magnitude must always be on the order of a unit.

The accurate determination of the magnitude of q necessitates a similitude of the neutron source spectrum and the spectrum of neutrons escaping from the assembly. Usually utilized is a source — — a spectrum fission simulator.

Table 5.3 presents the results of experimentally determining the interaction parameter of metallic spheres.

3. An Evaluation of the Interaction Parameter on the Basis of Measurements of Neutron Multiplication in Assemblies

It is usually adequate to point out that the determined magnitude of is less than a certain value of q_{cr} following the criticality of a system. This can be done by neutron multiplication measurements, provided the neutron source is constant. In the determinate equation $q = PM_s$ it is also necessary to determine P - - the probability of fallout and M_s - - the surface multiplication factor.

An Unreflected System

For unreflected assemblies, the magnitude P is determined by the solid angle between two assemblies. To evaluate the surface multiplication factor for an unreflected assembly - - the factor $M_{sb}Q$, we will consider a spherical assembly with a point neutron source of intensity Q. We will locate the source on the surface of a spherical assembly. The number of neutrons falling into the assembly is 1/2 Q. These neutrons will cause fission and yield 1/2 $M_{sb}Q$ of the neutrons emerging from the assembly. Total neutron emission is 1/2 (M_{sb} + 1) Q. When this source is located in the center of the spherical assembly, total neutron emission will be $M_{cb}Q$, since M_{cb} is the multiplication factor of neutrons from the central source.

It is possible to formulate the equation:

$$\frac{1}{2} (M_{sb} + 1)Q = M_{cb}Q\eta,$$

$$M_{sb} = 2\eta M_{cb} - 1,$$

where η is the function determining the relative value of the neutrons admitted into the reactor's surface and center.

Figure 5.11

The relationship between $1/M_s$ and the linear dimensions of an assembly
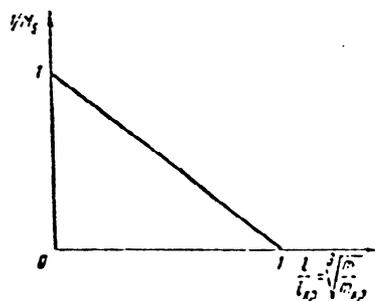
For systems with dimensions on the order of a single free-path length, there are no special points at which neutron multiplication is great, and the magnitudes η, M_{cb} and M_{sb} are on the order of a unit. For systems with dimensions on the order of several free-path lengths, a diffusion approximation can be applied. It is possible to demonstrate that the number of neutrons U (a, r) which are generated in a sphere with radius a is due to the presence of an isotropic spherical unit source located at the radius r $(\leq a)$, which is proportional to $\frac{\sin \varkappa r}{\varkappa r}$, $where$ $\varkappa^2$

is the material parameter.

Since M $(a, 0) = M_{cb}$ and M $(a, a) = 1/2$ $(M_{sb} + 1)$, then according to the diffusion theory $\eta = \frac{\sin \varkappa a}{\varkappa a}$. For an unreflected assembly, we

thus have the basis to assert that η is always less than a unit and

$$M_{sb} \leqslant 2M_{cb} - 1,\qquad\qquad (5.25)$$

since the value of the neutrons admitted into the center of the system is greater than the value of the neutrons fallen on its surface.

<u>Other Methods of Evaluating M_s</u> $\underline{/}$ 51-53 $\underline{/}$.

It is possible to evaluate the interaction parameter by assuming that the magnitude of the inverse surface multiplication factor $(1/M_s)$ is changed linearly in accordance with the cubic root of the magnitude of the mass, i.e., with the linear dimensions of a system. If the magnitude $1/M_s$ is constructed in accordance with the linear dimension which is divided into the critical mass, then $1/M_s$ must be equal to a unit with $\mathcal{Z} = 0$ and $1/M_s = 0$ for $\mathcal{Z} = \mathcal{Z}_{cr}$: in the critical state of the system. Then, if the relationship is linear, we obtain a graph for the rapid evaluation of the surface multiplication factor $\underline{/}$ Figure 5.11 $\underline{/}$. Computations indicate that such an evaluation is conservative; it results in overstating the value of M_s. Thus, if either the critical mass or the size of the sphere or the size of the cylinder for a fissionable material of given composition is known, $1/M_s$ can be evaluated by the equation

$$\frac{1}{M_s} = 1 - \sqrt[3]{\frac{m}{m}},\qquad\qquad (5.26)$$

where m_{ν} is the critical mass, m is the mass for which we determine the M_s factor. Specifically, for a sphere

$$\frac{1}{M_s} = 1 - \frac{r}{r_{cr}}.$$

The interaction parameter q is equal to the multiplication surface factor multiplied by the solid angle connecting two assemblies:

$$q = M_s \cdot \frac{\Omega}{4\pi} = \frac{r_{cr}}{r_{cr} - r} \cdot \frac{\Omega}{4\pi} = \frac{r_{cr}}{\Delta r} \cdot \frac{\Omega}{4\pi}. \tag{5.27}$$

The multiplication surface factors of metallic spheres and spheres with a solution of $UO_2(NO_3)_2$ are presented in Figures 5.12 and 5.13.

Figure 5.12

Inverse surface multiplication of $1/M_s$ in accordance with the radius of a metallic uranium, unreflected sphere. Enrichment of uranium is 93 percent.
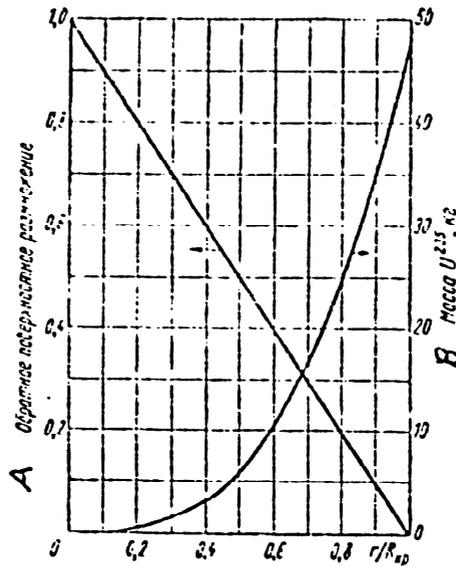
Legend: A = Inverse surface multiplication, B= Mass of U^{235} in kg.

- 128 -

Figure 5.13

Inverse surface multiplication of $1/M_s$ in accordance with the radius of an unreflected sphere with an aqueous solution of $UO_2(NO_3)_2$. The enrichment of uranium is 93 percent. The concentration of uranium is in g/l: 40 (●), 60-80 (○), >100 (X)
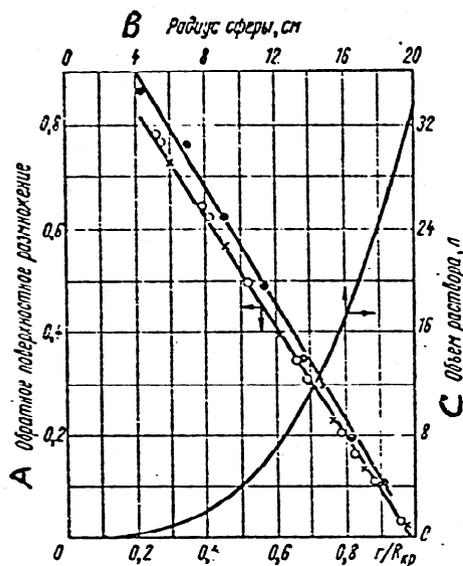
Legend: A = Inverse surface multiplication, B = Radius of sphere in cm, C = Volume of solution in l.

4. Interaction Parameter Method in Matrix Form

In the interaction parameter, the equality of zero to the determinant is the criterion of criticality. This criterion, however, has certain shortcomings which lead to a lack of definitiveness in evaluating the criticality of interacting systems.

1. The zero equality of the determinant is a necessary but inadequate condition for the criticality of a system. It is possible to demonstrate that, when the determinant is equal to zero, a system may be critical or supercritical. When the determinant does not equal zero, a system may be either supercritical or subcritical.

2. The numerical solution of the determinant is a difficult and prolonged procedure.

3. It is of interest to know not only the criticality or sub-criticality of a system but also the magnitude of subcriticality, which cannot be determined by solving the determinant.

The criterion of criticality formulated below $/\overline{52}/$ is free of these shortcomings.

Criterion of Criticality.

We will consider the system n of subcritical assemblies. The characteristics of this system will be analogous to those of part 1.

In a critical system neutron fluxes are maintained constant, without an extraneous neutron source, and are expressed in the form of

$$F_i = \sum_{j=1}^{n} M_{ij} P_{ij} F_j \quad (i = 1, 2, \ldots, n),. \tag{5.28}$$

where $q_{ij} = M_{ij} P_{ij}$.

The equation (5.28) can be written most conveniently in matrix form:

$$
\begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ F_n \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{bmatrix} \cdot \begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ F_n \end{bmatrix} \tag{5.29}
$$

Having specified the matrix of interaction parameters through $/\overline{Q}/$ and the vector of fluxes through $/\overline{F}/$, we will obtain

$$[F] = [Q][F]. \tag{5.30}$$

In a critical situation, this equation is satisfied by the vector $/\overline{F}/$, every term of which is positive. Consequently, the characteristic equation

$$\lambda [F] = [Q][F] \tag{5.31}$$

must have the eigenvalue $\lambda = 1$. This is inadequate to evaluate the criticality of a system, since the eigenvector $\underline{/F\underline{/}}$ corresponding to $\lambda = 1$ must be composed of only positive terms. It is possible to demonstrate that for positive matrixes the eigenvalue connected with the eigenvector with the positive elements is the greatest eigenvalue of the matrix $\underline{/52\underline{/}}$. This eigenvalue λ_{max} is real and positive, and modules of other eigenvalues do not exceed λ_{max}.

Consequently, in addition to the requirement that the matrix $\underline{/Q\underline{/}}$ have an eigenvalue equal to a unit, it is essential to require that this value be maximum.

Thus, the criterion of criticality can for formulated thus:

a) a system of interacting subcritical assemblies is critical only in case the maximum eigenvalue of the matrix of interaction parameters is equal to a unit;

b) if $\lambda_{max} > 1$, then the system is supercritical;

c) if $\lambda_{max} < 1$, then the system is subcritical.

In a certain sense, λ_{max} is a measure of criticality. For example, in a supercritical system for which $\lambda_{max} = 2$ a two-fold reduction of all the interaction parameters makes the system critical.

The degree of the criticality of a system is equal to α if, having changed all the interaction parameters by the factor $1/\alpha$ we will obtain a critical system. The magnitude α is determined by the maximum eigenvalue of the matrix of interaction parameters λ_{max}.

Previously, in part 1, the criterion of criticality (5.23) was presented in the form of

$$D \equiv \det |Q - I| = 0,$$

since I is the unit matrix of the n order. This condition is equivalent to the requirement that Q have an eigenvalue of $\lambda = 1$. If, however, $\lambda = 1$ is not the greatest eigenvalue, then the determinant can be equal to zero, and the system to supercriticality.

