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GENERAL APPLICATION OF A THEORY OF NEUTRON INTERACTION

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A B S T R A C T

Experimental data for critical systems of interacting containers have been compared with simple 2-group calculations in which the interaction average fractional solid angle between components is determined as a function of the multiplication factor, k_1, of each container in the array. The results reveal that useful accuracy can be obtained for all of the available experimental conditions, which include unreflected systems of cylinders and slabs containing high assay uranium solutions at hydrogen to U-235 atomic ratios varying from 44 to 337. The minimum value of k_1 for which a pair of containers can be made critical with a fractional solid angle of 8% between them was found to be 0.94, and that for a fractional solid angle of 24% was 0.86.

GENERAL APPLICATION OF A THEORY OF NEUTRON INTERACTION

INTRODUCTION

As a result of the fact that individually subcritical containers of fissionable material can become critical even though physically separated, the specification of adequate "safe" spacings for such containers is a continual problem for any organization handling fissionable materials, and a realistic method of evaluating the criticality hazards of various arrangements and nuclear fuel concentrations is an obvious necessity for efficient and safe operation. However, although many organizations throughout the atomic energy industry have established their own criteria for meeting their own problems, no generally standard methods for such determinations have been established. The methods actually used vary from comparatively rigid adherence to limited experimental data concerning a few special cases of particular interest to one installation, all the way to general determinations of spacing for large numbers of heterogeneous conditions based more or less closely on available experimental or theoretical information; for this latter case, practices vary from use of a standard spacing criterion based on some measurable parameter to individual consideration of each case as a special situation. For example, the Oak Ridge Gaseous Diffusion Plant follows the basic practice of requiring a minimum separation of 1 foot between individual components of a system of containers and also of so limiting the spacing between these components that the total fractional solid angle subtended at any component does not exceed 8%.[1]

In recognition of the potential problems posed by this "interaction", which may be rather loosely defined as the effect upon criticality of neutron exchange between physically separated containers, several workers, a fairly representative but not necessarily complete list of which is given,[2-7] have made calculations of critical conditions for systems of containers of fissionable materials. However, in general the methods employed are essentially applicable only to a limited number of conditions, the calculations themselves are rather difficult for specific situations of interest, or they do not readily lend themselves to the determination of criteria which may be used for a large variety of conditions. It may be noted that where efforts have been made to establish any general parameter as a basic criterion for spacing, the solid angle has usually been the one chosen although there have been some criteria set up on the basis of an "average" density.[8,9]

Recently Pond[10] has presented a method of calculating interaction which is an improvement on that developed earlier at the Oak Ridge Gaseous Diffusion Plant[3] and which has the basic advantage that it is relatively simple to calculate with the interaction solid angle, Ω, appearing as a function of the multiplication factor, k_1,* of one of the components of the system; thus, it is applicable to a wide variety of conditions, since k_1 is a rather well known characteristic of a reactor. In addition, the calculations, which use readily available nuclear constants, are relatively simple and the system can be simply

* In the literature, this factor is normally identified as k_{eff}.

extended to include rather complex geometrical arrangements of direct interest. The method does have the disadvantage, which is shared by most of the others, in that it is generally applicable only to bare and comparatively well-moderated systems.

This study was initiated to apply Pond's basic interaction methods, accompanied by modifications developed at the Oak Ridge Gaseous Diffusion Plant for more complex geometries, to available experimental data in order to see if basic generalized criteria for specifying spacing between individual containers might be developed for application to systems for which no experimental data are available.

SUMMARY

The validity of a general theory of interaction in which the critical interaction solid angle for a system of containers is developed as a function of the theoretical multiplication factor for one of the components of this system has been checked against available experimental data and found to be essentially valid for bare containers over a moderation range, defined by the H/U-235 atomic ratio, of 44 to 337; this range approximately includes the moderations of minimum geometry and minimum mass. From the available data, 0.94 is the minimum value of k_1 for which a pair of containers can be made critical with a fractional solid angle of 8% between them, and 0.86 is the corresponding value for a fractional solid angle of 24%. Where systems involve more than 2 containers, the consideration that the interaction solid angle at one component due to the other units is the simple sum of the contributory solid angles from these other components is a conservative factor.

A brief check of experimental data also revealed no case where neutron multiplication from a centrally placed neutron source was less than 2.0 for a container with a multiplication factor, k, of at least 0.90. It is recognized that these are qualitative measurements only.

CALCULATING METHODS

All calculations given in this report were done by methods of previous reports,[3,10] and extensions of these methods have been made to include other geometries. The calculations involve the use of a simple two-group theory giving the multiplication factor, k, in terms of quantities that are readily determined from the geometric buckling of the containers and readily available nuclear constants. The general formula is

$$k = n\, f\, U_t\, U_f$$

where n is the number of neutrons released per thermal neutron captured in the fuel, f is the thermal utilization or the probability that a thermal neutron capture is a capture by the fissionable material, U_t is the probability that a thermal neutron does not escape before being captured, and U_f is the probability that a fission neutron does not escape before becoming thermal; the values of the quantities used in this equation are readily obtained by the methods given by Pond.[10] This simple relation normally applies to unreflected and well-moderated solutions of high assay, the useful range certainly including H/U-235 atomic ratios from about 40 up to 2000, and U-235 assays above about 50%. Thus, it is particularly applicable to calculations for systems which may have a minimum critical mass or minimum critical geometry. Lower assays may be treated reasonably by including a factor for the U-238 resonance escape probability in the critical equation, and poorly moderated systems may also be calculated by using a reasonable fast reactor equation instead of the above expression.

The effective multiplication factor, K, for a system of identical containers is expressed in terms of the multiplication factor, k_1, of each container and the neutron interaction probability, V, by the relation

$$K = k_1(1 + pV)$$

where p is a constant determined by the configuration concerned and V is defined as

$$V = r\,\overline{\Omega}\,(1 - U_f).$$

In this expression for V, r is the ratio of the lateral (interacting surface) buckling to the total buckling, $\overline{\Omega}$ is the average geometric fractional solid angle between the containers, and $1 - U_f$ is the probability that a fast neutron will escape from its container. In other words, V represents the probability that a fast neutron escaping one container will strike the other container. The average fractional solid angles between identical cylinders as computed by Pond,[11] and those between identical slabs, as computed by Burton,[12] are shown graphically in the appendix.