We will assume that $\underline{/Q\underline{/}}$ has an eigenvalue of $\lambda = 2$ and $\lambda = 1$ (generally speaking) has n of eigenvalues of the matrix. Since $\lambda = 1$, $D = 0$ and if condition (5.23) is accurate, the system would be critical. However, $\lambda = 2$ is also an eigenvalue and therefore

$$\det |Q - 2I| = 0. \qquad\qquad (5.32)$$

This equation is accurate, provided each element of the determinant is reduced by a half so that

$$\det \left[\frac{1}{2} Q - I \right] = 0. \qquad (5.33)$$

In accordance with condition (5.23), the system is still critical, although all the interaction parameters have been halved. Thus, the equation (5.23) is not an adequate condition for the criticality of a system. The eigenvector $\underline{/F\_/}$ corresponding to $\lambda = 1$ has positive and negative terms, which do not correspond physically to the realized system in which all the fluxes are positive.

Neutron Multiplication in an Interacting System Located in a Nonabsorbent Environment. An External Neutron Source.

Let the surface multiplication factor M_{ij} not depend on the neutron spectrum. Then $M_{ij} = M_i = M_s$ is the surface multiplication factor of uniform assemblies.

We will consider a situation in which not a single neutron from a source escapes from the system, does not fall into any assembly. We will designate the source by the vector $\underline{/S\_/} = \left\{ S_1, S_2, \ldots, S_n \right\}$, since S_i is the proportion which has fallen into the assembly i. Then

$$\sum_{i=1}^{n} S_i = 1, \qquad (5.34)$$

and the matrix equation for neutron fluxes in a stationary state is written in the form of

$$[F] = [Q][F] + \begin{bmatrix} M_1 & S_1 \\ M_2 & S_2 \\ & \vdots \\ M_n & S_n \end{bmatrix}. \qquad (5.35)$$

Then, it is possible to determine for the given source $\underline{/S\_/}$ and the matrix $\underline{/Q\_/}$ the neutron fluxes in the system of assemblies:

$$[F] = [I - Q]^{-1} \begin{bmatrix} M_1 & S_1 \\ M_2 & S_2 \\ & \vdots \\ M_n & S_n \end{bmatrix}. \qquad (5.36)$$

After computing the fluxes in the system, the number of neutrons escaping from the system can be calculated.

The number of neutrons escaping from the assembly j and falling into other assemblies is equal to

$$\sum_i P_{ij} F_j.$$

The number of neutrons from j escaping from the system is equal to

$$F_j - F_j \sum_i P_{ij}.$$

The total number of neutrons escaping from the system is equal to

$$\sum_j \left(F_j - \sum_i P_{ij} F_j \right) = \sum_j F_j - \sum_i \sum_j P_{ij} F_j. \qquad (5.37)$$

From the equation (5.35) we have

$$F_i = \sum_j q_{ij} F_j + M_i S_i = M_i \sum_j P_{ij} F_j + M_i S_i,$$

так что
So that

$$\sum_j P_{ij} F_j = \frac{1}{M_i} (F_i - M_i S_i).$$

Having placed this value in the expression (5.37), we will obtain, for the number of neutrons escaping from the system,

$$\sum_i F_j - \sum_i \frac{1}{M_i} (F_i - M_i S_i) = \sum_i \left(1 - \frac{1}{M_i} \right) F_i + 1,$$

Since
так как $\sum_i S_i = 1.$

Thus, when all the neutrons of the source fall into an assembly, the multiplication M_A of neutrons in the system is equal to

$$M_A = 1 + \sum_{i=1}^{n} \left(1 - \frac{1}{M_i} \right) F_i. \qquad (5.38)$$

The multiplication M_A can be obtained following the determination of the magnitude $\underline{/F\underline{/}}$ of the equation (5.36) and the placement of this magnitude in the expression (5.38).

If the proportion (1 - P) of neutrons of a source escapes from the system without interacting with assemblies, the multiplication of neutrons in such a system M_{AP} will be equal to

$$M_{AP} = P\left[1 + \sum_{i=1}^{n}\left(1 - \frac{1}{M_i}\right)F_i\right] + (1 + P);$$

$$M_{AP} = 1 + P\sum_{i=1}^{n}\left(1 - \frac{1}{M_i}\right)F_i. \qquad (5.39)$$

The exact calculation of the value P is quite difficult; however, reasonable evaluations for nuclear safety can be made through the computation of solid angles.

Multiplication in Systems of Uniform Assemblies M_A

M_A in a system of uniform assemblies with surface multiplication M_S can be computed without resorting to the evaluation $\left[F\right]$ of the equation (5.36).

We will again consider a system of uniform assemblies. For it, $\lambda_{max}[F] = [Q] \cdot [F]$ *with* $\lambda_{max} < [$ We will add $(1 - \lambda_{max})[F]$ to the left and to the right, $\left[F\right]$ is the eigenvalue corresponding to λ_{max}. It is then possible to write

$$[F] = [Q][F] + (1 - \lambda_{макс})[F]. \qquad (5.40)$$

This is the equation for neutron flux in a subcritical system of uniform assemblies with source $\left[S\right]$ equal to

$$M[S] = (1 - \lambda_{макс})[F]. \qquad (5.41)$$

Так как $\sum_i S_i = 1$, то $\sum F_i = \dfrac{M}{1 - \lambda_{макс}}$.
Since

Its replacement in equation (5.38) yields

$$M_A = 1 + \frac{M-1}{M} \cdot \frac{M}{1 - \lambda_{макс}} = \frac{M - \lambda_{макс}}{1 - \lambda_{макс}}$$

$$\frac{M-1}{M_A - 1} = 1 - \lambda_{макс}. \qquad (5.42)$$

н.пи

Surface system multiplication M_A, which depends on assembly surface

multiplication M_s and the degree of the criticality of the system, is useful for evaluating reflector effect. When the proportion R of neutrons escaping from a system are returned to it via the reflector and distributed among the assemblies as in the equation (5.41), the reflected system will be subcritical, critical, or supercritical if $M_A R$ is less than 1, equal to 1, or more than 1, respectively.

If the neutrons returning to a system are distributed in a manner differing from that of the equation (5.41), the effect of the reflector can be evaluated by considering it as an additional assembly in the system. Returning to an evaluation of system multiplication in a situation in which of the neutrons of a source escape from it, we have

$$M_{AP} = PM_A + (1 - P),$$
$$M_{AP} - 1 = P(M_A - 1).$$

Applying the equation (5.42), we obtain $\dfrac{M-1}{M_{AP}-1} = \dfrac{1-\lambda_{\text{макс}}}{P}$.   (5.43)

From this equation, if follows that a multiplication system of uniform assemblies $(M > 1)$ may be less than individual assembly multiplication.

M_{AP} of a system is less than M, provided $P < 1 - \lambda_{max} (\lambda_{max} < 1 - P)$. Thus, when the proportion of neutrons of a source – – the proportion which passes through the system of individual assemblies without interaction – – is greater than the degree of criticality, system multiplication is less than individual assembly multiplication. The above-described matrix criterion of criticality of a system of interacting assemblies is necessary and adequate for evaluating the criticality of the interaction parameter method. The criterion of growth in evaluating the degree of criticality of the system also makes possible the computation of neutron multiplication in a system enclosed in a nonabsorbent environment with a source of neutrons.

The following is exceptional for a system composed of uniform assemblies: multiplication is determined by individual assembly multiplication, the proportion of neutrons of a source not interacting with the system, and the degree of the subcriticality of the system.

5. Application of the Criticality Matrix Criterion to the Computation of Spheres Deployed in the Air / 52 /

We will assume that, for a system of uniform spheres deployed in the air, neutron emission from each sphere is distributed evenly along the surface of spheres, that the distribution angle of neutron emission at each point of a sphere is isotropic, and that the surface multiplication of each sphere is uniform and does not depend on the energy and distribution angle of the falling neutrons.

For such a system P_{ij} is equal to the solid angle (standardized at 2π) subtended by the i sphere on the surface of the j sphere. Then

$$q_{ij} = M_i \Omega_{ij}.$$

Approximation 1. An analytical evaluation of Ω_{ij} is difficult
in the general case of the partial screening of the i sphere from the j
sphere by the sphere spaced between them. For the purposes of simplification,
partial screening can be disregarded. When three spheres are arranged
in a single line, the spheres on the two ends do not interact with each other
directly because of the complete screening provided by the center sphere,
and the angle between them is equal to zero. The average solid angle
between the two spheres of radius r at d distance between their centers
depends only on the relationship of r/d (this relationship is shown in
Figure 5.18 by the unbroken line).

Approximation 2. It is assumed that the average solid angle
between the two spheres of radius r at d distance between their centers is
changed by the $\Omega = K (r/d)^2$-law, where K is a certain constant. The
magnitude K is determined for each lattice in such a way that the solid
angle between the two closest spheres has an exact value, and the solid
angles between other pairs of spheres are oversized. If we will now
designate the spacing of the lattice, i.e., the distance between the
centers of the proximate spheres via d and the distance betwwen the centers
of the i and j spheres via d_{ij}, then having applied the second approximation,
we can write

$$\Omega_{ij} = K \left(\frac{r}{d_{ij}} \right)^2 = K \left(\frac{r}{d} \right)^2 \left(\frac{d}{d_{ij}} \right), \quad (5.44)$$

or

$$\Omega_{ij} = \Omega_0 \left(\frac{d}{d_{ij}} \right)^2, \quad (5.45)$$

is the average solid angle between the proximate spheres.
[The relationship between Ω_0 and the ratio r/d is shown in Figure 5.18
(Ω_0 and Ω_1.] After measuring the coordinates of the lattice points in
units d, we will obtain the i sphere coordinates x_i, y_i, and z_i. In
this manner,

$$\Omega_{ij} = \Omega_0 a_{ij}, \quad (5.46)$$

where

$$a_{ij} = \frac{1}{[(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2]}$$

or

$$a_{ij} = 0$$

$$\Big\} \quad (5.47)$$

depending on whether or not there is a complete screen between the spheres.

The interaction parameters can be written in the form of

$$q_{ij} = M\Omega_0 a_{ij} = q_0 a_{ij},\qquad\qquad (5.48)$$

since $M\Omega_0 = q_0$ is the interaction parameter between the two proximate spheres. Each interaction parameter d_{ij} is simply proportional to the interaction parameter q_0 and the proportionality factor depends only on the relative position of the i and j spheres.

Figure 5.14

The number of critical spheres n in a cubic lattice in accordance with the interaction parameter
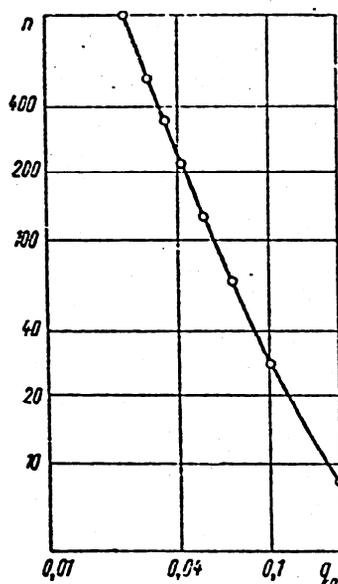
The Criticality Criterion Expressed via the Interaction Parameter.

When all the interaction parameters are known, it is an easy matter to evaluate the state of an interacting system. It is least difficult to compute the magnitude q_0 at which a given system appears critical.

We will assume that λ'_{max} is the greatest eigenvalue of the matrix $[A]$ with the elements a_{ij}. Since $q_{ij} = q_0 a_{ij}$ then the matrix of the interaction parameter $[Q] = q_0 [A]$ and the greatest eigenvalue is, consequently,

The previously written criticality criterion yeilds

$$q_0 = \frac{1}{\lambda_{макс}}.$$ (5.49)

It is possible to write matrix $\underline{/A\underline{/}}$ for any system and then calculate the magnitude q_0, at which the system is critical.