It should be noted that the value of k_1 as used in this relation obviously depends upon the neutron flux distribution in the interacting containers; for comparatively widely spaced cylindrical containers, the value of k_1 for any unit very closely approximates the corresponding value for an isolated container of the same dimensions, but for closely spaced containers where nuclear interactions will alter the flux distribution, the effect of this perturbation may require compensation. This consideration is obviously of particular importance for systems involving interacting thin slabs, even at comparatively large spacings.

Since effects upon the flux distribution of these types are usually easily compensable by consistent variations in extrapolation length and since this is the calculation method usually employed in reactor determinations, the average extrapolation lengths used in making computations in this report were considered to depend upon the interaction solid angle by the empirical relation:

$$d = d_0 + (d_W - d_0)\,\overline{\Omega}$$

where d is the extrapolation length used, d_0 is the extrapolation length in air, d_W is the extrapolation length in the solution (which was considered equivalent to that in water), and $\overline{\Omega}$ is the average fractional solid angle of interaction. In this report, d_0 is taken as 2.5 cm. and d_W as 7.0 cm.[13] It may be noted that a method for correcting the extrapolation length in a special case of interaction consideration was earlier used by Macklin.[4]

As an extension of his method, Pond also gave relations for a straight line of equally spaced identical containers. In each expression, V is calculated using the value of $\overline{\Omega}$ between any 2 adjacent containers. The resulting expressions are shown in table 1.

TABLE 1

FORMULAS FOR A LINE OF EQUALLY-SPACED IDENTICAL CONTAINERS

Number of Containers in Line	Formula
2	$K = k_1 (1 + V)$
3	$K = k_1 (1 + \sqrt{2}\ V)$
4	$K = k_1 \left[1 + \frac{1 + \sqrt{5}}{2} V \right]$
5	$K = k_1 (1 + \sqrt{3}\ V)$

Since these methods of calculations are fairly simple and straight-forward, several expressions in addition to the above have been derived, using the same basic procedure. The derivations are given in the appendix, and the resulting expressions are shown in table 2.

TABLE 2

FORMULAS FOR VARIOUS GEOMETRIES OF EQUALLY-SPACED IDENTICAL CYLINDERS

Description of System	Formula
3 cylinders in equilateral triangle	$K = k_1 (1 + 2V)$
7 cylinders in a close hexagonal array	$K = k_1 \left[1 + (1 + \sqrt{7})V \right]$

Many situations of practical interest involve interaction between unlike containers. Following the same basic procedure as noted above, 2 expressions for systems of unlike containers have been derived. In these expressions, k_1 and k_2 are the respective multiplication factors of containers 1 and 2, while V_1 is the fraction of fast neutrons escaping from container 1 to strike container 2, and V_2 the fraction from container 2 striking container 1. The resulting expressions are given in table 3, and their derivations are included in the appendix. Obviously, obtaining the values of fractional solid angles between 2 containers of unlike geometry, and thus determining the correct values of V_1 and V_2, would require solid angle calculations for the differing systems similar to those developed for identical units as shown graphically in the appendix.

TABLE 3

FORMULAS FOR SYSTEMS WITH UNLIKE CONTAINERS

Geometry of Array	Formula
2 unlike containers	$K = \frac{k_1 + k_2 + \sqrt{(k_1 - k_2)^2 + 4\, k_1 k_2\, V_1 V_2}}{2}$
3 containers in line, end containers identical and center container different	$K = \frac{k_1 + k_2 + \sqrt{(k_1 - k_2)^2 + 8\, k_1 k_2\, V_1 V_2}}{2}$

RESULTS

In order to check the accuracy and range of usefulness of the expressions discussed in the previous section, the multiplication factor, K, has been calculated for various assemblies which have been experimentally determined to be just critical and thus obviously having an experimental value of unity. Experiments on the interaction between 2 identical cylinders have been reported by Callihan and others,[14] whereas unpublished data[15] include arrays of 3 to 7 identical cylinders in various geometrical arrangements. Fox and Gilley[16] give results for interacting slabs in groups of 2 or 3, some of which consist of unlike slabs in a given system. In these calculations, when values of K which are less than unity are obtained, it is indicated that the theory underestimates the possibility of criticality occurring; that is, criticality actually occurs at separations greater than those predicted for a given system. Thus, such calculations give what are called non-conservative values from the standpoint of nuclear safety. Similarly, results of calculations which give $K > 1$ are considered to be conservative.

Table 4 shows the results of calculations for pairs of identical cylinders, along with the multiplication factor, k_1, of the individual containers and the average fractional solid angle between the cylinders.* It may be noted that the maximum error in the calculations is about 3%, this difference occurring for the cylinders with the largest diameters and the smallest height, where the fractional experimental uncertainties are the largest and where the effect of experimental errors in critical height determinations has the greatest effect on the calculations. However, for spacings of 1 foot or greater, the error is no more than about 2%. In addition, it appears that variation of the H/U-235 atomic ratio of the solution in the experimental range considered has very little effect on the result.

* These results are taken directly from reference 10. Check calculations give slightly different results, but no consistent bias is indicated.

TABLE 4

COMPARISON OF CRITICAL EXPERIMENTS WITH CALCULATIONS

TWO IDENTICAL UNREFLECTED CYLINDERS

H/U-235	INCHES DIAMETER	CM. HEIGHT	CM. EDGE TO EDGE SEPARATION	MULTIPLICATION FACTOR OF SINGLE CYL. k_1	AVERAGE FRACTIONAL SOLID ANGLE Ω	MULTIPLICATION FACTOR OF SYSTEM K
44.3	8	68.6	0.4	0.9552	0.177	1.040
169	10	28.7	0.3	0.9526	0.166	1.0105
		30.7	2.3	0.9657	0.137	1.0150
		32.8	6.1	0.9824	0.104	1.0206
		34.3	9.9	0.9889	0.082	1.0197
		35.8	15.9	0.9961	0.060	1.0189
		37.2	24.9	1.0077	0.040	1.0232
		38.2	34.2	1.0108	0.0285	1.0218
		39.1	50.3	1.0137	0.0174	1.0205
	15	17.3	0.6	0.9738	0.140	0.9991
		17.8	5.3	0.9864	0.090	1.0031
		18.0	15.3	0.9908	0.045	0.9992
		18.3	50.3	1.0038	0.039	1.0111
	20	14.7	0.3	0.9611	0.132	0.9750
		14.8	5.3	0.9638	0.080	0.9723
		14.8	20.3	0.9638	0.031	0.9670
329	10	40.8	0.3	0.9317	0.172	0.9938
		44.9	2.2	0.9472	0.147	1.0013
		50.0	5.1	0.9574	0.122	1.0034
		54.8	8.3	0.9648	0.103	1.0044
		64.7	16.9	0.9826	0.072	1.0111
		74.4	31.6	0.9891	0.046	1.0073
		80.1	43.6	0.9923	0.038	1.0058
	15	20.1	0.5	0.9581	0.146	0.9858
		20.8	5.3	0.9781	0.096	0.9969
		21.0	10.0	0.9812	0.069	0.9948
		21.3	31.6	0.9862	0.0237	0.9909
		21.5	50.3	0.9893	0.0129	0.9919
	20	16.7	0.3	0.9526	0.136	0.9677
		17.0	10.3	0.9617	0.060	0.9685
		17.3	25.3	0.9756	0.0278	0.9788

Table 5 gives the results of calculations for somewhat more complex arrangements of identical cylinders which are stacked in equally spaced groups of 3 to 7 in various geometries. It may be noted that the largest errors in these calculations are for the closest spacings in the triangular and hexagonal arrays, these errors being as much as 7.5%, and in the non-conservative direction. For wide spacings, the results, in general, are conservative, and for the cylinders in a straight line, all of the results are conservative.

Table 6 shows calculations for various arrangements of slabs. It should be noted that in some of these tests, the "6-inch slab" actually consists of two 3-inch slabs in close contact. It is significant that the critical heights of the two 6-inch slabs, even at separations up to 5-1/2 ft., are less than the critical height for an isolated 6-inch slab, demonstrating that interaction may be a factor of concern over fairly large distances.

The results for the slabs are non-conservative, with the largest errors being for the closest spacings. The values of K calculated for the 3 identical 3-inch slabs in a line are in error by as much as 3.5%; however, if a constant extrapolation distance of 2.5 cm. is used, errors as large as 38% result for the 3 thin slabs. It may be noted that for thicker slabs, the use of the constant extrapolation distance of 2.5 cm. for close separations introduced a maximum error of only about 8% as compared with experiment, and this dropped to 3% for separation distances of about 1 foot.

For convenience in calculations of cylindrical containers, a constant value of the extrapolation length of 2.5 cm. was used; an estimate of the error thus introduced is indicated by the fact that for two 10-inch cylinders containing material at an H/U-235 atomic ratio of 169 and in contact, a difference of 2% in the values of k_1 was found by calculations using the constant and solid angle-dependent values of the extrapolation length; for a separation of 6 inches, the difference was less than 1%. It may be noted that use of the constant extrapolation length for air in these calculations normally gives lower, and thus more non-conservative, results than the solid angle-dependent value considered more nearly applicable. The fairly large discrepancy in the results for the closely-spaced hexagonal array of cylinders is very likely due, in part at least, to the use of a constant extrapolation distance in these calculations.

In addition to the experiments with unlike slabs, one measurement has been made for the critical parameters of an interacting 6-inch slab and 10-inch cylinder in contact.[15] Since exact values of the average fractional solid angles between these containers were not readily available, the calculations were based on somewhat arbitrarily determined solid angles, estimated to be 10% and 35%, and the value of 0.934 obtained for the effective multiplication factor of the system can be considered indicative only.

Figure 1, in which the critical solid angles are plotted as a function of k_1 summarizes the data from tables 4, 5, and 6. It should be noted that, in line with Oak Ridge Gaseous Diffusion Plant operational practices, the "total" fractional solid angle for systems of more than 2 containers as shown in this figure is determined by merely adding all of the solid angles which are subtended at the central container by all the surrounding containers instead of

TABLE 5

COMPARISON OF CRITICAL EXPERIMENTS WITH CALCULATIONS

UNREFLECTED MULTIPLE CYLINDER ARRAYS

H/U-235 = 44.3 DIAMETER = 8"

NUMBER IN LINE	INCHES HEIGHT	INCHES EDGE TO EDGE SEPARATION	MULTIPLICATION FACTOR OF SINGLE CYLINDER k_1	AV. FRACTIONAL SOLID ANGLE BETWEEN ADJACENT PAIRS $\overline{\Omega}$	MULTIPLICATION FACTOR OF SYSTEM K
			STRAIGHT LINE OF CYLINDERS		
3	18.2	0.15	0.9231	0.173	1.035
3	51.0	3.0	0.9796	0.114	1.060
4	16.5	0.15	0.9157	0.172	1.040
4	38.0	3.0	0.9644	0.111	1.052
5	15.8	0.15	0.9079	0.172	1.039
5	31.0	3.0	0.9622	0.108	1.053
			7 CYLINDERS IN HEXAGONAL ARRAY		
	7.2	0.15	0.7448	0.159	0.926
	10.1	2.0	0.8343	0.102	0.976
	13.1	4.0	0.8836	0.075	0.998
	16.4	6.0	0.9146	0.062	1.015
	22.0	9.0	0.9425	0.043	1.016
			3 CYLINDERS IN TRIANGULAR ARRAY		
	10.6	0.3	0.8444	0.159	0.9676
	13.8	1.0	0.8919	0.136	1.0078
	17.8	2.0	0.9237	0.116	1.0288
	22.0	3.0	0.9433	0.100	1.0372
	27.0	4.0	0.9572	0.092	1.0456
	42.0	6.0	0.9732	0.080	1.0542

TABLE 6

COMPARISON OF CRITICAL EXPERIMENTS WITH CALCULATIONS

UNREFLECTED SLAB ARRAYS

H/U-235 = 337

2 "IDENTICAL" UNREFLECTED SLABS, 6" THICK x 47.5" WIDE

INCHES EDGE TO EDGE SEPARATION	INCHES HEIGHT	MULTIPLICATION FACTOR OF SINGLE SLAB k_1	AVERAGE FRACTIONAL SOLID ANGLE $\overline{\Omega}$	MULTIPLICATION FACTOR OF SYSTEM K: VARIABLE EXTRAPOLATION LENGTH	MULTIPLICATION FACTOR OF SYSTEM K: CONSTANT EXTRAPOLATION LENGTH
2	10.2	0.8270	0.305	0.9912	0.9161
6	13.1	0.8835	0.205	1.0113	0.9504
15	17.7	0.9305	0.112	1.0087	0.9704
20	19.8	0.9434	0.092	1.0087	0.9770
30	23.5	0.9593	0.055	1.0005	0.9800
48	28.8	0.9730	0.036	1.0004	0.9869
66	32.5	0.9791	0.024	0.9978	0.9884