Number of Critical Spheres in Accordance with the Density of a System and the Surface Multiplication of Each Sphere.

A calculation of a matrix for a series of interacting systems - - a calculation made on an electronic computer - - has been presented in another work $\underline{/52\underline{/}}$. For a system with fixed dimensions and shape, the magnitude $q_0 = M\Omega_0$ at which a system is critical can be obtained from graphs (see Figures 5.15 and 5.16).

When the surface multiplication factor of a sphere is known, the determination of the magnitude Ω_0 is easy. Ω_0 yields immediately the ratio r/d, i.e., the density of the system, since Ω_0 is the solid angle between spheres at distance d (see Figure 5.18).

Figure 5.15
The number of critical
spheres n in a square
lattice in relation to the
interaction parameter
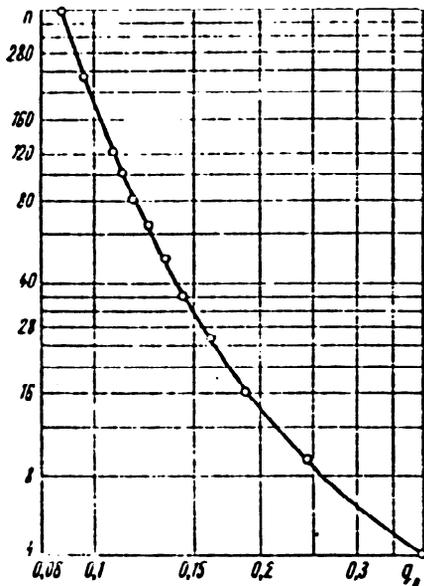
Figure 5.16
The number of
critical spheres
n in relation to the
density of the cubic
lattice r/d.
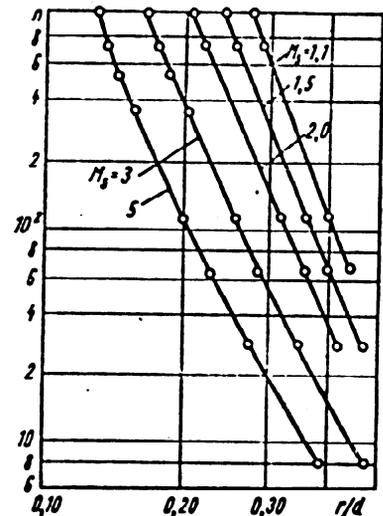
Minimal Multiplication M_o at Which a System Is Critical.

The criticality condition of a system is

$$q_0 = M\Omega_0 = 1/\lambda_{макс.} \qquad (5.50)$$

The maximum solid angle of contacting spheres is equal to $\Omega_0 \leqslant 0.8$. Since

$$\Omega_0 \leqslant 0.08, \quad \frac{q_0}{M} \leqslant 0.08,$$

consequently,

$$M \gg q_0/0.08. \qquad (5.51)$$

For a system of spheres, $M_0 = q_0/0.08$ is the lowest value of M_s at which a system is critical. As regards a cubic lattice, the magnitude M_0 must be less than a unit for a system of at least 48 spheres.

6. A Consideration of the Hypotheses Made during the Computation of Interaction

In analyzing interacting systems, the following five basic hypotheses are employed in all the methods of computing interaction.

1. Neutron emission changes along the surface of a sphere.

In the formulation of the equation, it has been assumed that neutron emission along the surface of a sphere is uniform, which, however, is not always so. This lack of uniformity is a result of the dimensions of a sphere and the surface multiplication M_s of a sphere.

Neutrons falling into a small sphere whose dimensions are comparable with the free-path length of a neutron usually escape from the opposite side of a sphere (forward scattering). With an increase in sphere dimensions, an increasingly large proportion of neutrons begin to escape from the sphere's side on which they had initially fallen; and for large spheres whose diameter is much greater than the free-path length of a neutron, backward scattering predominates.

When surface multiplication becomes great, neutron emission along a surface is more uniform. Spheres found at the edge of a system can return, depending on the size-shape factors and the magnitude of surface multiplication, to the system a number of neutrons discernibly different from that number which would be returned to a system on the basis of the hypothesis of uniform neutron emission along the surface of a sphere. Thus, the computation overstates the reactivity of a system composed of small spheres

and can understate the reactivity of a system composed of large spheres.

2. <u>Angular distribution of neutron emission</u>. It is reasonable to assume that neutron emission is isotropic at all the points of the surface of a sphere. Then, all the interaction parameters are simply proportional to the average solid angle. It is impossible to say definitely that the angular distribution of neutrons escaping from a sphere constitute the maximum proportion P, i.e., the number of neutrons escaping to another sphere. If such a distribution exists, it will extend in a radial direction (Figure 5.17, a).

Figure 5.17

Angular distribution of neutrons escaping from the surface of a sphere: a is distribution with escape in a radial direction, b is the uni-directional escape along the radius.
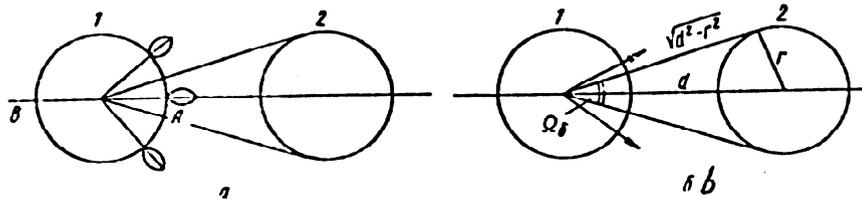
In this situation, a large number of neutrons escaping from field A of sphere 1 fall into sphere 2, while neutrons escaping from sphere 1 in field B are less likely to fall into sphere 2. Usually, the solid angle is averaged according to all the points of both fields, and the opposite effects try to compensate for each other and have little influence on the magnitude of the effective average solid angle

When angular distribution is directed completely along a radius, the magnitude Ω is determined by the solid angle between the center of one sphere and the surface of the other (see Figure 5.17, b):

$$\Omega = \Omega_s \frac{2\pi \left[1 - \sqrt{\frac{d^2 - r^2}{d^2}} \right]}{4\pi} = \frac{1}{2} \left[1 - \sqrt{\left(1 - \frac{r^2}{d^2} \right)} \right] .$$

A comparison of Ω_s and the average solid angle Ω indicates that the use of Ω in place of Ω_δ yields a more reactive system, i.e., a more intense interaction.

In Table 5.4, the magnitude K_1 is selected in such a manner that Ω_1 is the accurate average solid angle between the contacting spheres

the value $\frac{r}{d}$ is exact for a distance when $\frac{r}{d} = 0.4$.

3. **Surface multiplication of a sphere.** The surface multiplication of each sphere in a system is assumed to be uniform. In a real system, M_s

depends not only on the nature and size of the spheres themselves, but also on the spectrum and angular distribution of the falling neutrons and, consequently, is changed with the position of a sphere in the system. In order to be on the safe side, it is reasonable to choose the surface multiplication factor maximally realized in a system.

Table 5.4

A Comparison of Solid Angles

r/d	Ω	$\Omega_1 = K_1 (r/d)^2$	$\Omega_2 = K_2 (r/d)^2$	Ω_b
0,50	0,080	0,080	—	0,067
0,45	0,060	0,065	—	0,054
0,40	0,015	0,051	0,015	0,042
0,35	0,033	0,039	0,031	0,032
0,30	0,024	0,029	0,025	0,023
0,25	0,016	0,020	0,018	0,016
0,20	0,010	0,013	0,011	0,010
0,15	0,005 (7)	0,007 (2)	0,006 (3)	0,005 (6)
0,10	0,002 (5)	0,003 (2)	0,002 (8)	0,002 (3)
0,05	0,0006 (25)	0,0008	0,0007	0,0006

4. **The hypothesis of partial screening.** The solid angle Ω_{ij} is evaluated by proceeding from the hypothesis that a partial screening of spheres by other spheres does not exist. Because of this, the solid angles assumed in computations are frequently greater than those in a real system. The magnitude of partial screening is reduced with an increase in the distance between spheres. A disregard of partial screening leads to an overestimation of some interaction parameters and, consequently, to an overestimation of the reactivity of a computed system.

5. **Calculation of solid angles.** The hypothesis to the effect that solid angles are changed inversely with the square of the distance between assemblies yields safe results. By an appropriate choice, the value K of the equation $\Omega = K (r/d)^2$ is made precise for proximate spheres (at distance d from each other).

For spheres spaced at great distances, the solid angle $\Omega = K (r/d)^2$ overstates the real solid angles.

In Figure 5.18, Ω is the exact solid angle, Ω_{l} is computed according to the equation $\Omega_{l} = K_{l}(r/d)^{2}$ since the value of K_1 is chosen in such a

manner that Ω_{l} is the exact angle for spheres touching each other $(r/d = 0.5)$

The solid angles are $\Omega_{2} = K_{2}(r/d)^{2}$ since the value of K_2 is chosen in such a

manner that the solid angle is exact for spheres with $\frac{r}{d} = 0.4$.

Of the hypotheses considered above, the first obviously yields a "safe" result for systems composed of small spheres and a dangerous result for systems composed of large spheres. The second hypothesis insignificantly effects the evaluation of criticality, and the third-fifth hypotheses yield "safe" results, i.e., they overstate system reactivity.

Figure 5.18

Change of the solid angle Ω according to the lattice density r/d: 1 is $\Omega_{l}=K_{l}(r/d)^{2}$, 2 is $\Omega_{2}=K_{2}(r/d)^{2}$; 3 is the exact solid angle.
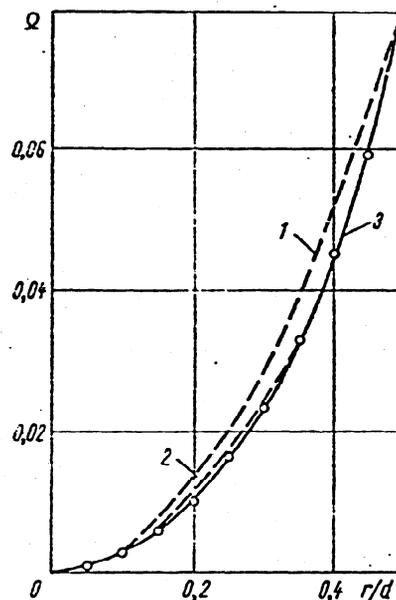
7. Comparison of the Solid Angle and the Interaction Parameter Methods

A comparison of the solid angle and the interaction parameter methods makes clear that the solid angle Ω and the multiplication factor K $_{eff}$

correspond to the fall probability P (which is equal to Ω for unreflected systems in the air) and the surface multiplication factor M_s. It is of interest to obtain simple and reliable correlations between M_s and K_{eff}.