1 - 3" SLAB AND 1 - 6" SLAB UNREFLECTED, 47.5" WIDE

SEPARATION	HEIGHT	k_1(3")	k_2(6")	$\overline{\Omega}$	K (VARIABLE)	K (CONSTANT)
6	17.7	0.4801	0.9400	0.234	-	0.9527
12	22.9	0.4902	0.9666	0.163	-	0.9731
18	26.9	0.4946	0.9784	0.122	-	0.9822
30	32.7	0.4986	0.9891	0.072	-	0.9904

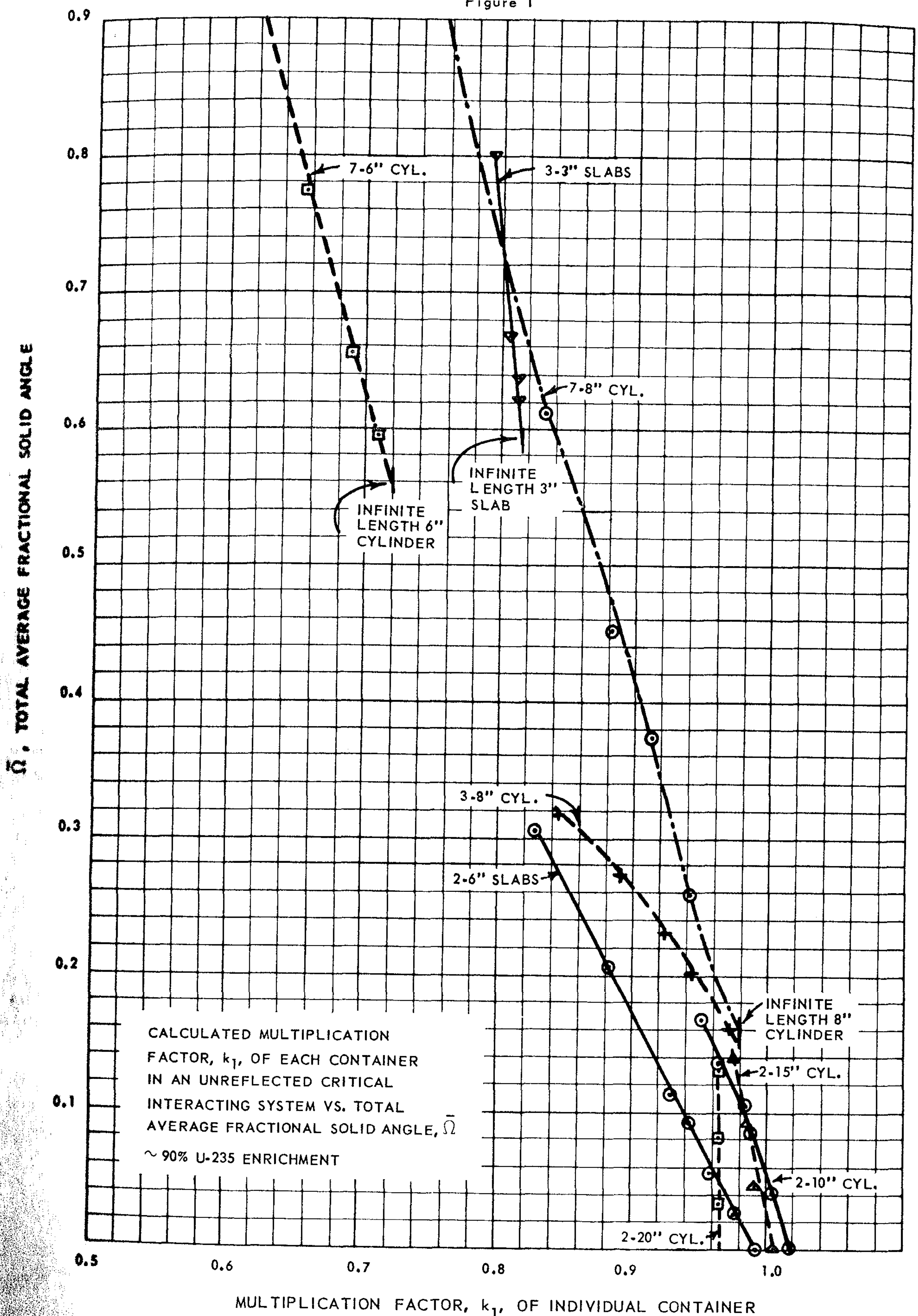
3 IDENTICAL SLABS IN LINE, 3" THICK x 47.5" WIDE

SEPARATION	HEIGHT	k_1	$\overline{\Omega}$	K (VARIABLE)	K (CONSTANT)
1	13.6	0.7930	0.400	0.9663	0.6286
3	23.1	0.8043	0.333	0.9710	0.6404
4.5	33.7	0.8105	0.318	0.9764	0.6464
5.5	42.3	0.8123	0.312	0.9773	0.6473
6	47.4	0.8126	0.310	0.9774	0.6488

3", 6", AND 3" SLABS IN LINE, 47.5" WIDE

SEPARATION	HEIGHT	k_1(3")	k_2(6")	$\overline{\Omega}$	K (VARIABLE)	K (CONSTANT)
10	17.4	0.4793	0.9195	0.160	-	0.932
20	24.6	0.4924	0.9531	0.110	-	0.959
32	32.1	0.4983	0.9685	0.070	-	0.971

Figure 1


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.5
0.6
0.7
0.8
0.9
1.0
$\bar{\Omega}$, TOTAL AVERAGE FRACTIONAL SOLID ANGLE
MULTIPLICATION FACTOR, k_1, OF INDIVIDUAL CONTAINER
7-6" CYL.
3-3" SLABS
7-8" CYL.
INFINITE LENGTH 3" SLAB
INFINITE LENGTH 6" CYLINDER
3-8" CYL.
2-6" SLABS
INFINITE LENGTH 8" CYLINDER
2-15" CYL.
2-10" CYL.
2-20" CYL.
CALCULATED MULTIPLICATION FACTOR, k_1, OF EACH CONTAINER IN AN UNREFLECTED CRITICAL INTERACTING SYSTEM VS. TOTAL AVERAGE FRACTIONAL SOLID ANGLE, $\bar{\Omega}$
~ 90% U-235 ENRICHMENT

using the more exact relations given in tables 1 and 2; thus, it is obvious that the lowest value of k_1 for any given solid angle is obtained for a system of only 2 containers. It may be noted that for fractional solid angles of 8%, the calculated multiplication factor is greater than 0.94 for all of the situations shown, and for a fractional solid angle of 24% it is greater than 0.86.