Upper limit of the magnitudes M_s and $\mathcal{P}$ expressed through the multiplication factor K_{eff}

It is known that neutron multiplication in a system is equal to $1/(1 - K_{eff})$. Using the designation of section B, it is possible to write that the proportion of fast neutrons $(1 - U_f)$ leaks from the system, and the proportion U_f remains in the system. In the thermal range, the proportion of neutrons $U_f U_t$ is captured in the fuel core and the proportion of neutrons $U_f (1 - U_t)$ escape from the system. The total number of neutrons escaping from the system is one generation is equal to

$$(1 - U_f) + U_f(1 - U_t) = 1 - U_f U_t = 1 - \frac{K_{\text{эфф}}}{\nu\theta} .$$

Thus the number of neutrons escaping from a sphere per one neutron falling into it, with the presence of the eigenfunction of neutron distribution, is equal to

$$\overline{M} = \frac{1 - (K_{\text{эфф}}/\nu\theta)}{1 - K_{\text{эфф}}} . \tag{5.52}$$

Let an isotropic neutron source be enclosed in a sphere. Then the number of neutrons escaping from the sphere per one source neutron is equal to M_c , provided the source is in the center, and $\dfrac{M_s + 1}{2}$ provided the source is on the surface. Under these conditions, the average multiplication M satisfies the inequality

$$M_c \geqslant \overline{M} \geqslant \frac{1}{2}(M_s + 1). \tag{5.53}$$

Отсюда
Hence

$$M_s \leqslant \frac{1 + \left[1 - \left(\frac{2}{\nu\theta} \right) \right] K_{\text{эфф}}}{1 - K_{\text{эфф}}} . \tag{5 54}$$

For assemblies with highly enriched uranium $\nu\theta \approx 2$, $M_s \leqslant \dfrac{1}{1 - K_{eff}}$. In a general situation,

$$M_s \leqslant \frac{1 + \left[1 - \frac{2}{K_\infty} \right] K_{\text{эфф}}}{1 - K_{\text{эфф}}} .$$

- 143-

For assemblies in the air, the fall probability P is equal to the solid angle Ω, and then

$$q \leqslant \frac{1 + \left[1 - \frac{2}{K_\infty}\right] K_{эфф}}{1 - K_{эфф}} \cdot \frac{\Omega}{4\pi} = M_s P. \qquad (5.55)$$

Examples. 1) We will consider two uniform assemblies containing a solution of uranium with $\ell_H/\ell_s = 93.75$. This magnitude has been chosen on the basis of the condition that $\nu\theta = 2$ and $q \leqslant \frac{1}{1 - K_{eff}} \cdot \frac{\Omega}{4\pi}$ In a critical state $q = 1$ and the solid angle $\frac{\Omega}{4\pi} \gg 1 - K_{eff}$.

Curve 2 in Figure 5.9 in which the solid angle is equal to $1 - K_{eff}$ is the line of safety for solutions with $\nu\theta = 2$ $\left(\frac{\ell_H}{\ell_s} \approx 100\right)$ critical and subcritical systems lie above and below curve 2, respectively. Experimental points lie above curve 2 and confirm its "safety."

2) We will consider two nonuniform assemblies with the values $K_{eff} - K_1$ (type 1) and K_2 (type 2) and express the solid angle by the areas of the cross section of the assemblies (A_1 and A_2) divided by

$$\Omega_1 = \frac{A_1}{4\pi d_{12}^2}, \quad \Omega_2 = \frac{A_2}{4\pi d_{12}^2}.$$

On the basis of the expressions (5.21) and (5.54), it follows that the two nonuniform containers are safe at the distance d_{12} , which satisfies the inequality

$$\frac{1 + \left[1 - \left(\frac{2}{\nu_1\theta_1}\right)\right] K_1}{1 - K_1} \cdot \frac{A_1}{4\pi d_{12}^2} + \frac{1 + \left[1 - \left(\frac{2}{\nu_2\theta_2}\right)\right] K_2}{1 - K_2} \cdot \frac{A_2}{4\pi d_{12}^2} < 1.$$

For subcritical systems composed of two uniform assemblies of type 1 or type 2, the corresponding magnitudes be less than a unit, and the corresponding distances d_1 and d_2 must satisfy the inequality

$$\frac{1 + \left[1 - \left(\frac{2}{\nu_i\theta_i}\right)\right] K_i}{1 - K_i} \cdot \frac{A_i}{4\pi d_i^2} < 1 \; \textit{for} \; i = 1, 2.$$

- 144 -

Hence, $d\frac{2}{12} > d_1 d_2$, i.e., the distance between nonuniform assemblies

must be greater than the average geometric distance. The empirical rule presented previously requires that distance d_{12} be greater than the average arithmetical distance. It is known that the arithmetical average is always greater than the geometric, and the empirical rule is more conservative.

D. Homogenization Method

1. Description of the Method

The search for a purely empirical macroscopic method of computing interaction has led to the creation of the so-called analog density method or, more precisely, the homogenization method $\underline{/}$54, 55$\underline{/}$.

The method is based on the assumed similitude of a relationship between critical mass and density of homogeneous assemblies and a relationship between critical mass and porosity (density) of spatial heterogeneous systems composed of subcritical assemblies deployed in air. An attractive characteristic of homogeneous reactors is the relationship between critical mass and density, which is described by the relationship $m_{cr} \sim (\rho)^{-s}$ (ρ is the density of the fuel core, s is a constant which does not change in a broad range of densities. For example, s = 2 for unreflected assemblies, s = 1.2 for assemblies with a natural uranium reflector). Figure 5.19 shows the behavior of s with a change of the reflecting capability of a reflector. The indicator of the degree of s aims at its limit, which is equal to 2, as the reduction of reflector effectiveness proceeds.

Figure 5.19

Indicator of the degree s for reflected, metallic spheres ($R_{cr.r}$ is the critical radius of the reflected sphere, R_{cr} is the critical radius of an unreflected sphere): $\square$ is uranium of 93.5 percent enrichment, O is Pu^{239} in α-phase.
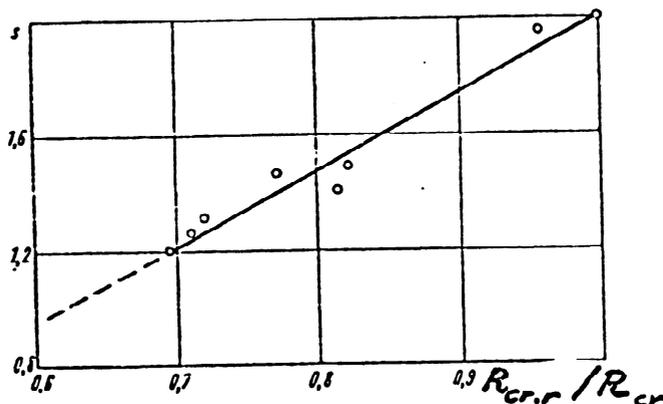
Like a homogeneous situation, the critical data on the entire series of cubic systems of subcritical assemblies with various distances separating the assemblies (which corresponds to a different density) demonstrate that the magnitude s is constant within a broad range of system density (porosity).

As s = 2 for an unreflected homogeneous cube, the analog s aims at the value 2 for a cubic heterogeneous system of very small, subcritical, unreflected assemblies. Thus, the method provides an opportunity for the reasonable extrapolation of experimental data on heterogeneous systems. For determining a series of systems, the degree s is a convenient intermediate parameter which, analogous to a homogeneous situation, is reduced commensurately with the increase in the effectiveness of the system reflector and depends intensely on the degree of the subcriticality of individual assemblies composing the interacting system.

<div align="center">

Figure 5.20

Critical systems composed of metallic uranium spheres.
The enrichment of uranium is 93.5 percent:
O is a 20-kg sphere in a cubic lattice with a concrete reflector; $\square$ is a
32-kg sphere in a square lattice on a plane, without a reflector; $\bullet,\blacksquare,\blacktriangle$
are the more reliable points.

</div>
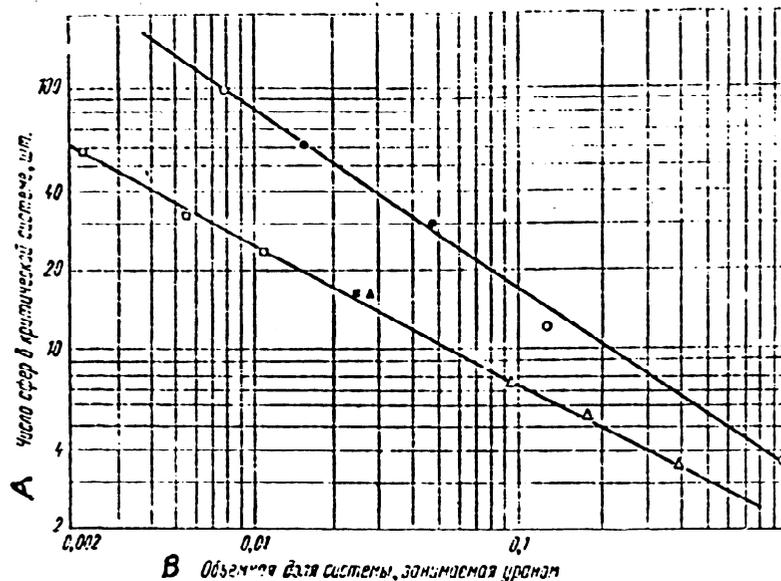
Legend: A = Number of spheres in the critical system, units; B = System's volumetric proportion of uranium.

A work $\underline{/50\_7}$ described a series of experiments with a system composed of metallic spheres of 93.5 percent enrichment with U^{235}. The weight of the spheres amounted to 20 kg. Twenty-seven spheres were arrayed in a cubic lattice within cubic concrete boxes with a size of 1.5, 1.2, and 0.9 meters. The thickness of a wall was 30 cm. The critical charges were obtained by extrapolation, and errors in their determination constituted about ten percent. The experimental results are presented in Table 5.5.

Table 5.5

Critical Quantity of Twenty-Kilogram Metallic Spheres in
a Cubic Lattice
(the uranium is of 93.5 percent enrichment)

A Сторона бетонного кубического бокса, м	B Расстояние между центрами сфер, см	C Плотность решетки, сфер/м³	D Количество сфер, шт.	
			E сферы без отражателя	F две полусферы рядом
1,5	50,8	7,66	99	—
1,22	40,5	14,9	61	73
0,915	28	45,9	30	45
0,61	20	119	12	16
3,05	Вплотную	907	3,6	—

Legend: A = Side of the concrete, cubic box, in meters; B = Distance separating the centers of the spheres, in centimeters; C = Density of the lattice, in spheres per cubic meter; D = Number of spheres, in units; E = Unreflected spheres; F = Two hemispheres in a row; 1 = Contact

In all the cases, the dimensions of a box exceeded the dimensions of the lattice by three times. The data of the table and the results of experimentation with 32-kg metallic spheres located on a plate at a height of 25 cm from the concrete floor are shown graphically in figure 5.20. Figure 5.21 present the results of experiments with metallic plates $\underline{/55\_7}$. Along the axis of the abscissa is presented the porosity of the system P, i.e., the system's volumetric proportion of metallic uranium; along the axis of the ordinate is presented the number $n_{c/}$ of subcritical metallic assemblies composing the critical system at an assigned porosity. From th··e data is also chosen the analog s which will be applied to a system of an··.ogous or other assemblies. The negative inclination of straight lines in the logarithmic coordinates determine s, since