In addition to the interaction experiments with unreflected containers, a number of experiments have also been done for completely water-reflected systems.[14,15,16] The values of the multiplication factor for each container in table 7 are those for an unreflected container having the same dimensions as the critical reflected container concerned; thus, all values are less than unity and the difference between k_1 and unity for a single container may be interpreted as an indication of the reactivity that must be added by reflection and interaction for criticality. The effective K for the reflected system was not calculated, since the methods used in this report are not applicable to such calculations. It may be noted that the effect of interaction between reflected containers drops off rapidly for separations beyond a few inches, and the interaction appears to be quite small for separations greater than 8 inches. This effect is to be expected, since the attenuation of neutrons by water is sufficient to reduce interaction effects rapidly with an increased thickness of water between the components.

It was felt that the experimentally measured neutron multiplication of an individual container could offer a possible means of evaluating the nuclear safety of various arrays of these containers, at least in the sense that it might indicate some usable limiting value of the multiplication factor that would result for a given experimental neutron multiplication. Thus, calculations of the multiplication factor of single containers were made and compared with the experimentally measured neutron multiplication, M.[17] These experiments were made with 2 or more detectors placed around a container with a neutron source located in its center. The results for calculated values of k of 0.90 and 0.95 are given in table 8. It may be noted that there is considerable variation in the neutron multiplication for a given value of the calculated k, but in no case where the value of k was as much as 0.90 was an experimental multiplication less than 2 found. In general, the larger measured multiplications are those for larger diameter cylinders. As has been previously noted for other calculations, there appears to be no significant effect due to variation in the H/U-235 ratio of the solutions. However, experiments for the measurement of the neutron multiplication are subject to a number of uncertainties that give these measurements a qualitative value only in estimating the critical parameters.

TABLE 7

CALCULATED VALUES OF THE MULTIPLICATION FACTOR OF BARE CONTAINERS THAT HAVE THE SAME DIMENSIONS AS CRITICAL REFLECTED CONTAINERS IN INTERACTING ARRAYS

UO_2F_2 SOLUTIONS, ~90% U-235 ASSAY

REFLECTED PAIRS - CYLINDERS

INCHES DIAMETER	H/U-235	CM. HEIGHT	CM. SEPARATION	MULTIPLICATION FACTOR OF EACH BARE CONTAINER k_1	AVERAGE FRACTIONAL SOLID ANGLE $\overline{\Omega}$
5	52.9	36.4	0.2	0.5286	0.179
↓	↓	56.2	3.3	0.5491	0.125
		76.0	4.2	0.5569	0.117
6	52.9	21.0	0.0	0.5891	0.175
↓	↓	24.0	2.9	0.6110	0.122
		40.7	8.7	0.6706	0.082
		52.0	13.0	0.6866	0.067
		70.9	∞	0.6991	0.000
8	52.9	13.0	0.0	0.6244	0.159
↓	↓	13.7	1.8	0.6432	0.119
		15.2	4.2	0.6792	0.0895
		16.6	6.8	0.7092	0.071
		17.6	8.6	0.7276	0.063
		18.95	14.7	0.7502	0.040
		19.5	∞	0.7587	0.000
10	52.9	10.3	0.2	0.6180	0.148
↓	↓	11.2	3.0	0.6573	0.096
		12.2	7.0	0.6970	0.081
		12.7	10.5	0.7156	0.047
		12.9	13.0	0.7228	0.038
		13.0	20.0	0.7263	0.026
		13.4	∞	0.7398	0.000
15	52.9	7.3	0.0	0.5371	0.121
↓	↓	7.6	2.9	0.5586	0.079
		7.65	5.8	0.5621	0.052
		7.7	11.6	0.5657	0.029
		7.75	∞	0.580	0.000

(TABLE 7 CONTINUED)

REFLECTED PAIRS - SLABS (47.5 IN. WIDE)
NO TOP REFLECTOR

INCHES THICKNESS	H/U-235	INCHES HEIGHT	INCHES SEPARATION	MULTIPLICATION FACTOR OF EACH BARE CONTAINER k_1	AVERAGE FRACTIONAL SOLID ANGLE Ω
3	337	9.05	0	0.4198	0.500
↓	↓	9.67	1	0.4333	0.430
↓	↓	16.58	3	0.4767	0.310
↓	↓	25.63	4	0.4935	0.305

REFLECTED MULTIPLE SYSTEMS - CYLINDERS
NO TOP REFLECTOR

NUMBER OF CYLINDERS	GEOMETRY	INCHES DIAMETER	H/U-235	INCHES HEIGHT	INCHES SEPARATION	MULTIPLICATION FACTOR OF EACH BARE CONTAINER k_1	AVERAGE FRACTIONAL SOLID ANGLE BETWEEN ADJACENT CONTAINERS Ω
3	Triangle	6	44.3	29.0	∞	0.706	0.000
↓	↓	↓	↓	12.2	3	0.6603	0.082
↓	↓	↓	↓	7.0	0.15	0.5709	0.149
7	Hexagon	6	44.3	29.0	∞	0.706	0.000
↓	↓	↓	↓	29.0	15	0.706	0.026
↓	↓	↓	↓	25.8	9	0.700	0.043
↓	↓	↓	↓	18.8	6	0.687	0.055
↓	↓	↓	↓	12.0	4	0.648	0.067
↓	↓	↓	↓	5.2	0	0.493	0.160
3	Triangle	8	44.3	8.9	∞	0.8320	0.000
↓	↓	↓	↓	8.9	6	0.8320	0.043
↓	↓	↓	↓	7.8	3	0.7684	0.074
↓	↓	↓	↓	6.9	2	0.7343	0.087
↓	↓	↓	↓	6.1	1	0.6913	0.110
↓	↓	↓	↓	5.7	0.15	0.6676	0.153
7	Hexagon	8	44.3	8.9	∞	0.8320	0.000
↓	↓	↓	↓	8.9	9	0.8320	0.028
↓	↓	↓	↓	8.6	6	0.7948	0.043
↓	↓	↓	↓	6.9	3	0.7343	0.060
↓	↓	↓	↓	5.1	1	0.6278	0.110
↓	↓	↓	↓	4.7	0.15	0.5973	0.150

(TABLE 7 CONTINUED)

REFLECTED MULTIPLE SYSTEMS - SLABS (47.5 IN. WIDE)
NO TOP REFLECTOR

NUMBER OF SLABS	GEOMETRY	INCHES THICKNESS	H/U-235	INCHES HEIGHT	INCHES SEPARATION	MULTIPLICATION FACTOR OF EACH BARE CONTAINER k_1	AVERAGE FRACTIONAL SOLID ANGLE BETWEEN ADJACENT CONTAINERS Ω
3	Line	3	337	6.81	0.0	0.3858	0.500
↓	↓	↓	↓	7.53	1.0	0.4010	0.410
				12.96	3.0	0.4607	0.285
				24.0	4.5	0.4917	0.291
				43.97	5.5	0.5026	0.315

TABLE 8

EXPERIMENTALLY MEASURED NEUTRON MULTIPLICATION, M
COMPARED WITH CALCULATED k

H/U-235	INCHES DIAMETER	CM. HEIGHT	CALCULATED k	M	
44	8	37.0	0.90	2.00	Minimums
↓	8	60.0	0.95	2.27	Minimums
	9	23.5	0.90	2.94	
	9	29.0	0.95	4.35	
	10	19.5	0.90	4.00	
	10	22.3	0.95	5.56	
86	8	44.5	0.90	2.38	
↓	8	-	0.95	-	
169	10	24.0	0.90	2.13	
↓	10	28.6	0.95	3.57	
175	9	33.6	0.90	3.57	
↓	9	46.6	0.95	5.00	
	10	24.1	0.90	3.85	
	10	29.2	0.95	8.33	
	12	17.8	0.90	4.55	
	12	20.8	0.95	10.00	
329	10	33.8	0.90	5.26	
↓	10	46.3	0.95	8.33	
	15	17.8	0.90	6.25	Maximums
	15	19.6	0.95	14.29	Maximums
404	15	20.8	0.90	3.85	
↓	15	23.5	0.95	5.00	
AVERAGES			0.90	3.71	
			0.95	7.00	

CONCLUSIONS

The calculating methods used in this report yield usable results for relating the theoretical multiplication factor, k_1, of a container to its critical solid angle of interaction with another similar container over a fairly wide range of conditions, and the fractional solid angle between members of an interacting array appears to be a consistent and reliable criterion of the interaction probability. In addition, the following general conclusions may be drawn:

1. Variation in the H/U-235 atomic ratio from 44 to 337 shows no marked effect on the accuracy of the results.

2. Although use of a solid angle-dependent value of the extrapolation length gives excellent agreement with experiment for all configurations reviewed, a constant value of this length may be used with very little error in all cases except for slabs, and the differences become significantly appreciable in these cases only for thin slabs which are individually far from critical.

3. For cylinders, the maximum error, as compared with experimental results was found to be about 3%, and for slabs, it was about 4%; for hexagonal arrays of cylinders, an error of as much as 7% was found. In all cases, the errors decreased as the separations increased.

4. For pairs of similar containers with a fractional solid angle of 8% between them, the multiplication factor, k_1, of an individual container is 0.96 or greater, and for a solid angle of 24%, k_1 is 0.86 or greater for all of the cases studied.

5. The measured neutron multiplication, M, corresponding to a given calculated value of the multiplication factor varies rather widely, being larger for larger diameter cylinders, but in none of the cases checked was M less than 2.00 for a calculated value of k_1 of 0.90.

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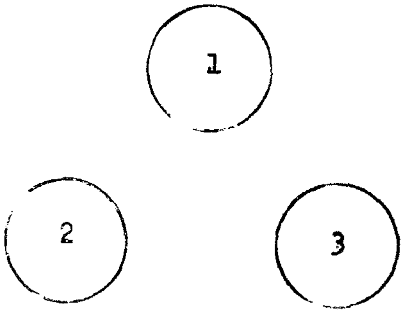
APPENDIX

The derivations of the expressions for a number of identical parallel cylinders in a straight line are given by Pond.[10] Following the same general procedure, expressions for other arrangements of containers can be obtained. It should be recalled that k_1 is the multiplication factor of a single container of the group and V is the probability that a fast neutron escaping from one container will strike another container.

IDENTICAL CONTAINERS

In a system of identical, equally-spaced containers, it is readily seen that the k_1 of each container is the same and the V between any 2 adjacent containers is the same.

Equilateral Triangular Array



First, the case of 3 identical cylinders in an equilateral triangular array will be considered. Let n_1, n_2, and n_3 be the total number of fast neutrons in the 3 respective cylinders. For the given arrangement, $n_1 = n_2 = n_3$ because of symmetry. The effective multiplication factor, K, of this system can be expressed as follows:

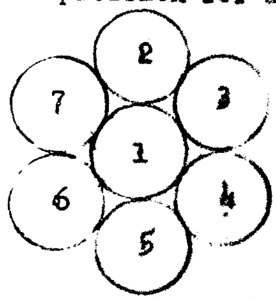
$$n_1 \, k_1 + 2 \, n_2 \, k_1 \, V = K \, n_1 \qquad (1)$$

This expression can be readily simplified, to give

$$K = k_1 \, (1 + 2V). \qquad (2)$$

It may be noted that this equation has the same form as the expression for an infinitely long line of equally-spaced parallel cylinders.

Hexagonal Array



In a close-packed hexagonal array of cylinders, the symmetry is such that $n_2 = n_3 = n_4 = n_5 = n_6 = n_7$.