- 147 -

Figure 5.21
Critical systems composed of
metallic uranium plates. Uranium
enrichment is 93 percent. The
dimensions of a plate are 2.54 x
20.32 x 25.4 cm; the weight is
24.5 kg: ▲ , △ = no moderator
between plates; 0 = plexiglass
moderator of 2.54-cm thickness
between plates, 0;▲ = critical
experiments, △ = curve extrapolation
of inverse multiplication;
n_{cr} = number of assemblies
in the critical system, F = system
porosity.
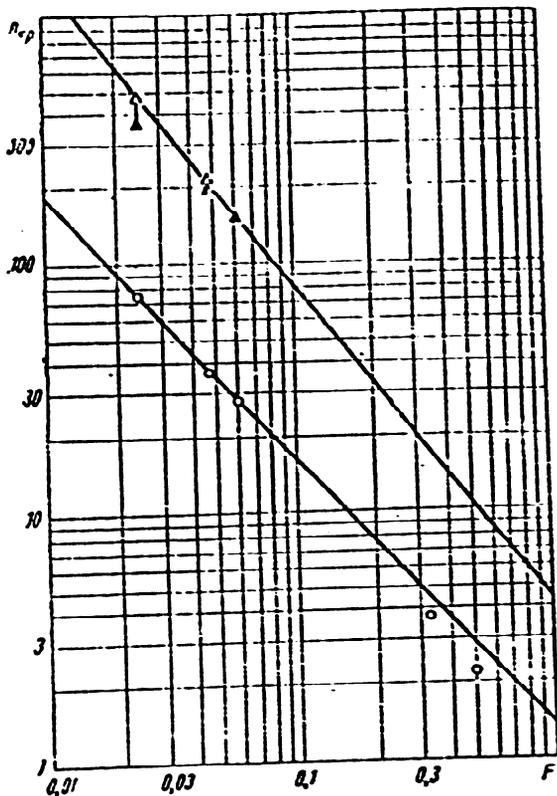
Figure 5.22
Indicator of the degree s in the relationship
between volumetric proportion and the un-
reflected critical sphere f:
0 = concrete reflected sphere, ◇ = partially
reflected system, ▯ = unreflected system, 1 = 20-
kg sphere of 93 percent enriched uranium, 2 =
32-kg sphere of 93 percent enriched uranium,
3 = 24.5-kg uranium plate with a size of 2.54
x 20.32 x 25.4. Enrichment of uranium is 93
percent. 4 = assembly of 5-kg uranium pieces
with a porosity of 40 percent and a total
weight of 30 kg. Enrichment of uranium is
93 percent; 5 = cylinder of 15.24-cm diameter
with a solution with $\frac{\ell_H}{\ell_S} = 44$; 6 = cylinder of

20.32-cm diameter with a solution with $\frac{\ell_H}{\ell_S} = 44$.

7 = cylinder of 22.32 cm with a solution with

$\frac{\ell_H}{\ell_S} = 307$. 8 = cylinder of 24.13-cm diameter with

$\frac{\ell_H}{\ell_S} = 297$. 9 = 3 plates of 7.62-cm thickness with

solution.
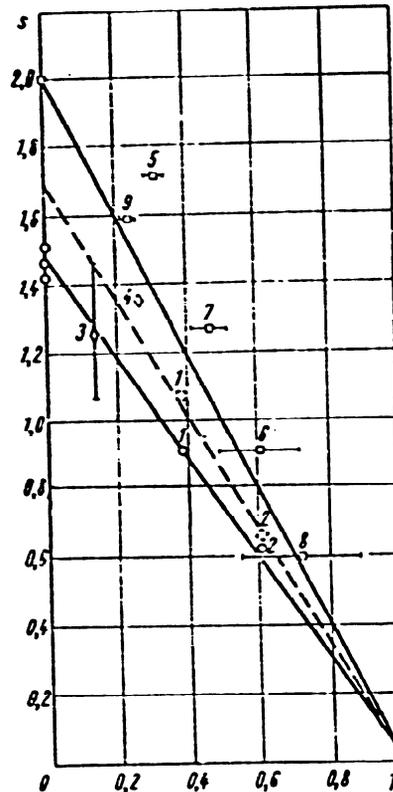
When experimental data are known for a single porosity (at one assigned distance between assemblies), the straight line for determining s can be drawn through this known point, so that this line will intersect the abscissis with F = 1.

The figures make evident that s depends on the presence of a reflector and the reactivity (multiplication factor) of a system's individual assemblies. In accordance with reflector effectiveness, as was previously pointed out, s is changed from 1.2 to 2.

Subsequently, it is essential to find the relationship between s and the reactivity of isolated assemblies composing the system. The most convenient magnitude to which s has been successfully connected is the proportion f, weight or volumetric, constituted by an individual isolated assembly, compared with the critical assembly of this same material, i.e., $f = m_{i.cr}/m_{cr}$, where $m_{i.cr}$ is the mass of an isolated assembly composing the system, and is the mass of an unreflected critical assembly of this same material.

For a spherical assembly, f is the ratio of the mass of the assembly to the mass of the critical spherical assembly of the same material. For an assembly having another shape, the mass of the sphere equivalent in reactivity to the real assembly is assumed to be the real mass. Obviously, the magnitude λ is a certain measure of the reactivity of an assembly. Figure 5.22 shows the relationship between the magnitudes s and f. The relationship is the generalization of experimental data on the criticality of systems of assemblies.

An evaluation of the number of critical, uniform assemblies with mass m in a cubic lattice having a common, complete reflector is made in the following manner.

Determined is f of an isolated assembly; this is followed, with a consideration of f, by the establishment of the value s along the bottom straight line on Figure 5.22. The critical mass m_0 of the material of the spherical assembly is computed when it is closely surrounded by the lattice reflector. The value of the desired porosity (lattice density) F is established, i.e, the ratio of the volume of the assembly to the volume of the cell found in one assembly. The mass of the critical lattice is determined: $M = m_0 F^{-s}$ (the value s has been determined above). The number of critical assemblies with mass m is

$$n_{\text{кр}} = \frac{m_0 F^{-s}}{m}.$$

(5.56)

- 149 -

In a majority of cases in practice, the critical mass of fissile materials required to evaluate f and m_0 can be found in this handbook.

An example. To compute the number of critical, metallic, 20-kg spheres of U^{235} laid in an accurate cubic lattice with a general concrete reflector.

We will determine f :

$$f = \frac{m}{m_{\text{кр}}} = \frac{20 \; \kappa 2}{48 \; \kappa 2} = 0,417,$$

where m_{cr} is the critical mass of metallic uranium without a reflector. Then, Figure 5.22 makes evident that $S \approx 0.85$.

The critical mass of U^{235} with a concrete reflector is equal to 22 kg, if one takes into account the fact that a concrete reflector is no more effective than an aqueous one.

We will compute a system with the porosity $P = 0.1$ and $P = 0.01$:

$$M = m_6 F^{-s} = 22 \cdot 0{,}1^{-0.85} = 156$$

$$n_{\text{кр}} = \frac{156}{20} = 7{,}8 \; \text{сферы при пористости} \; F = 0{,}1:$$

$$M = 22 \cdot 0{,}01^{-0.85} \approx 1080 \; kg;$$

$$n_{\text{кр}}(F = 0{,}01) = \frac{1080}{20} = 54 \; spheres.$$

For such a system lacking a concrete reflector, we obtain

$$M = m_0 \cdot F^{-s} = 48 \cdot 0{,}1^{-0.85} = 340 \; kg;$$

$$n_{\text{кр}}(F = 0{,}1) = \frac{340}{20} = 17 \; spheres.$$

$$M = 48 \cdot 0{,}01^{-0.85} = 2360 \; kg;$$

$$n_{\text{кр}}(F = 0{,}01) = \frac{2360}{20} = 118 \; spheres$$

At this point we have only considered systems with assemblies placed in air. Experiments indicate that the presence of hydrogen-containing materials between metallic assemblies in a system sharply reduce the number of critical assemblies. Figure 5.21 presents the results of introducing plexiglass plates of 2.5-cm thickness in a space between intersecting metallic plates. The bottom curve on the figure attests to the fact that the introduction of such plates reduces the number of critical assemblies by five or ten times. It should be noted that plexiglass of ?.5-cm thickness has nearly a maximum effectiveness (a subsequent increase of thickness has very little effect on the change of

the quantity of the plates), and it is more effective between metallic plates than between assemblies. With thickness of water or plexiglass of over 5 cm, system reactivity begins to drop, and with a thickness of about 20 cm interaction almost completely ceases.

2. Comparison of the Results of Experiments and Computations

Table 5.6 presents the results of experiments with 6-liter assemblies in an unreflected, spatial lattice and the results of computations by the interaction parameter method in matrix form - - computations made by homogenization and equivalent dimensions methods (for data on the deployment of the 6-liter containers, see Table 5.1).

Table 5.6

The Number of Critical Assemblies n_{cr} in an Unreflected Cubic Space Lattice
(experiment and computation)

A Пористость решетки	B Эксперимент [1]	C Расчет методом эквивалентных размеров	D Расчет методом параметра взаимодействия в матричной форме		E Расчет методом гомогенизации
			M_s	n	
0,373	23	12	3 2,5 2	10 15 20	15,5
0,329	39,5∓0,3	12	3 2,5 2	12 19 28	28
0,278	54∓0,5	18	3 2,5 2	14 23 30	24
0,256	72∓5	27	3 2,5 2	18 25 34	32
0,235	80∓0,5	36	3 2,5 2	20 25 40	36

Legend: A = Lattice porosity, B = Experiment, C = Computation by the equivalent dimensions method, D = Computation by the interaction parameter method in matrix form, E = Computation by the homogenization method.

Since the equation (5.54) yields the upper range of the magnitude M_s, results with several values of M_s are presented for the purpose of indicating the relationship between n_{cr} and the surface multiplication factor M_s. One can see from the equation that the three methods yield a reserve of $\sim$ 2 in favor of nuclear safety.

Figure 5.23

Unreflected critical systems of cylinders containing aqueous solution of $UO_2(NO_3)_2$ salts. The enrichment of uranium is 92.6 percent, concentration is 410 g/l. The cylinders are made of polyethylene of 13.65 cm-diameter. Volume is 12 *l*:

Δ is the triangular lattice, the height of the solution in a cylinder is 112.4 cm, volume is 12.76 *l*.

Square lattice

Height of solution in a cylinder, cm	Volume, *l*
◘ 56.5	5.9
▲ 84.8	9.3
▣ 112.4	12.76

O is the height of a solution in a cylinder ~ ~ 112.4 cm, volume is 12.76 l. The cylinders are placed at 2 levels, the distance between ends of the cylinders is 14.2 cm. The graph shows the number of cylinders at one level, i.e., half of the total number.