Thus, the 2 expressions for K of the system become:

$$n_1 \, k_1 + 6 \, n_2 \, k_1 \, V = K \, n_1, \qquad (3)$$

$$n_2 \, k_1 + n_1 \, k_1 \, V + 2 \, n_2 \, k_1 \, V = K \, n_2. \qquad (4)$$

Those equations can be arranged to give:

$$n_1 \, (k_1 - K) + n_2 \, (6 \, k_1 \, V) = 0 \qquad (5)$$

$$n_1 \, (k_1 \, V) + n_2 \, (k_1 + 2 \, k_1 \, V - K) = 0. \qquad (6)$$

These equations obviously have the trivial solution of $n_1 = n_2 = 0$, which is not reasonable for any multiplying system. The non-trivial solution, in terms of K, is

$$K^2 + K\,(1 + V)(-2k_1) + (k_1)^2\,(1 + 2V - 6V^2). \tag{7}$$

Equation (7) is quadratic in K, and will yield 2 solutions. Since a value of k_1 greater than unity means supercritical individual units, the solution for k_1 less than unity is used, as follows:

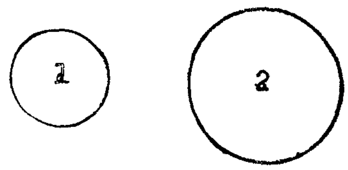
$$K = k_1 \left[1 + (1 + \sqrt{7})\, V \right]. \tag{8}$$

DISSIMILAR CONTAINERS

In a system of containers with 2 unlike containers present, the value of the multiplication factor, k_1, will be different for each set of unlike containers, and the neutron exchange probability, V, will vary similarly. For this type of system, k_1 and k_2 will represent the multiplication factors of containers type one and type two, respectively. Similarly, V_1 is defined as the probability that a fast neutron leaking from a type one container will enter a type two container, and conversely, V_2 is defined as the probability that a fast neutron leaks from a type two to a type one container. It should be noted that the containers may be unlike in geometry, content, or both.

Two Dissimilar Containers

For two dissimilar containers, two expressions for K can be obtained, as follows:



$$n_1\, k_1 + n_2\, k_2\, V_2 = K\, n_1, \tag{9}$$

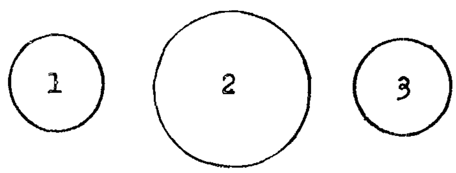
$$n_2\, k_2 + n_1\, k_1\, V_1 = K\, n_2. \tag{10}$$

Solving these equations, as before, gives the following results for K:

$$K = \frac{k_1 + k_2 + \sqrt{(k_1 - k_2)^2 + 4\, k_1\, k_2\, V_1\, V_2}}{2} \tag{11}$$

Three Containers, Two Alike

A line of equally spaced containers, with the end containers identical and the center container different from the ends will be considered next. From the symmetry of this array, it is easily seen that $n_1 = n_3$, $V_1 = V_3$, and $k_1 = k_3$. The use of these identities leads to the following



expressions:

$$n_1 k_1 + n_2 k_2 V_2 = K n_1; \qquad (12)$$

$$n_2 k_2 + 2 n_1 k_1 V_1 = K n_2. \qquad (13)$$

Equations (12) and (13) can be solved by the procedure outlined previously, giving the following results for K:

$$K = \frac{k_1 + k_2 + \sqrt{(k_1 - k_2)^2 + 8 k_1 k_2 V_1 V_2}}{2} . \qquad (14)$$

AVERAGE FRACTIONAL SOLID ANGLES

Figures 2 and 3 are charts from which the average fractional solid angle between identical cylinders and identical slabs may be obtained. Calculations for these charts were done by Pond[11] for the cylinders and Burton[12] for the slabs.

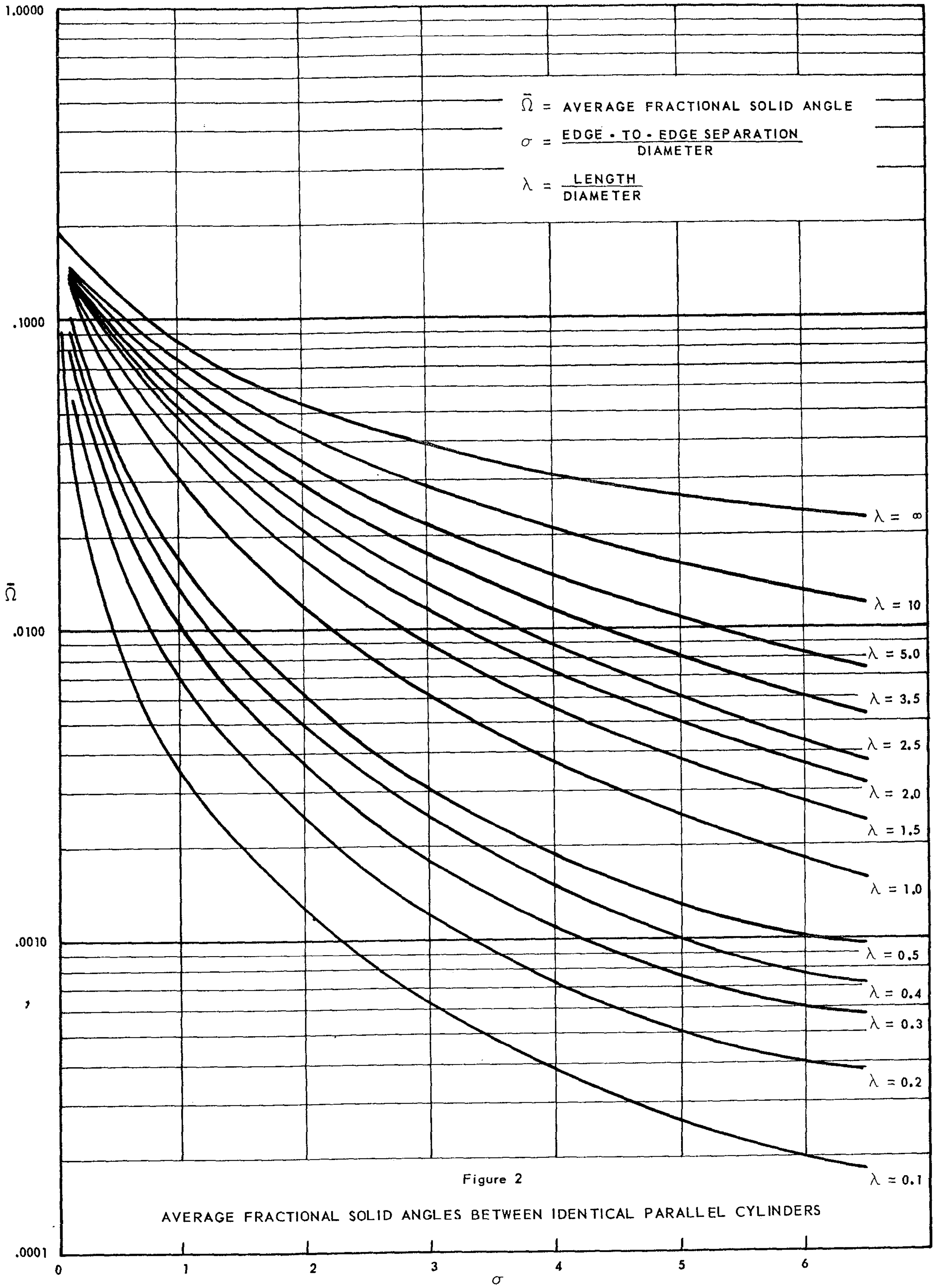


Figure 2

AVERAGE FRACTIONAL SOLID ANGLES BETWEEN IDENTICAL PARALLEL CYLINDERS

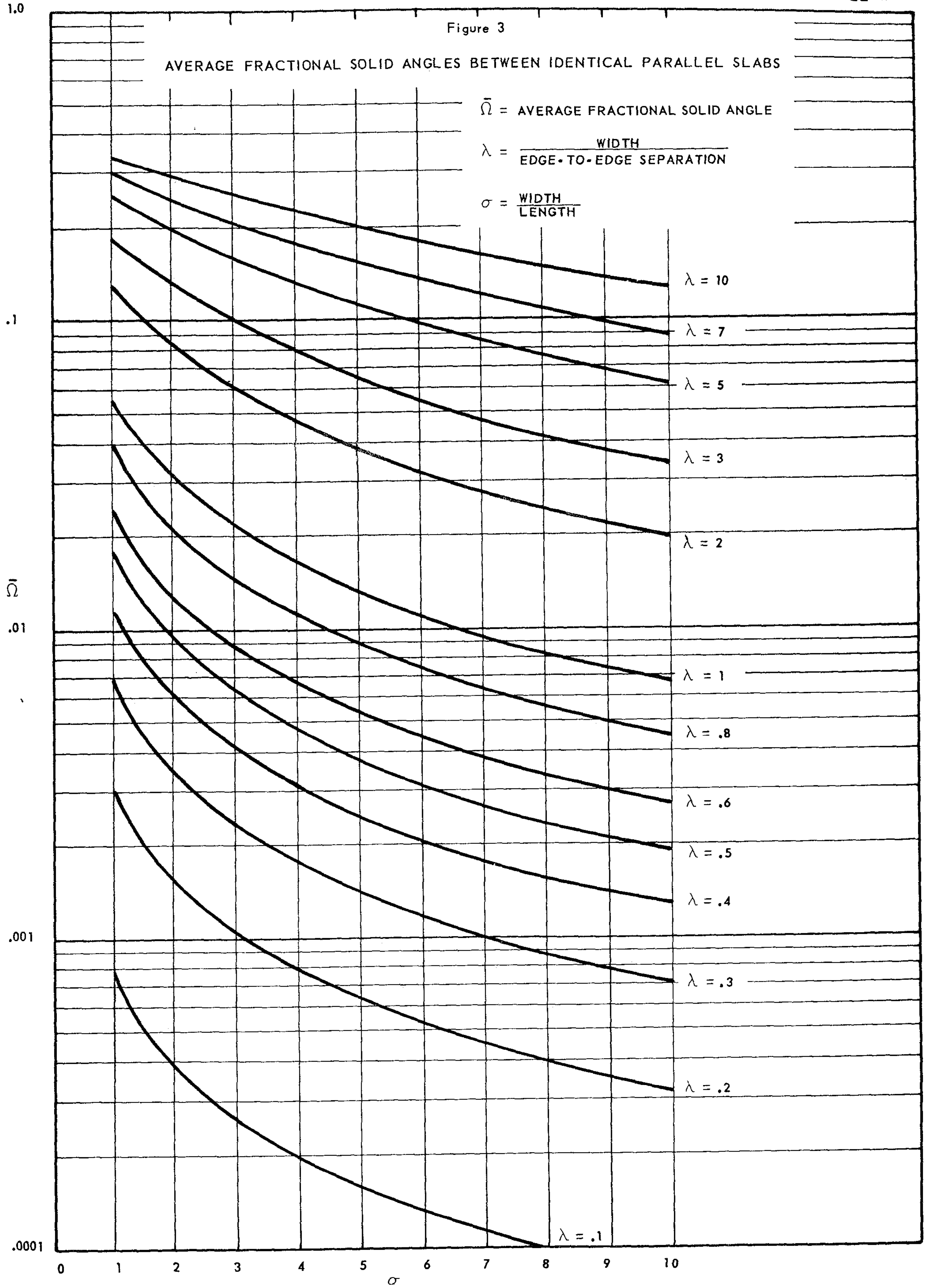


Figure 3
AVERAGE FRACTIONAL SOLID ANGLES BETWEEN IDENTICAL PARALLEL SLABS
$\bar{\Omega}$ = AVERAGE FRACTIONAL SOLID ANGLE
$\lambda = \frac{\text{WIDTH}}{\text{EDGE-TO-EDGE SEPARATION}}$
$\sigma = \frac{\text{WIDTH}}{\text{LENGTH}}$
λ = 10
λ = 7
λ = 5
λ = 3
λ = 2
λ = 1
λ = .8
λ = .6
λ = .5
λ = .4
λ = .3
λ = .2
λ = .1
1.0
.1
.01
.001
.0001
$\bar{\Omega}$
0
1
2
3
4
5
6
7
8
9
10
σ

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