A = Number of cylinders in the system, units; B = Distance between the surfaces of the cylinders, cm.
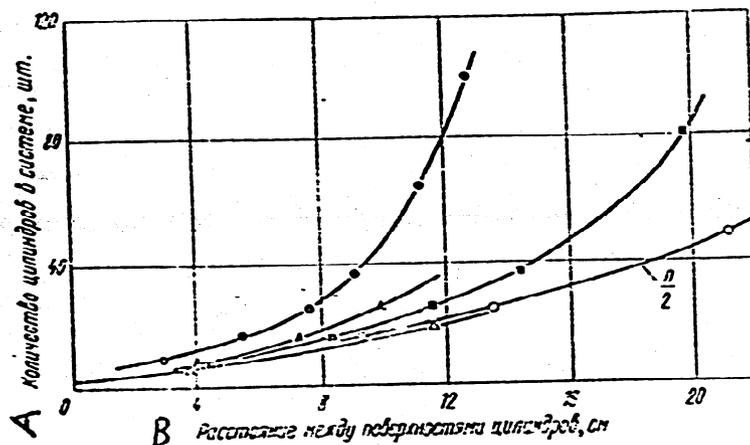
Figure 5.24

Interaction of 32-kg metallic spheres drawn up in a linear or flat lattice.
Uranium enrichment is 93 percent:
1 = flat lattice, the distance between the centers of the spheres is 40.6,
50.8, and 76.2 cm; 0 = the distance between the centers of the spheres,
63.5 cm; *, +, constitute the linear lattice, the distance between the
centers of the spheres is 40.6 cm.
A = Inverse multiplication, B = Number of spheres.
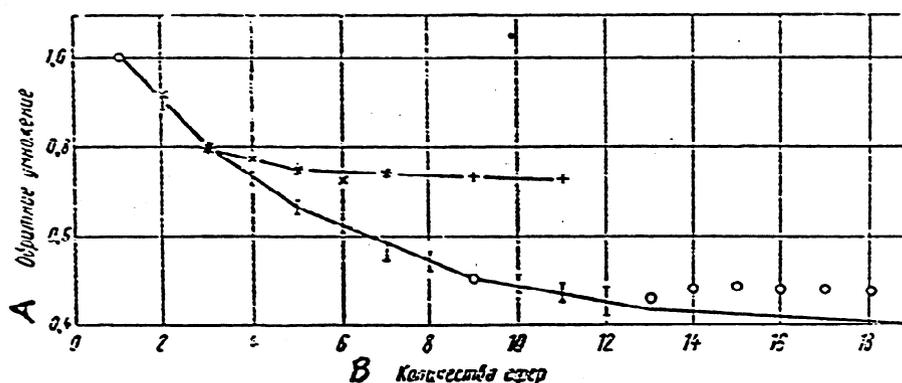
B. Some Experimental Results

Figure 5.23 points out the results of experiments of polyethylene
cylinders filled with an aqueous solution of salt of $UO_2(NO_3)_2$ $\underline{/}3\underline{/}$.

Figure 5.24 shows the inverse multiplication curves in accordance with the
number of 32-kg metallic spheres in a linear or flat system $\underline{/}55\underline{/}$. The
values of the effective multiplication factors for aqueous solution of salt
of $UO_2(NO_3)_2$ and UO_2F_2 - - values which can be useful in computing interaction - -
are presented in Table 5.7 $\underline{/}49\underline{/}$.

Figure 5.25

Critical height of aqueous solutions of UO_2F_2 in aluminum cylinders of 15-cm diameter in accordance with the distance between cylinders. The enrichment of uranium is

a = seven cylinders in a hexagonal lattice: 1 = cadmium-coated cylinder placed in water, 2 = cadmium-coated cylinder placed in air, 3 = cadmiumless cylinder placed in air, b = hexagonal and triangular lattices: 4 = seven cadmium-coated cylinders placed in water, 5 = three cylinders in water, 6 = seven cylinders in water, 7 = solution level in one isolated cylinder. A = Critical height, cm; B = Distance between cylinders, cm.
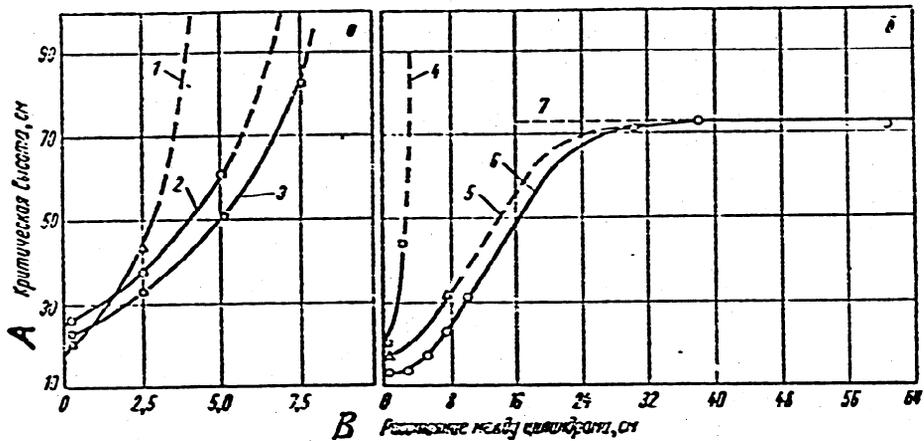
Figure 5.26

Critical height of an aqueous solution of UO_2F_2 in unreflected
aluminum cylinders in accordance with the distance between cylinders. The
enrichment of uranium is 93.2 percent.

1 = three cylinders in a triangular lattice (diameter of a cylinder is
20 cm), 2 = seven cylinders in a hexagonal lattice ($\triangle$ is the 15-cm diameter
of a cylinder, $\square$ is the 20-cm diameter of a cylinder).
A = Critical height, cm; B = Distance between cylinders, cm.
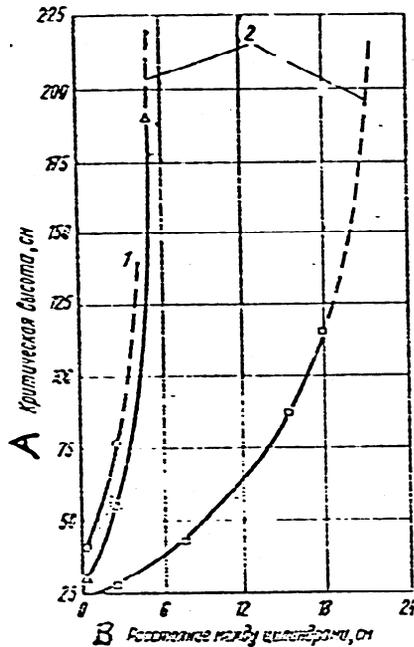
Experimental results $/\overline{8}/$ have been published on the criticality of
systems of aluminum cylinders filled with an aqueous solution of UO_2F_2 salt,
with uranium of 93.2 percent enrichment. The atomic ratio in the solution
is $\ell_H/\ell_S = 44.3$. The axes of the cylinders are parallel. The thickness of a
wall of the cylinders is 0.16 cm.

The experimenters summarized the results in the following manner.
It was established that a hexagonal system of seven (one in the center)
aqueously reflected, cadmium-coated cylinders of 15.2-cm diameter remain
subcritical at any solution height, provided the inside distance separating
the cylinders was no less than 5 cm. For a similar system of unreflected
cylinders of 20.3-cm diameter, a distance of 46 cm between cylinders was
necessary. Three unreflected cylinders of 20.3-cm diameter situated at
the apexes of an equilateral triangle were safe, provided the distance
between them was no less than 23 cm. An aqueously reflected, cadmium-
coated cylinder of 20.3-cm diameter became critical at an approximate height
of 62 cm.

Table 5.7

The Value of $K_{эф}$ for Unreflected Cylinders Containing Aqueous
Solutions of UO_2F_2 (uranium enrichment is 93 percent) and Aqueous Solutions
of $UO_2(NO_3)_2$ (uranium enrichment is 90 percent)

UO₂ F₂					UO₂ (NO₃)₂				
D, см	H, см	$\rho_{H:P_5}$	U^{235}, кг/л	$K_{эф}$	K_∞	τ, см²	L^2, см²	U^{235}, кг/л	$K_{эф}$
12,7	30,5	50	0,48	0,52	1,871	49,14	0,377	0,412	0,530
12,7	61	50	0,48	0,55	1,871	49,14	0,377	0,412	0,556
12,7	∞	50	0,48	0,57	1,871	49,14	0,377	0,412	0,569
12,7	30,5	150	0,17	0,47	1,846	33,30	0,801	0,140	0,468
12,7	61	150	0,17	0,50	1,846	33,30	0,801	0,140	0,493
12,7	∞	150	0,17	0,52	1,846	33,30	0,801	0,140	0,506
12,7	30,5	350	0,075	0,39	1,714	30,64	1,331	0,666	0,442
12,7	61	350	0,075	0,42	1,714	30,64	1,331	0,666	0,467
12,7	∞	350	0,075	0,43	1,714	30,64	1,331	0,666	0,478
20,32	15,24	50	0,48	0,68	1,871	49,14	0,377	0,412	0,653
20,32	15,24	150	0,17	0,63	1,846	33,30	0,801	0,140	0,587
20,32	30,5	150	0,17	0,81	1,846	33,30	0,801	0,140	0,725
20,32	15,24	350	0,075	0,54	1,714	30,64	1,331	0,666	0,559
20,32	30,5	350	0,075	0,70	1,714	30,64	1,331	0,666	0,691
20,32	61	350	0,075	0,77	1,714	30,64	1,331	0,666	0,749
30,5	15,24	150	0,170	0,83	1,846	33,30	0,801	0,140	0,738
30,5	15,24	350	0,075	0,72	1,714	30,64	1,331	0,666	0,704

Cylinders of 15.2-cm diameter arrayed in a line in water were effectively isolated from one another at a distance between them of no less than 30 cm. Two unreflected cylinders of 20.3-cm diameter remained subcritical at every solution height, provided the inside distance separating them was no less than 7.5 cm. When three cylinders of 20.3-cm diameter were arrayed according to an isosceles triangle, the total critical height remained constant, provided the angle at the apex exceeded 90°. It was established that this relationship between interaction and the angle is substantially greater in an unreflected system. In these experiments, the systems had no upper reflector.

The experimental results are presented in Figures 5.25 and 5.26.

THE BASIC NORMS FOR ENSURING NUCLEAR SAFETY

Pages 214-222

<u>Kriticheskiye Parametry Sistem</u>
<u>s Delyashchimisya Veshchestvami</u>
<u>i Yadernaya Bezopasnost'</u>

On the basis of the experimental data on criticality presented in this handbook, some basic generalizations can be made. The significance of these generalizations narrows down to the determination of the maximum permissible charges and volumes of fissionable materials, as well as the determination of the safe dimensions of systems and the distances between them. The maximum permissible charges of isolated homogeneous systems are determined with a consideration of a reserve factor somewhat exceeding 2; permissible volumes have a reserve factor of 1.3. The presented norms have been taken from other works $\boxed{4, 63, 64}$ and are only tentative. The possibility of accidental submersion of an entire consignment, as well as the merging of the shipment contained in two consignments, is permitted.

1. Basic Norms for Isolated Systems

Table 7.1

Permissible Masses, in Kilograms, for Isolated, Aqueously
Reflected Systems

x	A Металл, $\frac{\rho_H}{\rho_x} \leqslant 2$	B Смеси, $\frac{\rho_H}{\rho_x} \leqslant 20$	C Растворы	
			$\frac{\rho_H}{\rho_x} \leqslant 100$	$\frac{\rho_H}{\rho_x} \leqslant 2500$
U^{235}	11,0	2,5	0,8	0,35
Pu^{239}	2,6	2,2	0,5	0,25
U^{233}	3,0	1,3	0,48	0,25

Legend: A = Metal, B = Mixtures, C = Solutions

Table 7.2

Permissible Volumes, in Liters, of Isolated, Aqueously Reflected
Homogeneous Systems

x	A Растворы		
	$20 \leqslant \frac{\rho_H}{\rho_x}$	$400 \leqslant \frac{\rho_H}{\rho_x}$	$800 \leqslant \frac{\rho_H}{\rho_x}$
U^{235}	4,8	9,5	20,0
Pu^{239}	3,3	6,8	11,4
U^{233}	2,0	6,0	12,0

Legend: A = Solutions

Table 7.3

Safe Diameters of Cylinders of Aqueously Reflected, Isolated Systems, in Centimeters

x	A Металл при максимальной плотности	B Растворы		
		$20 \leqslant \dfrac{\rho_H}{\rho_x}$	$400 \leqslant \dfrac{\rho_H}{\rho_x}$	$800 \leqslant \dfrac{\rho_H}{\rho_x}$
U235	6,35	12,7	17,5	23,1
Pu239	3,55	11,4	15,5	18,7
U233	3,60	9,4	14,7	18,8

Legend: A = Metal at maximum density, B = Solutions.

Table 7.4

Safe Thicknesses of Plates for Aqueously Reflected, Isolated Systems, in Centimeters

x	A Металл при максимальной плотности	B Растворы		
		$20 \leqslant \dfrac{\rho_H}{\rho_x}$	$400 \leqslant \dfrac{\rho_H}{\rho_x}$	$800 \leqslant \dfrac{\rho_H}{\rho_x}$
U235	1,78	3,6	6,3	10,0
Pu239	0,50	3,8	6,3	8,3
U233	0,50	1,2	4,8	7,3

Legend: A = Metal at maximum density, B = Solutions.

Table 7.5

Maximum Permissible Values of the Height and Diameter of Cylinders for the salts and Aqueous Solutions of U^{235} (aqueous reflector, enrichment and any density of uranium of no more than 3.2 kilograms per liter)

A Диаметр, см	B Высота, см	A Диаметр, см	B Высота, см	A Диаметр, см	B Высота, см
12,7	∞	20,3	15,7	30,5	8,4
14,0	85,6	22,9	12,7	38,1	6,6
15,2	44,7	25,4	10,7	91,4	4,3
17,8	21,6	27,9	9,4	∞	3,8

Legend: A = Diameter, cm, B = Height, cm.

Table 7.6

Maximum Permissible Values of Mass, Volume, Safe Diameter of
Cylinders, and Safe Thickness of Plates for the Salts and Aqueous Solutions
of U^{235}.

(homogeneous systems, any ϱ_{H}/ϱ_{s})

A Обогащение урана, %	B Масса U235, кг	C Объём, л	D Безопасный диаметр цилиндра, см	E Безопасная толщина пластины, см
100	0,350	4,8	12,7	3,8
75	0,360	5,0	13,2	4,1
50	0,390	6,0	14,5	4,8
49	0,410	6,7	15,2	5,1
30	0,440	7,7	16,0	5,6
20	0,480	9,5	17,5	6,9
15	0,520	11,0	18,8	7,9
12	0,583	12,5	19,8	8,6
10	0,600	14,0	20,8	9,1
8,0	0,650	16,0	22,1	9,9
6,0	0,710	20,5	24,4	11,4
5,0	0,800	27,0	26,0	12,7
4,0	0,930	33,8	28,4	14,0
3,5	1,04	40,0	30,5	15,2
3,0	1,200	49,2	32,5	16,5
2,5	1,470	64,6	35,6	18,0
2,0	2,000	95,1	40,6	22,1
1,75	2,560	126,0	44,7	24,4
1,5	3,600	186,0	50,8	27,9
1,25	6,670	308,0	61,0	34,3
1,0	22,700	731,0	83,8	49,5
0,8	36,000	3917,0	147,3	91,4
0,72	∞	∞	∞	∞

Legend: A = Uranium enrichment, in percent; B = Mass U^{235}, kg.; C = Volume, 1; D = Safe diameter of a cylinder, cm; E = Safe thickness of a plate, cm.

Table 7.7

Maximum Permissible Values of Mass for the Salts and Aqueous
Solutions of U^{235} in Accordance with the ℓ_H/ℓ_S Relationship
(aqueous reflector, any uranium enrichment, density of uranium of no less
than 3.2 kg/l)

$\frac{\rho_H}{\rho_S}$	A Масса U235, кг	$\frac{\rho_H}{\rho_S}$	A Масса U235, кг	$\frac{\rho_H}{\rho_S}$	A Масса U235, кг
0.01	43.0	4.0	11.5	40.0	1.34
0.1	39.8	5.0	9.5	50.0	1.10
0.5	33.6	8.0	6.3	75.0	0.80
1.0	28.5	10.0	5.0	100.0	0.65
1.5	24.2	15.0	3.3	200.0	0.41
2.0	20.6	20.0	2.5	Без ограничений B	0.35
3.0	14.8	30.0	1.7		

Legend: A = Mass of U^{235}, kg; B = No limitation.

Table 7.8

Maximum Permissible Values of Mass for Metallic Rods of U^{235} with
a Density of 18.9 Grams per Cubic Centimeter in Accordance with the Enrichment
of Uranium
(aqueous reflector)

A Обогащение U235, %	B Масса U235, кг	C Объем, л	A Обогащение U235, %	B Масса U235, кг	C Объем, л
100	11.0	0.582	20	31.9	8.44
90	11.0	0.647	15	38.5	13.6
75	12.6	0.689	10	51.6	27.3
50	17.0	1.8	8	60.0	39.7
40	19.8	2.62	6	72.6	64.0
30	24.0	4.23	<5	∞	∞

Legend: A = Enrichment of U^{235}, in percent; B = Mass of U^{235}, kg.;
C = Volume, l.

Table 7.9

Permissible Values of a Mass of U^{235} for Mixtures of UF_6 and Hydrogen-Containing Materials with ℓ_H/ℓ_{5} <10 (any reflector)

A Максимальная плотность урана, г/см³	$\dfrac{\rho_H}{\rho_5}$	B Безопасная масса U²³⁵, кг	A Максимальная плотность урана, г/см³	$\dfrac{\rho_H}{\rho_5}$	B Безопасная масса U²³⁵, кг
1.8	10	5.0	3.0	1	28.5
2.3	5	9.4	3.2	0.1	39.8
2.6	3	14.3	3.2	0.01	43.0
2.8	2	20.0			

Legend: A = Maximum uranium density, g/cm^3; B = Safe mass of U^{235}, kg.

Table 7.10

Permissible Values of a Mass of Metallic Uranium in Water
with Any Volumetric Ratio of Water to Uranium
(aqueous reflector, blocks or pieces of uranium placed at random in a container with water)

A Обогащение урана, %	B Масса U²³⁵, кг	A Обогащение урана, %	B Масса U²³⁵, кг
3.0	1.04	0.9	12.5
2.0	1.36	0.8	30.0
1.0	6.80		

Legend: A = Enrichment of uranium, in percent; B = Mass of U^{235}, kg.

2. Conditions Not Requiring Limitations

No limitations are required for masses and volumes of aqueous solutions and homogeneous mixtures provided that:

a)   which corresponds to a concentration of $U^{235} \leqslant$ g/l;

b)   which corresponds to a concentration of $Pu^{239} \leqslant 7.3$ g/l;

c)   which corresponds to a concentration of $U^{233} \leqslant 10.9$ g/l.

These values contain almost no reserve factor; therefore, it is essential either to reduce the above-mentioned concentrations by 10 percent or to control them exactly.

Any mass of natural or depleted uranium homogeneously distributed in common water is safe.

No limitations of uranium with $\dfrac{\rho_5}{\rho_8} \leqslant 0.05$ are required, provided the

uranium is in the form of a:

a) metal without hydrogen-containing admixtures,
b) chemical materials without hydrogen,
c) homogeneous mixtures with any element having the atomic number $Z > 13$, provided that $\rho_Z/\rho_5 \leqslant 100$.

3. Conditions Requiring Special Consideration

The basic norms are not applicable to such systems as mixtures of fissile materials containing heavy water, beryllium, and graphite (in which the ratios ρ_D/ρ_5, ρ_{Be}/ρ_5 or ρ_C/ρ_5 are equal to about 100) or to systems with a thick reflector made of these materials, or natural uranium, or tungsten.

The norms are also inapplicable in case the densities of fissile materials are higher than normal densities. If the density of the fissile materials is greater than the density of γ_0 (Figure 7.1), the permissible charges must be multiplied by $(\gamma_0/\gamma)^2$ the volume by $(\gamma_0/\gamma)^3$, and the linear dimensions of a system by γ_0/γ.

Figure 7.1

The Relationship between the maximum densities of Pu^{239} (1), and U^{233} (2), and U^{235} (3) and the atomic ratio of
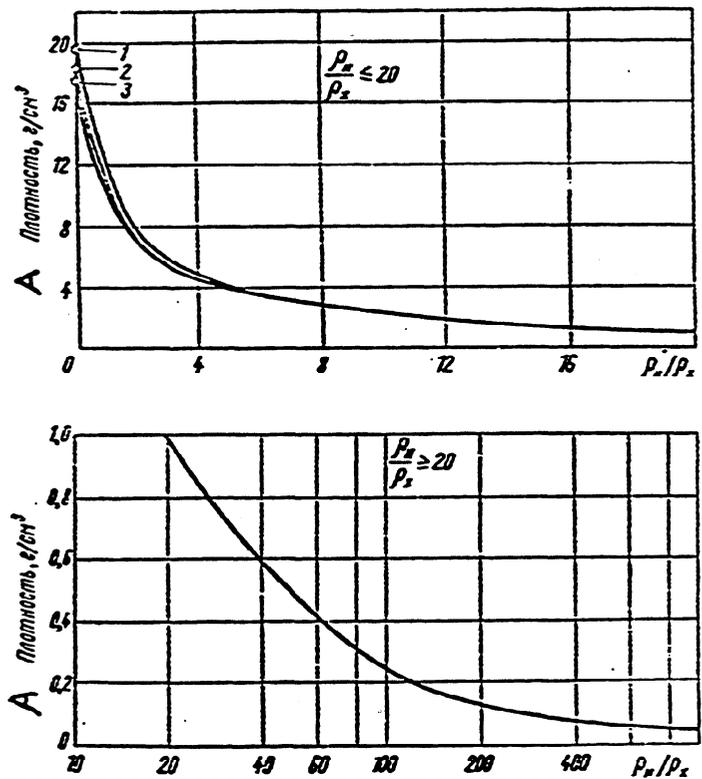
Legend: A - Density, g/cm^3.

4. Conditions Permitting an Increase of the Basic Norms

Table 7.11 presents the factors of a form for metallic, aqueously reflected U^{235} rods of cylindrical configuration with any uranium enrichment. Figure 7.2 indicates the factor K_ρ by which it is necessary to multiply the permissible values of the masses (see Table 7.1) of U^{235}, U^{233}, and Pu^{239} in metallic form, provided their density γ_x is less than the normal γ_0. The normal

densities of metals in g/cm^3 are as follows: U^{235} - - 17.6, Pu^{239} - - 19.6, and U^{233} - - 18.3. The factor K_ρ is applied only for aqueously reflected metals.

Table 7.11

The Factor of Form K for Metallic U^{235} Rods of Cylindrical Configuration
(water reflector, any enrichment)

H/D	K	H/D	K	H/D	K
4,0	1,515	1,5	1,085	0,3	1,220
3,5	1,435	1,0	1,010	0,2	1,435
3,0	1,350	0,8	1,0	0,15	1,650
2,5	1,255	0,6	1,025	0,10	2,030
2,0	1,170	0,4	1,115		

Note. Multiplied by the factor of the form are the magnitudes of the permissible masses presented in Table 7.8; H/D is the ratio of the height of a cylinder to its diameter.

Figure 7.3 shows the factor K_x ; when the permissible masses of fissionable materials (see Table 7.1) are multiplied by this factor they can be increased, provided the fissile materials are homogeneously mixed with some dilutions. This factor cannot be applied if the dilututions are hydrogen, deuterium, or beryllium. In Figure 7.3, γ_x is the density of a fissile material mixed with a dilution and γ_0 is the normal density of metallic U^{235}, U^{233}, and Pu^{239}. Curve 1 is applied when any element from sodium to bismuth is the diluent; curve 2 is applied when the diluents are carbon, nitrogen, oxygen, and the elements from sodium to bismuth, provided that for one atom of a fissile material there are no more than seven atoms of a diluent $(UC, UO_2, U_3O_8, UO_3, UO_2F_2, UF_4, UF_6)$.

Figure 7.2

The relationship between the factor K_p and the ratio of the densities γ_x/γ_0.
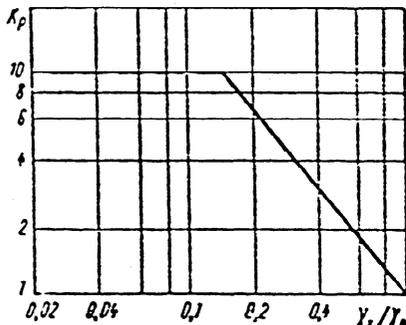
Figure 7.3

The relationship between the factor K_x and the ratio of the densities γ_x/γ_0.
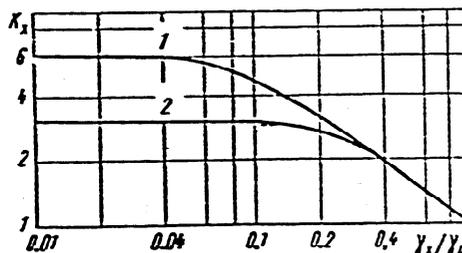
5. Some Norms for Interacting Assemblies·

Tables 7.12 and 7.13 present the norms for storing and transporting fissionable materials built up from the assemblies described in Table 7.14. The norms shown in Table 7.12 are applied for assemblies deployed in a concrete basement, whose lowest linear size is about three meters. The lattices of such assembles — — lattices which are safe in a concrete basement — — will also be safe in basements constructed of such materials as steel, wood, and earth.

Table 7.12

Norms of Fissile Materials during Storage
(the lattices are composed of the assemblies described in Table 7.14)

Тип решетки A	Минимальное расстояние между центрами соседних сборок в решетке [a], см B	Количество сборок в решетке, шт. C
1 Изолированная линейная или плоская решетка	⩾ 40	Без ограничений 4
2 Изолированная кубическая решетка	91,5 76 61 51	200 120 80 50
3 Две связанные плоские решетки	76 61 51	120 (всего 240) [б] b 90 (всего 180) [б] b 5 50 (всего 100) [б] b

Legend: A = Type of Lattice;   B = Maximum distance separating the centers of proximate assemblies in lattice [a], in cm,;   C = Number of assemblies in a lattice, in units.   1 = Isolated linear or flat lattice, 2 = Isolated cubic lattice, 3 = Two connected flat lattices, 4 = Without limitations, 5 — — words and numbers in the parentheses mean (in all 240), etc,   a = Distance separating the walls must be no less than 30 cm,   b = Such a total number is the limit for more than two connected lattices.

Each assembly must be composed of a number of smaller assemblies, provided the total dimensions determined in Table 7.14 are not exceeded. The lattices shown in Table 7.12 will be safe upon submersion in water, provided that the distances between the walls of individual assemblies constitute no less than 30 centimeters and that no more than ten percent of the volume of a separate assembly can be occupied by water.

Two lattices are isolated when they are separated by a concrete wall of about 20-centimeter thickness. Two flat or cubic lattices are also considered to be isolated if the distance separating the walls of any two assemblies of the various lattices is greater than the maximum size of one lattice or is 365 cm.

Two linear lattices can also be considered to be isolated, their length notwithstanding, if the distance between them is about 365 centimeters. Nonisolated flat lattices are considered to be connected if the minimal distance separating the walls of two assemblies from various lattices constitutes at least 230 centimeters.

Table 7.13 presents the norms for shipping assemblies of the fissile materials listed in Table 7.14. The maximum density permitted by a limiting form or shipping container is obtained with the deployment of the assembly in the center of a limiting cubic form or container of no less than a 51-cm side. The data in Table 7.14 is based on the hypothesis that the entirety of the forms and containers for shipping, as well as insulation from the fall of water inside the assembly, is maintained during shipping.

Table 7.13

Norms of Fissile Materials for the Shipment of Assembles Described in Table 7.14

A Состав делящихся веществ	B Максимальная плотность, допускаемая ограничительным каркасом или транспортировочным контейнером [a]			C Предельное количество делящихся веществ в партии (50 сборок, за исключением безопасных контейнеров [б])		
	U²³⁵	Pu²³⁹	U²³³	U²³⁵	Pu²³⁹	U²³³
1 Металлы и смеси при $\frac{CH}{Cx} < 2$; предел массы	142 кг/м³	35,5 кг/м³	35,5 кг/м³	925 кг/вагон [4]	225 кг/вагон [4]	225 кг/вагон [4]
2 Водородсодержащие смеси при $2 < \frac{CH}{Cx} < 20$; предел массы	35,5 кг/м³	35,5 кг/м³	17,7 кг/м³	225 кг/вагон [4]	225 кг/вагон [4]	125 кг/вагон [4]
3 Растворы или водородсодержащие смеси при $\frac{CH}{Cx} > 20$ в небезопасных контейнерах [в] . . .	28,4 л/м³	28,4 л/м³	14,2 л/м³	225 л/вагон [4]	225 л/вагон [4]	100 л/вагон [4]

Legend: A = Composition of fissile materials, B = Maximum density permitted by a limiting form or shipping container [a], C = Maximum number of fissile materials in a consignment (50 assemblies excepting safe containers [b]).
1 = Metals and mixtures with...the limit of the mass, 2 = Hydrogen-containing mixtures with...the limit of the mass, 3 = Solutions or hydrogen-containing mixtures with...in unsafe containers [c], 4 = freight car.

[a] The density is determined as the ratio $\frac{\text{of the mass (or volume) of the assembl}}{\text{the volume of the limiting form}}$

The limiting form or shipping container must provide a distance of no less than 30 cm between the sides of an assembly; the packed container must not admit water.

[b] During combined shipments, any combination of assemblies is possible, provided the number of units does not exceed 50.

[c] If assemblies in the form of solutions (see Table 7.14) are in safe cylinders, the maximum number of fissile materials during storage (see Table 7.12) can be retained during shipping, provided that the indicated distances are maintained during an accident.

Table 7.14

Maximum Parameters of Individual Assemblies [a]

Fissile materials makeup	U²³⁵	Pu²³⁹	U²³³
A Металлы и смеси при $\dfrac{\varrho_H}{\sigma_x} \leqslant 2$; максимальная масса, кг [б]	18,5 [в]	4,5 [г]	4,5
B Водородсодержащие смеси при $2 < \dfrac{\varrho_H}{\varrho_x} < 20$; максимальная масса, кг	4,5	4,5	2,5
C Растворы или водородсодержащие смеси при $\dfrac{\varrho_H}{\varrho_x} \geqslant 20$ в небезопасных контейнерах; максимальный объем, л [д]	4,0	4,0	2,0

Legend: A = Metals and mixtures with...maximum mass in kg [b], B = Hydrogen mixtures with...maximum mass in kg, C = Solutions or hydrogen-containing mixtures with...in unsafe containers; maximum volume in liters [e].

[a] When the density γ is greater than the value γ_0 in Figure 7.1, the maximum value of the mass is reduced by $(\gamma_0/\gamma)^2$, the volume by $(\gamma_0/\gamma)^3$.

[b] The volume of the packed material does not exceed 4.5 l.

[c] This corresponds to 20 kg of uranium of 93 percent enrichment.

[d] For a plutonium density of 19.6 g/cm³. With a plutonium density of 15.84 g/cm³, the maximum mass is 6 kg. [e] In case of the unsafe containers mentioned in Table 7.3, for solutions with $\varrho_H/\varrho_x \geqslant 20$, there is no limit on mass or volume.

Table 7.15

Minimal Safe Distances Separating Cylindrical
Containers Containing Dry Salts or Solutions of U^{235}
(the density of the uranium does not exceed 3.2 k/l)

A Цилиндр		B Безопасное обо-гащение урана, %	C Расстояние между контейнерами, см	
диаметр, см	высота, см		контейнеры расположены в линию	плоская квадратная решетка
1	2		3	4
12,7	70	100	38	45
12,7	120	100	45	60
20,0	120	12,5	105	137
25,0	120	5,9	120	150
30,0	100	3,75	120	150

Legend: A = Cylinder; B = Safe uranium enrichment, in percent; C = Distance separating containers, in cm. 1 = Diameter, in cm; 2 = height, in cm; 3 = containers arranged in line; 4 = flat square lattice.

Table 7.16

Minimal Permitted Distances Separating Containers of a Safe
Configuration - - Containers Containing Dry Salts or Solutions of U^{235}
(the density of the uranium does not exceed 3.2 kg/l)

A Контейнер	B $K_{эф}$	C Расстояние между двумя контейнерами, см	D Расстояние между двумя контейнерами, расположенными в линию, см	E Расстояние между контейнерами в квадратной решетке, см
1 Сфера объемом 4.8 л	0,71	30,5	30.5	30,5
2 Цилиндр диаметром 12,7 см	0,58	30,5	77,0	137,0
3 Пластина толщиной 3,5 см	0,31	30,5	215,0	—

Legend: A = Container; B = $K_{эф}$; C = Distance between two containers, in cm; D = Distance between two containers arranged in line, in cm; E = Distance separating containers in a square lattice, in cm. 1 = Sphere of 4.8-l volume, 2 = Cylinder of 12.7-cm diameter, 3 = Plate of 3.5-cm thickness.

Note. The infinite dimensions of the cylinders and plates are assumed to be 6 meters. The distance separating the containers is the distance separating the proximate walls.

Notes to Tables 7.15 and 7.16

1. Axes of the cylinders are parallel.

2. The cylinders are located vertically.

3. The centers of the containers lie on a plane.

4. When there are only two containers, a safe distance is 30 cm.

6. The Intersection of Pipes Conveying Solutions of Fissile Materials

Table 7.17 presents the safe intersections of pipes with aqueous solutions of U^{235}, U^{233}, and Pu^{239} salts. The tabular data is inapplicable to metals. When a pipe has several intersections, the distance between two axes of the proximate intersections must be no less than 45 cm.

Table 7.17

Permissible Internal Diameters of Pipes (in centimeters) for Intersections at an Angle of 90°

A Отражатель	U²³⁵	Pu²³⁹	U²³³
B Пересечения в виде буквы T			
1 Бесконечный водяной отражатель	8,9	8,1	6,6
a ≤2,5 см воды	10,4	10,0	8,4
b ≤3,0 см стали	11,9	12,2	10,0
C Пересечение в виде креста			
1 Бесконечный водяной отражатель	7,4	6,6	5,3
a ≤2,5 см воды	8,4	8,4	6,8
b ≤3,0 см стали	9,9	9,9	8,4

Legend: A = Reflector; B = T-shaped intersection; 1 = Infinite aqueous reflector; a = ...cm of water; b = ...of steel; C = Cross-shaped intersection.

- END -